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Is the value of humanity increasing? A critical-level enquiry

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Abstract

We assess whether the *value of humanity* (or global social welfare) has improved in the last decades despite (or because of) the substantial increase in global population sizes. We use for this purpose a relatively unknown but simple and attractive social evaluation approach based on *critical-level generalized utilitarianism* (CLGU). CLGU posits that social welfare increases with population size if and only the new lives come with a utility level higher than that of a *critical level.* Despite its attractiveness, CLGUposes a number of practical difficulties that may explain why the literature has left it largely unexplored. The most important of these difficulties deal with the choice of an individual welfare aggregation function and with the value of the critical level.

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… / … We address these difficulties by developing new procedures for making partial social orderings over classes of CLGU evaluation functions. These orderings are designed to be robust to choices of individual welfare aggregation functions (within certain classes of such functions) and to ranges of the critical level. The headline result is that we can robustly conclude that world welfare has increased between 1990 and 2005 if we judge that lives with *per capita* yearly consumption of more than $1,288 *necessarily increase* the value of humanity; the same conclusion applies to Sub-Saharan Africa if and only if we are willing to make that same judgement for lives with *any* level of *per capita* yearly consumption above $230. Otherwise, some of the admissible Paretian CLGU functions will judge the last two decades' increase in global population size to have lowered the value of humanity.

Keywords: Global welfare; Critical-level generalized utilitarianism; Social evaluation, Welfare dominance; Critical level; Population growth

JEL Codes: D31; D63; I32; O15; Q56

1 Introduction

It took roughly 250,000 years for humanity to reach 250 million individuals — *viz*, at around 1 AD. It took another 1800 years for the global population to reach 1 billion. Between 1800 and 1960, that level grew to 3 billion. Estimated global population size will reach 7 billion at the turn of 2011-2012 (see United Nations 2011); current 2020 projections of the size of humanity stand at about 7.6 billion.

These increases in global population sizes have been a frequent source of concerns. The most prominent of them reflects the fear that population size may be detrimental to the environment and to living standards, a concern frequently expressed for instance by personalities as diverse as Al Gore — "No goal is more crucial to healing the global environment than stabilizing human population." (Gore 1992, p. 380) — and the Dalai Lama:

> "One of the greatest challenges today is the population explosion. Unless we are able to tackle this issue effectively we will be confronted with the problem of the natural resources being inadequate for all the human beings on this earth." (Dalai Lama 2000)

Such views feed mainly on the Malthusian preoccupation that large populations can put unsustainable pressure on limited natural resources and fixed assets such as land (see for instance Ehrlich and Ehrlich 1990, Cohen 1995, Dasgupta 2010 and Eastin, Grundmann, and Prakash 2011 for instance), although it has been conversely argued that population growth can also serve as a vehicle for economic development by stimulating human ingenuity and technological progress and improving the effectiveness of the provision of public goods (see for instance Klasen and Nestmann 2006 for numerous references to the literature and Nerlove, Razin, and Sadka 1986 for a model of the overall trade-off).

While it is certainly useful to analyze population growth and living standards from a *causal* perspective (as has often been done: see Cassen 1994 and Birdsall, Kelley, and Sinding 2003 for a review), it would seem equally important to assess the joint *normative* effect of demographic growth and living standards on the *value* of societies. It is indeed such a normative assessment that should presumably guide demographic and development policies. A normative assessment of the joint impact of population sizes and living standards on societies raises fundamental ethical issues, however, and those issues have been somewhat neglected in the recent debates on global trends in welfare and poverty. It is our main objective in this paper to address them in an original and (we believe) persuasive normative setting.

There are three major existing normative measures of the impact of population growth and living standards on social welfare. All of them incorporate an implicit trade-off between the "quantity" and the "quality" of lives (the quality of lives being measured by their well-being, their utility, or their living standard — as in the case of our empirical application below). Two of them derive from the standard social evaluation approaches consisting of total and average utilitarianism.

Total (or *classical*) utilitarianism is the oldest form of utilitarianism. It values society's welfare by the sum of utilities and thus sets the government's objective function to the "greatest happiness of the greatest number" (in the words of the total utilitarians, see Burns and Hart 2000, p. 393). Sidgwick (one of the fathers of utilitarianism) clearly states the associated trade-off between the quantity and the quality of lives:

> "So that, strictly conceived, the point up to which, on utilitarian principles, population ought to be encouraged to increase, is not that at which average happiness is the greatest possible (...) but that at which the product formed by multiplying the *number* of persons living into the amount of *average* happiness reaches its maximum". (Sidgwick 1966, pp. 415-416; emphasis is ours)

The implications of total utilitarianism are clear: the quantity of lives can compensate for the quality of them. It has been convincingly argued, however, that this can lead to a "repugnant" trade-off, a term used in Parfit (1984)'s famous "repugnant conclusion". Parfit considers as a repugnant implication of total utilitarianism the fact that any sufficiently large population, even with a very low level of average utility, could be deemed preferable to any other smaller population with a relatively high level of average utility:

> "For any possible population of at least ten billion people, all with a very high quality of life, there must be some much larger imaginable population whose existence, if other things are equal, would be better, even though its members have lives that are barely worth living." (Parfit 1984, p.388).

A revised version of utilitarianism that avoids the repugnant conclusion is average utilitarianism. Edgeworth (1925) attributes it to John Stuart Mill, who indeed chose it to justify limits to population sizes,[1] although Say, Sismondi and Wicksell were probably earlier users of an average principle in the discussion of an optimal population size (see Guillaumont 1964, Sumner 1978 and Blackorby and Donaldson 1984). Average utilitarianism, however, also has "repugnant" implications. A policy designed on average utilitarianism will seek to maximize average utility, regardless of how small population size may result. A population with only a few individuals may be preferred to an arbitrarily larger one with almost the same average well-being.[2] The death of a

[1] "It is no accident that the average theory was devised strictly to handle questions of population" (Sumner 1978, p. 99).

[2] "An alternative with a population of any size in which each person is equally well off is ranked as worse than an alternative in which a single person experiences a trivially higher utility level" (Blackorby, Bossert, and Donaldson 2005, p. 143). Also consider the following recently estimated impact of AIDS on the distribution of income in Côte d'Ivoire: "We find that although the size of the economy in terms of total household income is reduced by about

person with below-average utility (as in the case of a relatively poor person) will increase the value of a society (see Cowen 1989, Broome 1992a and Kanbur and Mukherjee 2007). The replication of a population with no effect on average utility would also be a matter of social indifference.

Average utilitarianism can also lead to important (and sometimes disturbing) population policy implications. Take for instance China's 1979 implementation of the one-child policy, which has probably contributed to the remarkable increase in China's average living standards over the last three decades (see Hasan 2010 and Bussolo, De Hoyos, Medvedev, and van der Mensbrugghe 2010 for references and some evidence). The one-child policy has, however, caused an important reduction in population growth and contributed to levels of (sometimes forced) abortions of the order of 10 million per year.[3] Such effects on population size would, however, not be accounted for (at least directly) by average utilitarianism.[4]

The third major normative measure of the impact of population growth and living standards focusses exclusively on the quantity of lives — in complete contrast to average utilitarianism's exclusive focus on the average quality of lives. It is probably the oldest of the three social evaluation measures. It also has deep roots in political and moral discourse. One example is the historically frequent assessment of the strengths of nations on the basis of population sizes, as is clear from Jean Bodin's (a 16th-Century French philosopher) political thought:[5]

> "But one should never be afraid of having too many subjects or too many citizens, for the strength of the commonwealth consists in men." (*Six books of the commonwealth*, Book V, chapter II, translation in Tooley 1955)

Another example is the religiously-encouraged procreation to replenish the world, partly inspired by the Genesis, the first book of the Bible:

> "The value of the replication of God's image is the reason given for Man's creation. (...) Their number should be as large as possible so as to permeate the world with

6% after 15 years, average household income per capita, household income inequality and poverty remain almost unchanged" (Cogneau and Grimm 2008, p. 688). According to average utilitarianism, AIDS would then have had no effect on Côte d'Ivoire's social welfare.

[3] See http://www.tldm.org/News13/13MillionAbortionsPerYearInChina.htm. One outcome of this trade-off between the quantity and the quality of lives is that abortions of female fetuses are more common in China and elsewhere, largely explained by the perceived higher (private) cost/benefit ratio of raising a daughter — see Sen (2001) for a discussion. Klasen and Wink (2003) estimate for instance the number of "missing women" in the 1990s at nearly 41 million for China and 31 million for India.

[4] Policies aimed at producing the "greatest happiness" can be deemed ethically unacceptable for reasons of *procedural* justice (justice of *means*), as opposed to reasons of *consequential* justice (justice of *outcomes*, such as the achievement of greater average or total utility) — see for instance Rawls (1971). The judgements of procedural justice and consequential justice may also overlap, as in the case of forced contraception, infanticides, abortion and forced migration. We focus in this paper solely on assessments of consequential justice.

[5] A more recent example is from Michel Rocard, then French prime minister:

> "Most of Western European states are in the process of committing suicide, suicide by demography, without even noticing it." Rocard (1989)

God's image. (...) It is conscious procreation rather than simple biological propagation which is the object of the first (moral) duty". (Heyd 1992, p. 2)

A related supporting rationale for this is that being alive has a moral and social value that transcends the utilitarian value of the life being lived. Manifestations of this rationale can take several forms. Several of them were vividly present in a lively online debate of *The Economist*, entitled "Too many people? This house believes that the world would be better off with fewer people".[6] One contributor (Arturo Barrios) answered as follows:

> "So unlike the Economist reader elites who, having solved most of their existential problems, are constantly seeking problems to temper their well-being, most people in the third world are very happy to exist indeed, thank you very much. Being poor does not make one as unhappy as the Western elites imagine".[7]

Choosing one of these three measures of social evaluation is certainly difficult, and can certainly not be expected to generate consensus. We can, however, address the underlying fundamental trade-offs between the quantity and the quality of lives that these measures capture through the *critical-level generalized utilitarianism* (CLGU) framework proposed by Blackorby and Donaldson (1984). This has the advantage of presenting a framework that is both an alternative and a generalization of the above social evaluation approaches (those being total and average utilitarianism as well as population head-counting).

CLGU social evaluation functions are defined as the aggregation of the differences between individual welfare (or utility) and the welfare of someone with an income level equal to a *critical level*. The critical level is the minimum income needed for someone to add value to humanity. CLGU can thus serve to assess the value to humanity of adding a new life to an existing population. CLGU functions can also be expressed as the product of population size and the difference between average welfare and welfare at the critical level. CLGU thus provides an explicit framework for trading off average welfare and population size. Choosing a relatively high value of the critical level results in optimally smaller populations; choosing a lower value results in optimally larger populations.

Despite its attractiveness, CLGU poses important practical difficulties, which have impended its application and explained in large part its relative lack of popularity. The most salient of these are the choice of an individual welfare aggregation function and assigning of a value to the critical level. It is indeed difficult to agree on one precise form of a CLGU function. It is also difficult to agree on the appropriate value of the critical level. The level has to be high enough to avoid the

[6] The debate took place between August 21st 2009 and September 2nd 2009; the contributions can be found at http://www.economist.com/debate/debates/archive/page:10.

[7] 80% of the online voters in *The Economist*'s debate disagreed with Barrios' view, leading to the following disenchanted comment by another contributor: "For those that think we are too many, they can start by taking their own life. Nobody is in this world against their will and they should not be forced to stay".

repugnant conclusion; the level also has to be low enough not to rule out additions of lives that are manifestly worth living. In a world of heterogenous preferences and opinions, it is naturally difficult to envisage a wide consensus on something as fundamentally un-consensual as the *value of living*.

Our first main objective in this paper is hence to address these difficulties by deriving procedures for making partial social orderings over classes of CLGU evaluation functions. These orderings are designed to be robust to choices of individual welfare functions (within certain classes of such functions) and to ranges of the critical level.

In addition to being useful in themselves, these orderings resonate very well with an important aspect of recent debates on the evolution of global poverty. Consider for instance the following extract from Angus Deaton's 2010 presidential address to the American Economic Association (using a poverty line of $1.25 per person per day in 2005 international dollars):

> "[The figures] show the well-known reduction in the global headcount ratio, from 51.9 percent of the world's population in 1981 to 25.2 percent in 2005. In spite of growth in the world's population, the number of people in this kind of poverty has fallen by more than half a billion in the last quarter century. Much of this success comes from China, in the East Asia and Pacific region. The headcount ratio in sub-Saharan Africa has fallen only slowly, and there are 176 million more Africans in poverty in 2005 than in 1981. South Asia, dominated by India, is part success and part failure, and the Bank — and the government of India — estimate that, in spite of a falling headcount ratio, there has been a small increase in the numbers of Indians in poverty since 1981, in spite of India's relatively rapid growth in per capita GDP in recent years, and its relatively slow rate of population growth." (Deaton 2010, p.8)

Opposite movements of absolute and relative numbers of the poor emerge often in poverty comparisons. And when the numbers move in the same direction, they often do so at very different rates. This leads to a natural question: "If the absolute number of poor people goes up, but the fraction of people in poverty comes down, has poverty gone up or gone down?" (Kanbur 2005, p.228 and Mukherjee 2008, p. 97; see also Chakravarty, Kanbur, and Mukherjee 2006 and Pogge 2005.) Whether we should consider *absolute* (total population) indices or *proportional* (relative to total population size) indices to measure poverty would therefore seem important. Our second main objective in this paper is to show how this important question can be nicely associated to the resolution of the equally important inquiry into the value of societies.

Our third main objective is to use CLGU to assess empirically whether there has been an improvement in social welfare during the last decades. To do this, we compare the value of humanity between 1990 and 2005 from a national, regional and global perspective. We consider 173 countries (accounting for 95 percent of the global population in 2005), of which 114 are developing countries and 59 are high-income countries.

The most general result is that humanity in 2005 can be robustly considered to be better than in 1990 if we are willing to judge that lives with *per capita* yearly consumption of any level greater than \$1,288 *necessarily increase* the value of humanity. For some countries and groups of countries, particularly in Europe, Central Asia and in Sub-Saharan Africa, 1990 can conversely be deemed *better* than 2005 if we judge that lives with *per capita* yearly consumption lower than \$560 *necessarily decrease* the value of humanity — that threshold falls to \$300 for higher orders of CLGU dominance. Further regional and national comparisons illustrate how the trade-off between the quantity and the quality of lives is starker in some environments than in others. The results also demonstrate how a critical level framework assesses global social welfare differently from the traditional average and total utilitarian approaches.

The social evaluation questions addressed in the paper are at the heart of the optimal population size problem.[8] They also have considerable policy relevance. For instance, the process of demographic transition (through a reduction of both fertility and mortality) in which a large part of humanity has recently engaged is often rationalized as one that maximizes *per capita* welfare under resource constraints. It is unlikely for developed countries that this process also maximizes social welfare in a CLGU perspective. For developed countries, a CLGU perspective can most likely provide a rationale for promoting policies that encourage fertility, such as the provision of relatively generous child benefits for families with more children. Whether the current demographic transition is consistent with CLGU maximization in developing countries depends much on the value that is set for the critical level. A social planner would favor a population increase only if the additional lives enjoyed a level of income at least equal to that level. Indeed, if additional lives are below this level, policies could favor enhanced family planning and birth control. The paper's discussion and estimates of the ranges of *robust* critical levels for which the value of humanity has been changing in one direction or in another can be instructive in that planning exercise.

The rest of the paper runs as follows. Section 2 sets the basic CLGU analytical framework. Section 3 outlines the estimation procedures. Section 4 describes the data and presents the findings. Section 5 concludes briefly. The Appendix presents additional methodological precisions and results.

[8]The notion of an optimal size can be seen as going back to the time of Plato, who quantified the optimal size of a state to be 5,040 individuals. See also Meade (1955), Mirrlees (1967), Dasgupta (1969), Lane (1975), Samuelson (1975), and Gigliotti (1983) for influential contributions.

2 Social evaluations when population sizes differ

Consider two populations of different sizes. The smaller population of size M has a vector $\mathbf{u}$ of individual *incomes* (as a shorthand for well-being, living standards, or consumption) and the larger population with vector $\mathbf{v}$ is of size N, with $M < N$. Let $\mathbf{u} := (u_1, u_{2,...}, u_M)$, where u_i refers to the income of individual i, and $\mathbf{v} := (v_1, v_{2,...}, v_N)$, where v_j is the income of individual j. To assess the value of the two populations, we let the social evaluation functions of $\mathbf{u}$ and $\mathbf{v}$ take the form

$$W(\mathbf{u}; \alpha) = \sum_{i=1}^{M} (g(u_i) - g(\alpha)) \tag{1}$$

and

$$W(\mathbf{v}; \alpha) = \sum_{j=1}^{N} (g(v_j) - g(\alpha)), \tag{2}$$

where g is some increasing monotonic transformation of incomes and α is the critical level. The smaller population is socially better than the larger one given this if and only if $W(\mathbf{u}; \alpha) \geq W(\mathbf{v}; \alpha)$. It is clear from the above that the social value of a population remains unchanged if a new individual with income equal to α is added.

Blackorby and Donaldson (1984) show that (1) can be derived from a set of reasonable axioms for making social evaluations. Their first result is that continuous Paretian fixed-population social evaluation rankings can be based on functions of population size and a socially representative income. Under the additional axiom that the replication of a population, income by income, should not change that representative income — which is equivalent to imposing social separability of individual income utility valuations —, the representative income (for a fixed population) can be expressed as $g^{-1} \sum_{i=1}^{M} g(u_i)$. Adding to this the *critical-level population principle* that adding an individual with income α should not change the value of a society leads to the CLGU functions in (1) and (2).

By aggregating the differences between transformations of individual incomes and of a critical level, CLGU avoids the above-mentioned problems of both average and total utilitarianism. The addition of a new person will be socially profitable if that person's income is higher than the critical level, although that income may not necessarily be higher than *average income*. The "repugnant conclusion" is also avoided since it is socially undesirable to add individuals with incomes lower than the critical level, regardless of how many there may be of them.

The *critical level* is clearly a central feature of the CLGU social evaluation framework. It is called the "value of living" by Broome (1992b). It is described as follows in Trannoy and Weymark (2009):

> "The critical level is the level of income for which it is a matter of social indifference to add an additional person with this amount of income. For most societies, this level

will be below the observed average income of the population. It is also likely to be below what is regarded as an appropriate value for an absolute poverty line". (p.277)

Why societies should use such a level for social evaluation purposes is also suggested in John Stuart Mill's classical essay *On Liberty*:

> "The fact itself, of causing the existence of a human being, is one of the most responsible actions in the range of human life. To undertake this responsibility – to bestow a life which may be either a curse or a blessing – unless the being on whom it is to be bestowed will have at least the ordinary chances of a desirable existence, is a crime against that being." (Mill 1859 (1962), p.242)[9]

One common form of social evaluation functions is the negative of poverty evaluation functions (or poverty indices) — see for instance Atkinson (1987). This suggests that ranking the value of two populations can be made by ranking the poverty indices of these two populations. This can be done by focussing on those income values that fall below some *censoring point* or *poverty line* z. We make use of such a censoring point for two reasons. The first is that this makes it possible to integrate the special case of critical-level poverty measurement into the broader CLGU framework used in this paper. Censoring incomes at z is indeed a procedure that is often used in welfare economics to focus social evaluations on the poor — see for instance Blackorby and Donaldson (1980) for an early example. As we will see below, this enables us to interpret our social evaluation orderings as an intersection of poor-focussed CLGU rankings.

The second reason for using a censoring point is that the CLGU social evaluation rankings can then be tested in a dominance framework similar to the poverty dominance one found in the literature. To see this, suppose that z^- and z^+ are respectively the minimum and the maximum values of a range of censoring points that are considered. Consider $\mathbf{u}_\alpha := (\mathbf{u}, \alpha, ..., \alpha)$ as the vector $\mathbf{u}$ *expanded to size of population* $\mathbf{v}$ by adding $N - M$ α's to $\mathbf{u}$. For any choice of censoring point $z \in [z^-, z^+]$, define the well-known FGT (Foster, Greer, and Thorbecke 1984) poverty indices of parameter s ($s \geq 1$) for the population $\mathbf{u}$ as

$$P^s_{\mathbf{u}}(z) = M^{-1}\left(\sum_{i=1}^{M}(z-u_i)^{s-1} I\left(u_i \leq z\right)\right), \tag{3}$$

[9] Related to this is the analogous notion of a "restricted life" in Kavka (1982):

> "The vexed problem of whether average or total utility maximization is the appropriate goal remains unsolved. (...) One approach to evaluating the desirability of states of society seems especially promising, in the present context. Let us introduce the notion of a restricted life, a life that is significantly deficient in one or more of the major respects that generally make human lives valuable and worth living. (...) Now, suppose that we adopt the principle that, other things being equal, conditions of society or the world are intrinsically undesirable from a moral point of view to the extent that they involve people living restricted lives." (pp. 104-105)

where $z^- \leq z \leq z^+$ and $I(\cdot)$ is an indicator function that takes value 1 if its argument is true and 0 if not. The FGT indices for the expanded population $\mathbf{u}_\alpha$ are given by

$$P^s_{\mathbf{u}_\alpha}(z) = N^{-1}\left(\sum_{i=1}^{M}(z-u_i)^{s-1}I\left(u_i \leq z\right) + (N-M)(z-\alpha)^{s-1}I\left(\alpha \leq z\right)\right). \tag{4}$$

Note that the FGT of the expanded population,

$$P^s_{\mathbf{u}_\alpha}(z) = \frac{M}{N}P^s_{\mathbf{u}}(z) + \left(1-\frac{M}{N}\right)(z-\alpha)^{s-1}I\left(\alpha \leq z\right), \tag{5}$$

is a weighted average of the usual proportional FGT for the smaller population $\mathbf{u}$ and of the FGT for its expansion, $(z-\alpha)^{s-1}$. For $s=1$, we have:

$$NP^1_{\mathbf{u}_\alpha}(z) = MP^1_{\mathbf{u}}(z) + (N-M)\,I\left(\alpha \leq z\right), \tag{6}$$

which is the total poverty headcount in $\mathbf{u}$ plus the increase in population size, if $z \geq \alpha$.

Similarly, the FGT index for vector $\mathbf{v}$ is defined as

$$P^s_{\mathbf{v}}(z) = N^{-1}\sum_{j=1}^{N}(z-v_j)^{s-1}I\left(v_j \leq z\right) \text{ where } z^- \leq z \leq z^+. \tag{7}$$

The greater the value of $P^s_{\mathbf{v}}(z)$, the lower the social value of $\mathbf{v}$. We will se shortly that comparing $P^s_{\mathbf{u}_\alpha}(z)$ and $P^s_{\mathbf{v}}(z)$ will enable us to rank the two populations in a robust CLGU framework.

One difficulty with (2) is the form that g should take. We tackle this by considering classes of g functions. These classes are defined with respect to conditions of order s. Consider $\mathcal{C}^s$ as the class of functions $\mathbb{R} \longrightarrow \mathbb{R}$ that are s-times piecewise differentiable and let $\mathcal{F}^s_{z^-,z^+}$ be defined as

$$\mathcal{F}^s_{z^-,z^+} := \left\{ g^z \in \mathcal{C}^s \;\middle|\; \begin{array}{c} z^- \leq z \leq z^+, \\ g^z(x) = g^z(z) \text{ for all } x > z, \\ g^z(x) = g^z(z^-) \text{ for all } x < z^-, \\ \text{and where } (-1)^k \frac{d^k g^z(x)}{dx^k} \leq 0 \; \forall z^- < x < z^+ \text{ and } \forall k = 1, ..., s. \end{array} \right\} \tag{8}$$

Also denote W^s_{α,z^-,z^+} as the class of CLGU social evaluation functions with $g^z \in \mathcal{F}^s_{z^-,z^+}$ and critical level α. For any vector of income $\mathbf{v} \in \mathbb{R}^N$, $N \geq 1$, this class is formally defined as:

$$W^s_{\alpha,z^-,z^+} := \left\{ W \;\middle|\; W(\mathbf{v};\alpha) = \sum_{i=1}^{N}\left(g^z(v_i) - g^z(\alpha)\right) \text{ where } g^z \in \mathcal{F}^s_{z^-,z^+} \text{ and } \mathbf{v} \in \mathbb{R}^N \right\}. \tag{9}$$

The assumptions made on g^z and its derivatives enable us to have social evaluation measures that are sensitive to income disparities. The first-order class W^1_{α,z^-,z^+} uses non-decreasing functions g^z (see fourth line of (8)) for which an increase in any individual's income must (weakly) increase social welfare. The evaluation functions that are part of this class thus obey the Pareto principle in addition to being symmetric in income (since the form of g^z does not depend on i). The second-order class of indices must in addition obey the Pigou-Dalton principle of transfers, which postulates that a mean-preserving transfer of income from a higher-income person to a lower-income person constitutes a social improvement. This also corresponds to the familiar incorporation of inequality aversion into social evaluations, here expressed through the concavity of the g^z function in the fourth line of (8).

Social evaluation functions that are part of the third-order class of evaluation functions must also be sensitive to favorable composite transfers. These transfers are such that a beneficial Pigou-Dalton transfer within the lower part of the distribution, coupled with an adverse Pigou-Dalton transfer within the upper part of the distribution, will increase social welfare, provided that the variance of the distribution is not increased — see Kolm (1976), Kakwani (1980), Davies and Hoy (1994) and Shorrocks (1987) for formal characterizations of this transfer principle). Higher-order indices can be interpreted using the generalized transfer principles of Fishburn and Willig (1984). Fourth-order social evaluation functions, for instance, increase following a combination of a favorable composite transfer within a lower part of the distribution and of an unfavorable one within a higher part of the distribution. Generalized higher-order transfer principles essentially postulate that, as s increases, social evaluation functions become increasingly Rawlsian (Blackorby and Donaldson 1978.

Now define the partial CLGU welfare ordering $\succsim^{sW}_{\alpha,z^-,z^+}$ by

$$\mathbf{u} \succsim^{sW}_{\alpha,z^-,z^+} \mathbf{v} \Leftrightarrow W(\mathbf{u};\alpha) \geq W(\mathbf{v};\alpha)\, \forall W \in W^s_{\alpha,z^-,z^+}. \tag{10}$$

Also denote $\succsim^{sP}_{z^-,z^+}$ as a partial FGT poverty ordering defined by

$$\mathbf{u}_\alpha \succsim^{sP}_{z^-,z^+} \mathbf{v} \Leftrightarrow P^s_{\mathbf{u}_\alpha}(z) - P^s_{\mathbf{v}}(z) \leq 0 \text{ for all } z^- \leq z \leq z^+. \tag{11}$$

The partial orderings $\precsim^{sW}_{\alpha,z^-,z^+}$ and $\precsim^{sP}_{z^-,z^+}$ can be defined in the same manner as the inverse of the orderings $\succsim^{sW}_{\alpha,z^-,z^+}$ and $\succsim^{sP}_{z^-,z^+}$, respectively. Formally, we have that

$$\mathbf{u} \precsim^{sW}_{\alpha,z^-,z^+} \mathbf{v} \Leftrightarrow W(\mathbf{u};\alpha) \leq W(\mathbf{v};\alpha)\, \forall W \in W^s_{\alpha,z^-,z^+} \tag{12}$$

and

$$\mathbf{u}_\alpha \precsim^{sP}_{z^-,z^+} \mathbf{v} \Leftrightarrow P^s_{\mathbf{u}_\alpha}(z) - P^s_{\mathbf{v}}(z) \geq 0 \text{ for all } z^- \leq z \leq z^+. \tag{13}$$

It is demonstrated in the Appendix that the two partial dominance orderings $\succsim^{sW}_{\alpha,z^-,z^+}$ and $\succsim^{sP}_{z^-,z^+}$

are equivalent, for some given value for the critical level α:

$$\mathbf{u} \succsim^{sW}_{\alpha,z^-,z^+} \mathbf{v} \Leftrightarrow \mathbf{u}_\alpha \succsim^{sP}_{z^-,z^+} \mathbf{v}. \tag{14}$$

The two partial orderings $\precsim^{sW}_{\alpha,z^-,z^+}$ and $\precsim^{sP}_{z^-,z^+}$ are also analogously equivalent.

These equivalence results have a number of useful properties. First, they address explicitly the link between total poverty and the value of societies. Take (14) for instance. It says that, for the larger population to dominate the smaller one over all CLGU functions with critical level set to α and for all censoring points in $[z^-, z^+]$, total poverty in the larger population must be smaller than in the smaller population, when the smaller population is expanded with $N - M$ individuals of incomes α. Unless α is below z^-, therefore, this dominance condition demands that total poverty, over some range of poverty lines z, must be lower in the larger population than in the *non-expanded* smaller population. Population size increases must therefore be combined with sufficient *falls* in proportional poverty for social welfare to rise.

When $s = 1$, which corresponds to the most robust CLGU welfare orderings, this means that the total number of the poor must fall over some range of poverty lines $z \in [z^-, \alpha]$ for population size increases to lead to greater social welfare. Otherwise, some first-order CLGU indices will necessarily declare the smaller population to be better. A similar comment applies to higher values of s, simply by replacing the total number of the poor by the total amount of FGT poverty.

When the $P^s_{\mathbf{u}_\alpha}(z) \geq P^s_{\mathbf{v}}(z)$ condition in (13) is checked for $z > \alpha$, it is total poverty in $\mathbf{u}$'s expanded population that must be compared. In this case, for $s = 1$, it suffices that the total number of the poor in the smaller population be larger for the larger population to dominate — recall (6). For higher values of s, lower proportional poverty is not sufficient for the larger population to dominate: in (5), the FGT of the expanded population may be lower than the usual proportional FGT for the smaller population.

Linking social welfare and total/proportional poverty is also interesting from the converse perspective of establishing dominance of the smaller population. From (5), it is clear that it is *not enough* that proportional poverty be lower in the smaller population for this to happen. For $s = 1$ for instance, (6)says that it is not enough that the proportional poverty headcount — and thus that the total number of the poor — be larger in the larger population for the smaller population to dominate. It must also be that the cost of the $N - M$ additional lives in the larger population not be too low. This cost will be low if the $(z - \alpha)^{s-1} I\,(\alpha \leq z)$ term in (5) is large.

Alternatively, (5) can be understood as the weighted average of the poverty cost of the smaller population (measured in a total FGT fashion) and of the opportunity cost of having a lower population (measured by total FGT with incomes set to α). (5) is therefore a weighted average of the value (here measured negatively as a cost) of the quality and of the quantity of lives. It says that the smaller population will dominate if its higher quality of lives is sufficient to offset its lower quantity of them.

The equivalence results can also serve to show the tension that exists between total and average utilitarianism, and how CLGU helps ease such a tension, but also how CLGU cannot be viewed as a middle view between the two traditional approaches. To see this, consider the following decomposition of the difference between the FGT dominance curves:

$$P^s_{\mathbf{u}_\alpha}(z) - P^s_{\mathbf{v}}(z) = \frac{M}{N}\left\{\underbrace{(P^s_{\mathbf{u}}(z) - P^s_{\mathbf{v}}(z))}_{\text{proportional effect}} + \underbrace{\left(\frac{M-N}{M}\right)P^s_{\mathbf{v}}(z)}_{\text{total effect}}\right\} \tag{15}$$

$$+\left(1-\frac{M}{N}\right)\underbrace{\left\{(z-\alpha)^{s-1}I\left(\alpha \leq z\right)\right\}}_{\text{critical level effect}}. \tag{16}$$

Again, the combination of (15) and (16) is a weighted average of the value of population quality and quantity. Total and average utilitarianism clash when the quantity of lives varies. The tradeoff is shown on the right-hand-side of (15). The first term is a common-quantity effect, or a *proportional effect*: it measures the advantage of the larger population in terms of the quality of its population, ignoring differences in total population sizes. The second term in (15) is a common-quality effect, or a *total effect*: : it measures the poverty disadvantage of the larger population in terms of the quantity of its population, setting proportional poverty constant across the two populations. These terms can take different signs, in which case total and average utilitarianism (and total and proportional poverty) may rank the populations differently. The *total effect* is always negative: for a given proportional poverty, the welfare importance of that poverty is larger for larger population sizes. But the *proportional effect* can certainly be positive — implying that, were it not for population size differences, poverty in the larger population would be lower.

The (16) term shows how the *critical-level effect* (always positive) may tilt the balance in favor of the proportional effect, or may also go *against* both the proportional and the total effects. The lower the value of α, the more likely will the larger population tend to CLGU dominate the smaller one — this is true regardless of the contributions of the proportional and total effects. Moreover, even though a (negative) proportional effect may favor the smaller population (in addition to the negative total effect), it may still be that the larger population will CLGU dominate the smaller one. This would be because the case the valuation of the quantity of lives is sufficiently large. In such a case, both the proportional and the total views would be reversed by CLGU.

The above results can also be used to make a social welfare ordering based exclusively on population sizes — recall the quote from Heyd (1992) on page 4. Using only differences in population sizes to order distributions can be made by testing conditions (11) and (13) for $\alpha = z^- = 0$. This boils down to checking the sign of $N - M$. It is clear that the conditions make no use of the distribution of incomes, and thus of information on the quality of lives, which is consistent with the view that the number of lives is then the lexicographically prevailing criterion.

3 Robust ranges of critical levels

The previous section addresses the difficulty of specifying a form for the CLGU g function through an extension of relatively standard stochastic dominance techniques. It assumes, however, a particular value for the critical level α. As the literature provides little guidance on such a value, it is useful to extend the dominance techniques to assess over which values of α it is possible to rank the social value of two populations.

This we do by estimating the lower and upper bounds of ranges of critical levels over which a CLGU social ranking can be made.[10] The intuition is relatively simple. Assume that (14) holds for some value of $\alpha = \alpha_0$, and therefore that population $\mathbf{u}$ CLGU dominates population $\mathbf{v}$ at α_0. Since (7) is invariant with respect to α and since (4) is decreasing with α, (14) will also hold with higher values $\alpha > \alpha_0$. (14) may not, however, hold at values of α lower than α_0. The lowest value of α for which (14) holds will set a lower bound to the range of critical levels for which the smaller population dominates the larger one. An analogous procedure is used for estimating an upper bound to the range of critical levels for which the larger population CLGU dominates the smaller one.

To see this formally, define F and G as the distribution functions of the data processes that generate $\mathbf{u}$ and $\mathbf{v}$ respectively. Let α_s and α^s be defined respectively as follows:

$$\alpha_s = \max\{\alpha | P^s_{F_\alpha}(z) \geq P^s_G(z) \text{ for all } z^- \leq z \leq z^+\} \tag{17}$$

and

$$\alpha^s = \min\{\alpha | P^s_{F_\alpha}(z) \leq P^s_G(z) \text{ for all } z^- \leq z \leq z^+\}. \tag{18}$$

Defined as such, α_s is the maximum value of the critical level for which the larger population $\mathbf{v}$ dominates the smaller population $\mathbf{u}$ at order s, whereas α^s is the minimum value of the critical level for which the population $\mathbf{u}$ dominates the population $\mathbf{v}$ at order s.

The definitions (17) and (18) are illustrated graphically in Figures 1 and 2 for $s = 1$. Figure 1 supposes that the larger population $\mathbf{v}$ dominates the lower population $\mathbf{u}$ for a range of censoring points between 0 and α_1. This is equivalent to saying that the absolute poverty incidence curve (which gives the absolute number of poor individuals) is lower in the larger population for all poverty lines between 0 and α_1; this is also equivalent to finding that the cumulative distribution function G lies under the cumulative distribution function $\frac{M}{N}F$. At α_1, the two functions cross and $\mathbf{v}$ does not dominate $\mathbf{u}$ when the critical level is set to a value α that is greater than α_1 (such

[10]Using ranges of critical levels has also been suggested in Blackorby, Bossert, and Donaldson (1996) and Trannoy and Weymark (2009).

as α_2). For all lower values of α and for all censoring points up to z^+, $\mathbf{v}$ first-order dominates $\mathbf{u}$. Formally, this says that $\mathbf{u} \precsim^{1W}_{\alpha,0,z^+} \mathbf{v}$ for all $\alpha \leq \alpha_1$.

Figure 2 presents the symmetric case by supposing that the absolute of number of poor individuals is lower in the smaller population $\mathbf{u}$ than in the larger population $\mathbf{v}$ for a range of censoring points between 0 and z^+. That is, however, not sufficient for the smaller population to CLGU dominate the larger one: we also require that α not be lower than α^1. This is also equivalent to finding that the cumulative distribution function G lies above the cumulative distribution function F_α for all α larger than α^1. At $z = \alpha^1$, G and F_α cross. Hence, $\mathbf{u}$ first-order dominates $\mathbf{v}$ for all critical levels set above α^1 and for all censoring points up to z^+. Formally, this says that $\mathbf{u} \succsim^{1W}_{\alpha,0,z^+} \mathbf{v}$ for all $\alpha \geq \alpha^1$.

4 Application using PovcalNet data

4.1 Data description

The global assessment of poverty and welfare has generated much interest in the academic literature. This interest is nicely reviewed in Anand and Segal (2008), which also discusses the important measurement and data issues that must be dealt with. Much of the recent academic debate has usefully focussed on several of these issues, and explored how their treatment affects the portrait of global poverty. This includes the choice of an indicator of well-being (typically consumption and/or income, scaled for economies of scale, but ideally *functionings* and *capabilities*), adjusting for differences in prices and consumption behavior across time and space (using estimates of purchasing power parities and/or local consumption prices indices), the choice of a global poverty line, reliance on household survey data only (or on national accounts also), distinguishing between inequality and poverty and between absolute and relative poverty, and country weighting *versus* individual weighting of the poverty estimates. Some of the more recent contributions include Dikhanov and Ward (2001), Chen and Ravallion (2001), Milanovic (2002), Bourguignon and Morrisson (2002) Sala-i-Martin (2002), Chen and Ravallion (2004), Sala-i-Martin (2006), Chen and Ravallion (2010), and Deaton (2010).

In this application, we mostly abstract from these important conceptual and measurement issues, except for a rather fundamental one, which has generated both interest and a sense of insatisfaction. Much of the recent evidence indeed reports opposite trends in how the number of the poor changes *versus* how the percentage of the poor varies across time, globally or locally — see for instance Dikhanov and Ward (2001), Bourguignon and Morrisson (2002), and Chen and Ravallion (2010) for important examples. This is indeed a troubling outcome, which inevitably leads to some confusion when it comes, for instance, to evaluating the poverty effect of development. More fundamentally, and as discussed above, neither of these absolute/relative statistics may in fact be sufficient to assess how the value of humanity has been evolving from a social evaluation perspective.

Related to this is the important issue of whether poverty reduction objectives should be set on the basis of changes in proportions of the poor as opposed to changes in the numbers of them. This issue has in fact played an influential role in the establishment of the Millenium Development Goals, as stated in Pogge (2008):

> "At the 1996 World Food Summit (WFS) in Rome, 186 governments promised to halve, by 2015, the number of people in severe poverty. In the first UN Millennium Development Goal (MDG-1) they then promised to halve the 'proportion of the world's people' living in poverty. Later reformulations of MDG-1 backdate its baseline from 2000 to 1990 and also replace 'world's people' with the population of the developing countries.
>
> So there were three successive targets: (1) the WFS target: to halve, over 19 years, the number of poor worldwide, which implies a 3.58 per cent annual reduction in this number; (2) MDG-1 as adopted: to halve, over 15 years, the proportion of poor in world population, which implies a 3.40 per cent annual reduction in the number of poor; and (3) MDG-1 as reformulated: to halve, over 25 years, the proportion of poor in the developing world, which implies a 1.28 per cent annual reduction in the number of poor worldwide. The last and now official target is so much less ambitious because — thanks to 1990-2015 population growth of 45 per cent in the developing world – the number of poor needs to be reduced by only 27.5 per cent."

The CLGU framework developed can fortunately help address these difficulties in a consistent social welfare framework. The data we use come from the living standard household surveys carried out in most developing countries of the world during the last two decades. They are available on the World Bank's PovcalNet website in the form of grouped income distributions. We use the PovcalNet software tools to extract the grouped income distribution data for all available developing countries and then generate samples of individual-level microdata at the national level. This is done by means of Shorrocks and Wan (2009)'s algorithm (which is programmed in the freely available *Distributive Analysis Stata Program* — see Araar and Duclos (2007). A sample of 1,000 observations is generated for every dataset. A total of 173 countries (114 developing countries and 59 high-income countries) are included to estimate the world distribution of income/consumption (depending on the datasets) for 1990 and 2005. The Appendix draws up the list of the high-income countries that are included, the developing countries that are excluded, and those developing countries for which we have only one survey.

The income (for short, although consumption is more frequently used) levels are expressed in yearly *per capita* 2005 PPP (purchasing power parity) US dollars. Whenever a dataset for a particular country is not available for 1990 or 2005, the nearest dataset for that country is used and the income data are extrapolated or interpolated to 1990 or 2005 using the relevant GDP growth estimates found in http://databank.worldbank.org/ddp/home.do?Step=12&id=4&CNO=2

Whenever PovcalNet does not provide estimates of total population sizes, the information is obtained from http://www.indexmundi.com/. We sometimes regroup countries into World-Bank-defined regions, identified as East Asia and Pacific (EAP), Europe and Central Asia (ECA), Latin America and the Caribbean (LAC), Middle East and North Africa (MENA), South Asia (SA) and Sub-Saharan Africa (SSA). z^- is set to the minimum income of the two populations being compared. For the purposes of this paper, the upper bound of the censoring point (z^+) is often set to \$2,000, with the implicit assumption that the range $[\$0, \$2{,}000]$ contains the appropriate censoring points for assessing global welfare and poverty. When focussing on a particular country (or group of countries), z^+ is set to a value around or higher than the 80th quantile. When such a relatively high value of z^+ does not allow the computation of estimates α_s or α^s, it is decreased to check if such estimates do exist for lower values of z^+.

Using this, humanity's population size is estimated to be 5.03 billion in 1990 and 6.16 billion in 2005, and average income in the developing world is estimated to be \$1,068 in 1990 and \$1,524 in 2005. Estimated population sizes and average incomes by regions are shown in Table 1. The ECA and LAC regions are those with the higher average incomes. The 1990-2005 period saw a reversal of the average income rankings of EAP and SSA (due in part to China's higher growth). SA has had an average income growth rate four times higher than that of SSA. Except for MENA and SSA, all regions in Table 1 have seen average income growth rates at least no lower than population growth rates. The developing world has seen increases in population size (25%) and in average income (43%) that are both higher than for the entire world (22.5% and 30%, respectively).

(Milanovic-TrueWorlIncoDist:02). I then applied this value to incomes 1990. I did the same for incomes 2005 even if the coefficient was estimated using data nearest 1990 than 2005.

4.2 Dominance of large over small

Given this, can we tell whether the value of humanity has increased between 1990 and 2005? A first answer is given by simply drawing the *absolute* poverty incidence curves $MP^1_F(z)$ and $NP^1_G(z)$ over the range of censoring points specified above. This is done on Figure 3. The global absolute number of poor is lower in 2005 for all poverty lines up to \$1,288 (including at \$456, which corresponds to \$1.25 per day, and which is around the poverty line often used in international comparisons). Graphing the absolute number of the poor in the expanded 1990 population, $NP^1_{F_\alpha}(z)$, using a critical level set to $\alpha^1 = \$1,288$ shows that there is first-order CLGU dominance of humanity in 2005 over humanity of 1990. Keeping in mind the earlier discussion, this also says that all first-order CLGU functions with critical levels no higher than

\$1,288 will necessarily evaluate 2005 better than 1990. Formally, we have that 1990 $\precsim^{1W}_{\alpha,\$0,\$2000}$ 2005 for all $\alpha \leq \$1,288$.

This is a powerful result obtained simply from a straightforward inspection of the absolute poverty incidence curves. Table 2 repeats this exercise for the various regions and for various orders of CLGU dominance, namely, it provides estimates of the upper bounds of the ranges of critical levels for which 2005 dominates 1990 for more restricted classes of CLGU functions and for specific regions. We do not provide estimates for the ECA and MENA regions as there is no dominance relations between 1990 and 2005 for these regions. As seen with Figure 3, at any critical level lower than \$1,288, we can assert that global welfare has robustly increased between 1990 and 2005 in spite of the significant increase in world population size. Table 2 shows that the dominance of 2005 over 1990 is stronger for the EAP region and the entire world than it is for the LAC and the SSA regions. For instance, any critical level no greater than \$2,242 leads to first-order dominance of EAP in 2005 over EAP in 1990. To conclude that LAC in 2005 is better than in 1990 requires lower values of α: at first-order for instance, one would need to assume a critical level no greater than \$827.

As the order of dominance increases, the set of ordered distributions can be ranked becomes larger. Also, once a lower-order CLGU dominance ranking between two distributions is established, higher-order dominance between these two distributions also holds up to a higher upper bound for the range of critical levels. This is visible in Table Table 2 and also discussed briefly in the Appendix.

An example of a social evaluation index in the class $W^1_{\alpha,0,z^+}$ is the critical-level utilitarian social evaluation index defined as

$$W(\mathbf{u};\alpha) = \sum_{i=1}^{M} \left(u_i^z - \alpha^z\right) \tag{19}$$

and

$$W(\mathbf{v};\alpha) = \sum_{j=1}^{N} \left(v_j^z - \alpha^z\right). \tag{20}$$

The above notation x^z says that x is censored to z if x exceeds the censoring point z; otherwise, x remains unchanged. Table 4 shows values of U when the critical level is equal to $\hat{\alpha}_2$ and when it takes a value α above $\hat{\alpha}_2$. We see that for some of these higher values of the critical level, the world in 1990 has greater value than in 2005. However, the usual social evaluation functions based on total and average utilitarianism unambiguously declare that the world in 2005 is better than in 1990 — the estimates for 1990 and 2005 are respectively of \$4430 billion and \$7870 billion for 1990 and 2005 in the case of total utilitarianism, and of \$1,068 and \$1,524 for average utilitarianism.

4.3 Dominance of small over large

It can also be that population size increases leads to a worse social evaluation. This is the case for some groups of countries in ECA and SSA, where we can estimate an α^s critical level value above which 1990 necessarily dominates 2005. To show this, we consider a group of 15 countries in ECA and 10 countries in SSA. In ECA, this includes Belarus, Bulgaria, the Czech Republic, Estonia, Hungary, Kazakhstan, the Kyrgyz Republic, Latvia, Macedonia, Moldova, Poland, Romania, Slovakia, Slovenia and Uzbekistan. The 10 SSA countries are made of Burundi, Comoros, the Congo Republic, Côte d'Ivoire, Ghana, Liberia, Mali, Niger, Rwanda and Tanzania.

The results are shown in Table 3. Using a range of censoring points is set to $[0, \$3{,}000]$ for the joint set of those countries, Table 3 shows that all first-order CLGU evaluation functions fall in those countries between 1990 and 2005 for any critical level α^1 no lower than \$566 — this seems to be a relatively convincing case that social welfare can fall quite robustly in spite of a substantial increase in population size between 1990 and 2005. For all second-order CLGU functions (*i.e.*, those that penalize inequality), this is true for any critical level α^1 no lower than \$333.

The dominance of 1990 over 2005 is less strong for the group of 10 SSA countries. For a range of censoring points set to $[0, \$1{,}000]$, no critical level makes all first-order CLGU functions be larger in 1990 than in 2005. Restricting those functions to inequality-penalizing ones, however, makes 1990 better than 2005 for all critical levels larger than \$487. There is much stronger evidence that 1990 dominates 2005 for the group of 15 ECA countries; this is the case for all censoring points between $[0, \$4{,}500]$, for all first-order CLGU functions, and for all values larger than \$182. Hence, despite the finding that 1990 can reasonably be declared globally better than 2005, it is quite clear that social welfare in some groups of countries has deteriorated during the last decades.

This is also true for some individual countries. For most developing countries of the world, the Appendix provides the estimated values of the bounds of the ranges of critical levels for which 1990 dominates 2005, or the reverse. For some countries, such bounds cannot be estimated since a dominance relation does not exist.[11] There are also 17 countries in the developing world that have a larger population in 1990 than in 2005; these are Albania, Armenia, Belarus, Bosnia, Bulgaria, Crotia, the Czech Republic, Estonia, Georgia, Hungary, Kazakhstan, Latvia, Lithuania, Moldova, Romania, Russia and Ukraine. Our estimates strongly suggest that more than half of these population-declining countries have also experienced a fall in social welfare between 1990 and 2005.

[11] Altogether, this concerns 17 countries: Azerbaijan, Belarus, Macedonia and Russia in the ECA region; Guyana, Bolivia, Haiti, Honduras, Paraguay and Peru in the LAC region; Morroco in the MENA region; and Côte d'Ivoire, Ghana, Niger, Rwanda, and Tanzania in the SSA region.

4.4 Comparison between CLGU and traditional approaches

CLGU evaluations can also lead to social assessments that differ starkly from those of traditional approaches. Consider again the above group of selected ECA and SSA countries. As shown in Figure 4, the cumulative distribution function $\frac{M}{N}F$ lies everywhere under the the cumulative distribution function G. This says that the absolute number of poor people in 1990 is lower than the absolute number of poor people in 2005, suggesting that social welfare is higher in 1990. A similar conclusion applies when using a *per capita* approach: since F is everywhere under G, the proportional number of poor people is lower in 1990, implying that 1990 is again better than 2005.

Suppose instead that we use CLGU for social evaluation purposes. For any critical level value greater than \$566, we also conclude that there has been a normatively robust decline in social welfare between 1990 and 2005 in parts of the ECA and SSA regions. But this is not the case for critical level values lower than \$566. Some of the first-order CLGU social evaluation functions will indeed rank 2005 better if we levels lower than \$566 to the *value of living*. A similar conclusion applies to higher orders of dominance.

For instance, let us consider again the censored critical-level utilitarian social evaluation index, that is,

$$W(\mathbf{u};\alpha) = \sum_{i=1}^{M} \left(u_i^z - \alpha^z\right) \tag{21}$$

and

$$W(\mathbf{v};\alpha) = \sum_{j=1}^{N} \left(v_j^z - \alpha^z\right). \tag{22}$$

For some value of z no greater than z^+, suppose that $\sum_{j=1}^{N} v_j^z \geq \sum_{i=1}^{M} u_i^z$. For a value of α_0 such that $W(\mathbf{v};\alpha_0) \geq W(\mathbf{u};\alpha_0)$; we then have

$$\alpha_0 \leq \frac{1}{N-M}\left(\sum_{j=1}^{N} v_j^z - \sum_{i=1}^{M} u_i^z\right). \tag{23}$$

This relation gives in fact the set of values α_0 for the critical level for which the larger population has a higher censored critical-level utilitarian value. Because the maximum value of α_0 in (23) is $\frac{1}{N-M}\left(\sum_{j=1}^{N} v_j^z - \sum_{i=1}^{M} u_i^z\right)$, we can verify that $\hat{\alpha}^2 \geq \frac{1}{N-M}\left(\sum_{j=1}^{N} v_j^z - \sum_{i=1}^{M} u_i^z\right)$. Otherwise, $\hat{\alpha}^2$ could not be the lower bound of the range of critical levels that let the smaller population dominate the larger one at second-order — this is because the critical-level utilitarian index is a member of the second-order class of CLGU functions.

To illustrate these relations, consider again the case of the 10 earlier SSA countries. In Table

3, the value of $\hat{\alpha}^2$ is \$487. As discussed above, this says that 1990 is robustly better than 2005 only for critical levels higher than \$487, despite the fact that traditional *per capita* and total social evaluation approaches will rank 1990 unambiguously better at first-order of (censored) welfare dominance. For a censoring point z set to \$900, we find a range of critical levels α_0 equal $[0, \$467]$. This says that the (censored) critical-level utilitarian indices of the types (21) and (22) will rank 2005 better than 1990 for all critical levels lower than \$467. Moving the censoring point up to \$1,000 and still using a value of α_0 equal to \$467, total critical-level utility amounts to \$403 million for 1990 and \$408 million for 2005. Again, the selected SSA countries can be deemed better in 2005 than in 1990, although the 2005 population does not CLGU dominate the 1990 population. All this serves to illustrate how the usual (average and total) social evaluation (and dominance) rankings may *commonly* conflict with those obtained with CLGU.

Figure 5 illustrates another situation that often occurs in SSA. As shown by Chen and Ravallion (2010)'s empirical results, the proportional poverty rate has fallen recently in SSA while the absolute number of the poor has gone up — due the relatively high rate of population growth in SSA. Let us consider three such SSA countries, Côte d'Ivoire, Ghana and Mali. Over a range of censoring points z^+ up to \$1,000, we find that between 1990 and 2005 that the proportional number of poor people has declined, but that the absolute number of poor people has increased. The curves are shown in Figure 5. Robust CLGU evaluations declare that 1990 is better than 2005 for any critical level higher than \$1,000. This is in accordance to the total poverty view that the situation of some countries in SSA has deteriorated over the last decades because there are more poor people. For a critical level value lower than \$1,000, however, welfare in these three SSA countries can be shown to be higher in in 2005 for some total social evaluation indices. This is because values of $\alpha \leq \$1000$ would push the $P^1_{\mathbf{u}_\alpha}(z)$ curve (given by $M/NF(z) + (1 - M/N)I\,(\alpha \leq z)$ above $P^1_{\mathbf{v}}(z)$ (given by $G(z)$) in Figure 5.

To see CLGU valuations with critical levels below \$1000 may clash, let the function g in (1) be defined as $g\,(u) = \frac{u^{1-\varepsilon}}{1-\varepsilon}$ for any income u. (This is the well-known homothetic social utility function popularized by Kolm (1969) and Atkinson (1970).) $\varepsilon \geq 0$ provides the relative inequality aversion value. It is convenient to express social welfare in units of an *equally distributed equivalent income* (EDE), *viz*, the equally distributed level of income that gives the same level of social welfare.[12] For ε =0 and α =\$650, the social valuation of Côte d'Ivoire, Ghana and Mali

[12] The EDE for $\mathbf{u}$ and $\mathbf{v}$ are respectively defined as

$$EDE_{\mathbf{u}} = \left\{ \frac{1}{N} \sum_{i=1}^{M} u_i^{1-\varepsilon} + \left(\frac{N-M}{N} \right) \alpha^{1-\varepsilon} \right\}^{\frac{1}{1-\varepsilon}}$$

and

$$EDE_{\mathbf{v}} = \left\{ \frac{1}{N} \sum_{j=1}^{N} v_j^{1-\varepsilon} \right\}^{\frac{1}{1-\varepsilon}}.$$

equals -\$6 million for 1990 and -\$1 million for 2005. Expressed in EDE units, this gives \$639 for 1990 and \$650 for 2005. Hence, social welfare has increased. For $\varepsilon=$0.5 and $\alpha=$\$650, the EDE estimate is \$2,454 and \$2,441 for 1990 and 2005 respectively, saying that 1990 is then better than 2005. Incorporating aversion to inequality into utilitarian assessments of welfare gives relatively more importance to lower incomes and then gives preference to the earlier distribution (since it CLGU dominates the larger 2005 population at lower z). Hence, for a critical level value below the lower bound of \$1000, two different CLGU functions, both members of the class W^1_{α,z^-,z^+}, can give opposite rankings to 1990 and 2005, depending on the degree of aversion to the inequality of individuals below that critical value.

4.5 Decomposing the change in the absolute number of the poor

Many of the numerous techniques that have been applied to traditional poverty measures (such as for poverty profiling and decompositions, computations of growth/inequality elasticities, targeting algorithms, poverty impacts of price changes, *etc.*) can also be applied to the expressions introduced above, such as $MP^s_{\mathbf{u}}(z)$ and $NP^s_{\mathbf{v}}(z)$. One technique that has been quite popular is the growth-redistribution decomposition introduced by Datt and Ravallion (1992). It is useful to extend it here to a population-growth-redistribution setting in order to assess the respective effects of population/income/inequality changes on the above 1990-2005 CLGU dominance comparisons. We can for instance express sequentially the movement from $MP^1_{\mathbf{u}}(z)$ to $NP^1_{\mathbf{v}}(z)$ as

$$NP^1_{\mathbf{v}}(z) = MP^1_{\mathbf{u}}(z) + (N-M)P^1_{\mathbf{u}}(z) + \Delta_{upg}(z) + \Delta_{ig}(z) + \Delta_{uig}(z) + \Delta_{iri}(z), \tag{24}$$

where:

- $(N-M)P^1_{\mathbf{u}}(z)$ is the change in 1990's total number of the poor obtained by scaling it up by the 1990-2005 increase in world population (given by $N-M$);
- $\Delta_{upg}(z)$ is the change in that total number obtained once 1990-2005 differential population growth rates across countries are introduced;
- $\Delta_{ig}(z)$ is the subsequent change in the global number of the poor obtained when all incomes are uniformly scaled up by the global income growth rate observed between 1990 and 2005;
- $\Delta_{uig}(z)$ is the subsequent change obtained once 1990-2005 differential income growth rates across countries are introduced;
- and $\Delta_{iri}(z)$ is the final change obtained in the total number of the poor once 1990-2005 changes in within-country income distributions are introduced.

$(N-M)P^1_{\mathbf{u}}(z)$ gives a "pure population effect"; $\Delta_{ig}(z)$ provides a "pure income growth effect"; and $\Delta_{upg}(z)$, $\Delta_{uig}(z)$, $\Delta_{iri}(z)$ give a "redistribution effect", either over population shares, across-country income shares, or within-country income shares.

Table 5 shows the decomposition results. z is set to \$1288 since this is where the absolute numbers of the poor are equal in 1990 and in 2005. 3.33 billion individuals are then considered to be poor. Since the world population growth rate between 1990 and 2005 is around 22%, the world population effect scales that number up by 0.74 billion people. Population growth rates in poorer countries were, however, larger than those of richer ones, and this differential population growth effect has led to a further increase in the global number of poor of about 130 million people ($\Delta_{upg}(z)$). World *per capita* income grew by 30% between 1990 and 2005, and this has led to a fall in the number of poor individuals of about 600 million ($\Delta_{ig}(z)$). Poor countries grew on average more rapidly than richer countries, and this led to a fall in the global number of poor by 1 billion ($\Delta_{uig}(z)$). Finally, within-country income distribution have generally become less equal, leading to an increase in the global number of poor people of about 800 million ($\Delta_{iri}(z)$). It is interesting to note that the unequal income growth effect exceeds both the total population effect and the world income growth effect. The second largest effect is the within-country income redistribution effect. That effect exceeds the world income growth effect and is almost as strong as the sum of the two population effects.

5 Conclusion

This paper uses and extends an attractive but relatively little-known social evaluation approach to overcome the important flaws of traditional social assessments based on various forms of total and average utilitarianism. It develops dominance relations for critical-level generalized utilitarianism that are sufficiently general to allow for different classes of attitudes to inequality, different levels of focus on the poorer (through different extents of censoring of incomes), in addition to different views on what critical level (the so-called *value of living*) should be used to make social evaluations. A special case (total generalized utilitarianism) is obtained when that critical level is set to zero. That, however, makes social evaluations suffer from Parfit (1984)'s repugnant conclusion. Another important special case (average generalized utilitarianism) is obtained when differences in population sizes are ignored; this, however, can lead to anti-population-biased social assessments. A further special case is obtained when social assessment is based only on differences in population sizes; this, however, takes no account of the distribution of incomes, and can lead to the promotion of extreme instances of the repugnant conclusion.

The CLGU dominance conditions are nicely tied to total and proportional poverty dominance. As in the traditional poverty and social welfare dominance literature, the conditions allow dominance tests of arbitrary orders that involve ranges of possible choices of poverty lines (or censoring points) as well as (in a CLGU context) ranges of possible values for critical levels.

We apply this framework to data on the global distribution of income to assess whether the *value of humanity* can be persuasively shown to have increased between 1990 and 2005. The answer is *yes* if and only if we are willing to judge that lives with *per capita* yearly consumption of more than \$1,288 *necessarily increase* the value of humanity. The same conclusion applies to Sub-Saharan Africa if and only if we are willing to make that same judgement for lives with *any* level of *per capita* yearly consumption above \$230. If not, we arrive at the opposite conclusion that the value of humanity has decreased during this period for at least some of the admissible CLGU functions.

Whether these values of the critical levels are reasonable enough to make a firm judgement on the evolution of humanity is open to debate (see for instance Klugman, Rodríguez, and Choi (forthcoming)). For reference, note that Maddison (2010) uses 2005-PPP \$570-\$640 as a subsistence estimate of *per capita* income from 1 AD onwards, that Bairoch (1993) estimates a bare subsistence minimum of around 2005-PPP \$420, and that Becker, Philipson, and Soares (2005) calibrate the value of life expectancy using a level of income at which an individual would be indifferent between being dead and alive set to about 2005-PPP \$486 prices. This would support the view that the value of humanity has globally and robustly increased between 1990 and 2005 (Table 2), that there are some CLGU evaluation functions that would declare total social welfare to have fallen in sub-Saharan Africa (Table 2), and that the value of humanity has globally and robustly fallen between 1990 and 2005 for some countries in sub-Saharan Africa and Europe and Central Asia (Table 3).

We also examine how and why CLGU assessments and traditional total and *per capita* social evaluation approaches can conflict in theory, and do conflict in practice. Among other things, this rationalizes the important claim often made that the situation of some countries in the world may have deteriorated over the last decades because there are now more poor people than before, although their proportion in the total population may well have fallen.

Figure 1: Large dominates small: Poverty incidence curves with $\alpha = \alpha_1$ and adjusted for differences in population sizes

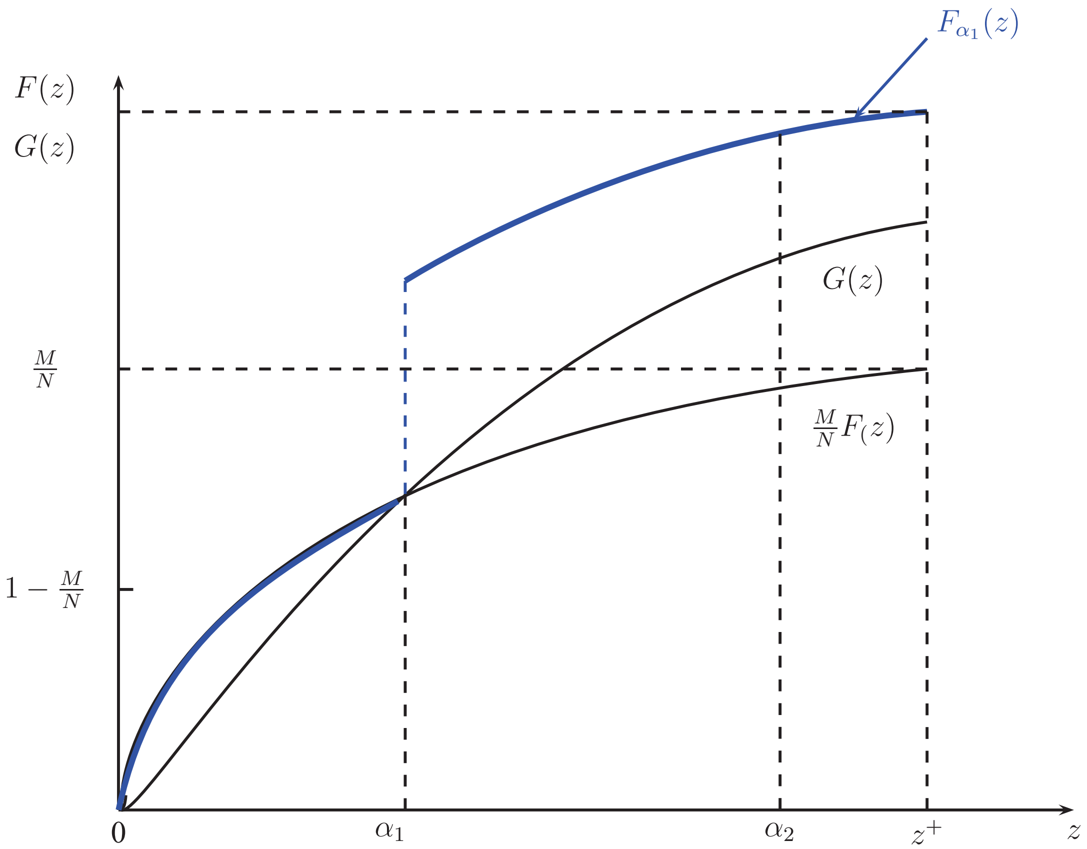

Figure 2: Small dominates large: Poverty incidence curves with $\alpha = \alpha^1$ adjusted for differences in population sizes

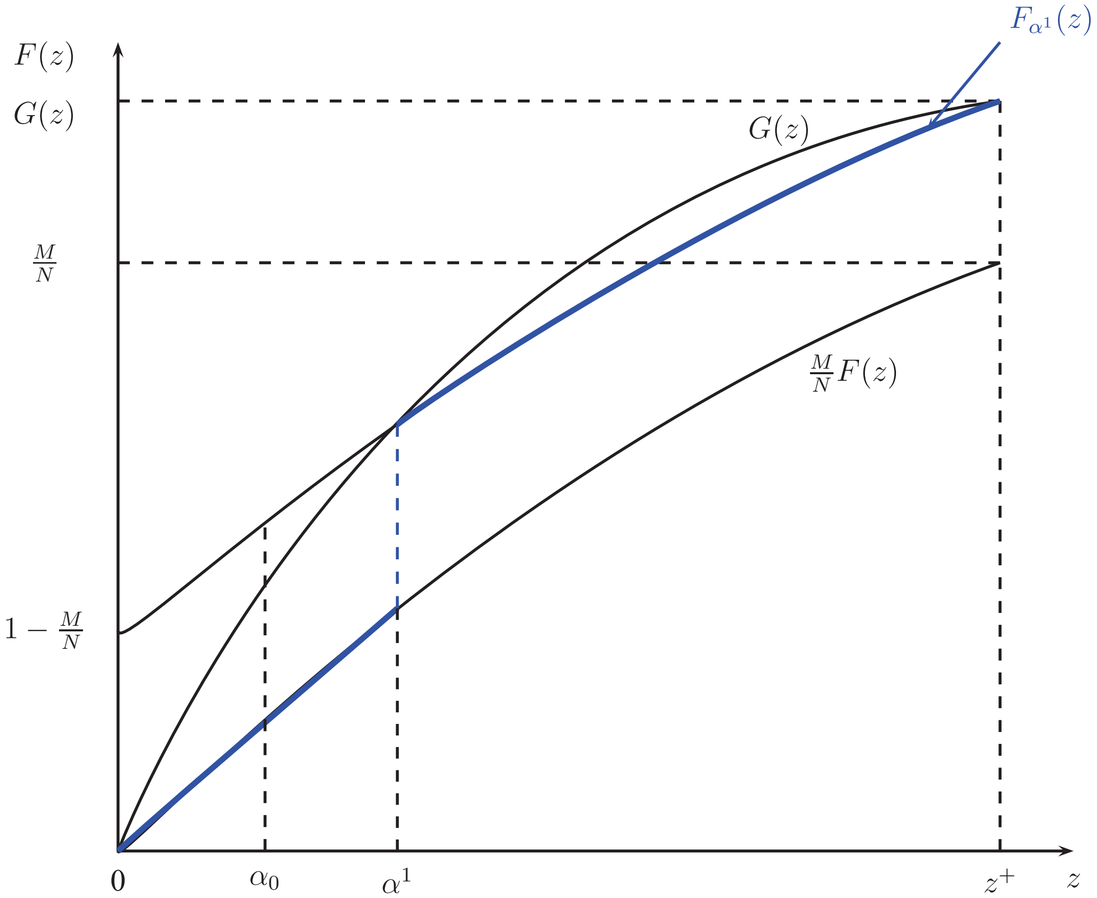

Figure 3: World in 2005 CLGU-dominates world in 1990

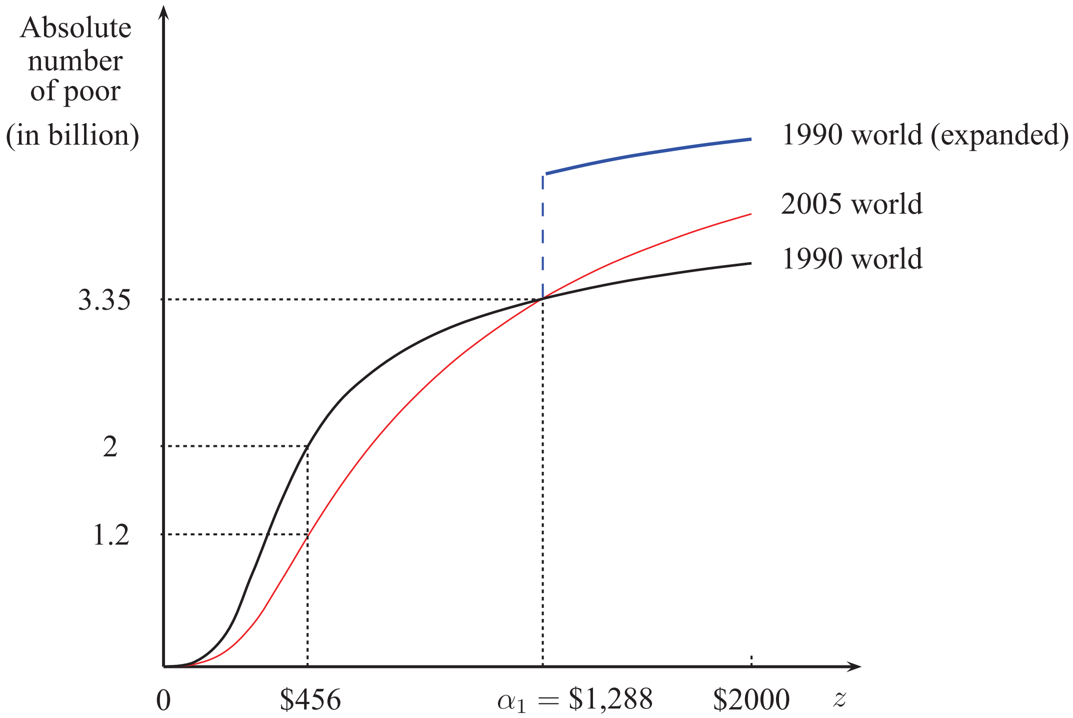

Figure 4: 1990 first-order CLGU dominates 2005 for a group of 15 ECA and 10 SSA countries, for all critical levels beyond \$566

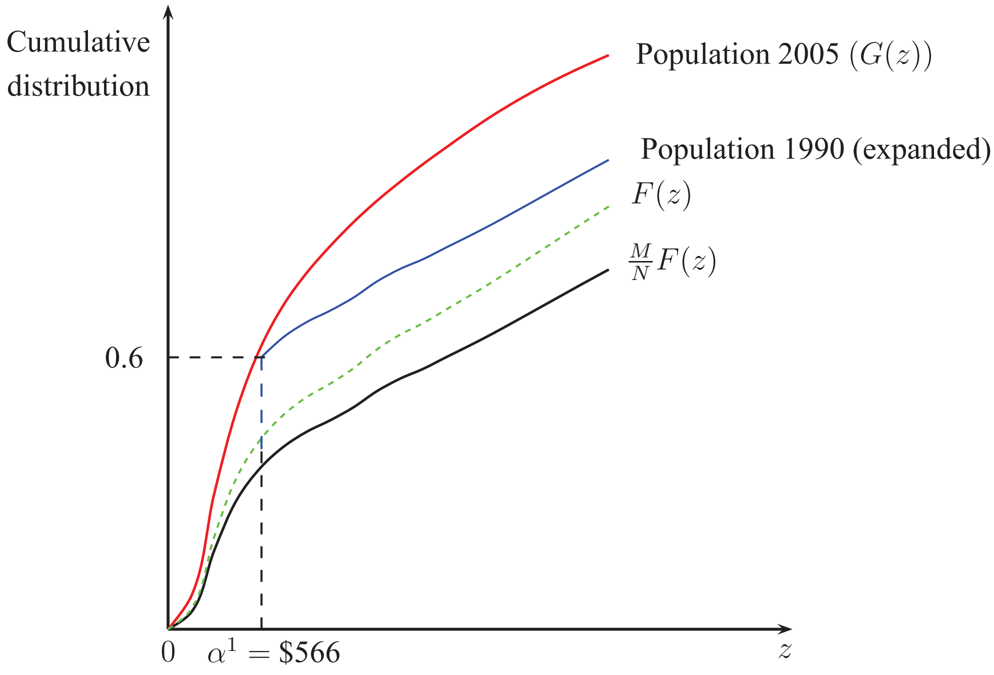

Figure 5: The increase in the absolute number of the poor leads to CLGU dominance of 1990 over 2005 in three SSA countries

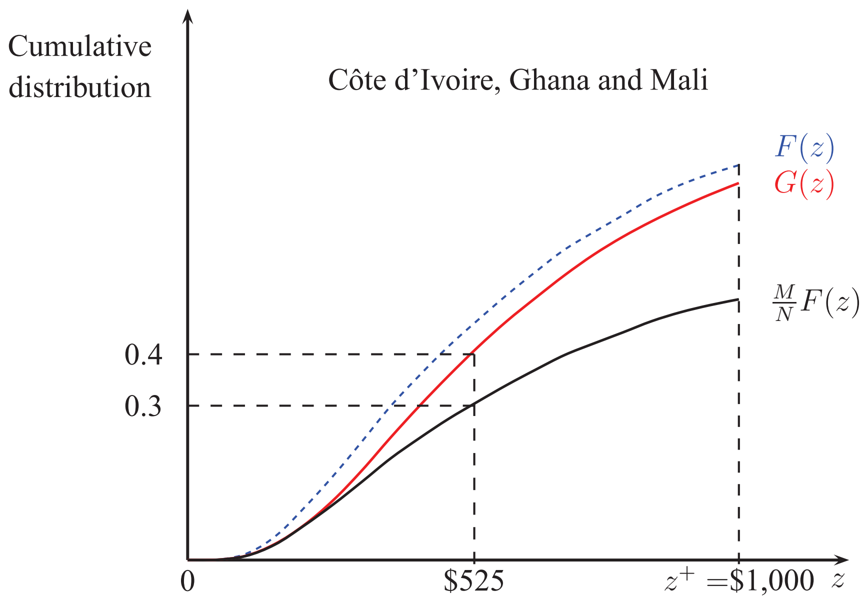

Table 1: Population sizes and average incomes by region, 2005 PPP USD

Regions	Population (in millions) 1990	Population (in millions) 2005	Growth in population size	Average income 1990	Average income 2005	Growth in average income
EAP	1,540	1,810	18%	580	1,520	162%
ECA	458	464	1%	2,577	3,181	23%
LAC	393	497	26%	2,388	3,020	26%
MENA	186	247	33%	1,776	1,856	5%
SA	1,110	1,450	30%	491	714	45%
SSA	463	695	50%	721	801	11%
Developing world	4,150	5,170	25%	1,068	1,524	43%
Entire world	5,278	6,468	22.5%	6797	8826	30%

Table 2: Estimates of the upper bounds of critical levels up to which 2005 dominates 1990, by region and order of CLGU dominance

s	EAP	LAC	SA	SSA	World
	$\hat{\alpha}_s$	$\hat{\alpha}_s$	$\hat{\alpha}_s$	$\hat{\alpha}_s$	$\hat{\alpha}_s$
$s = 1$	2,242	827	674	230	1,288
$s = 2$	6,620	1,148	1140	355	2,602
$s = 3$	10,000	1,497	1666	470	3,443

Note: All critical levels are in 2005 PPP US dollars.

$z^+ = \$10{,}000$ for EAP, $z^+ = \$3{,}500$ for LAC, $z^+ = \$3{,}500$ for SA, $z^+ = \$1{,}000$ for SSA and $z^+ = \$3{,}500$ for World.

Table 3: Estimation of lower bounds: small dominates large

s	ECA (15)	SSA (10)	ECA & SSA
	$\hat{\alpha}^s$	$\hat{\alpha}^s$	$\hat{\alpha}^s$
$s = 1$	182	-	566
$s = 2$	182	487	333
$s = 3$	136	394	303

Note: All critical levels are in 2005 PPP US dollars.

$z^+ = \$4{,}500$ for ECA(15), $z^+ = \$1{,}000$ for SSA(10), $z^+ = \$3{,}000$ for ECA(15)+SSA(10).

Table 4: Values of the utilitarian social evaluation index (in billion $)

Year	EAP		LAC		SSA		World	
	z^+=8,000		z^+ =1,800		z^+ =400		z^+ =3,000	
	$\hat{\alpha}_2$ =6,620	α =6,710	$\hat{\alpha}_2$ =1,148	α =1,700	$\hat{\alpha}_2$ =355	α =385	$\hat{\alpha}_2$ =2,602	α =2,730
1990	-9297	-9435	63	-154	- 18.38	-32,28	-7263	-7794
2005	-9279	-9443	120	-155	- 11.57	-32.41	-7142	-7804

Table 5: Total numbers of poor in 1990 and 2005, and changes in these numbers induced by population, income and redistributive effects (in billion)

	1990	Population effects		Income effect	Redistribution effects		2005
	$MP^1_{\mathbf{u}}(z)$	$(N-M)P^1_{\mathbf{u}}(z)$	$\Delta_{upg}(z)$	$\Delta_{ig}(z)$	$\Delta_{uig}(z)$	$\Delta_{iri}(z)$	$NP^1_{\mathbf{v}}(z)$
$z = \$1288$	3.334	0.746	0.127	-0.265	-0.686	0.806	3.337

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6 Appendix

6.1 Critical level bounds for developing countries

Country	Larger population	z^+	Estimated bound	Change in welfare
A				
Albania	1990	2000	$\hat{\alpha}^1 = 296$	Improvement if $\alpha \geq \hat{\alpha}^1$
Algeria	2005	2500	$\hat{\alpha}_1 = 2130$	Improvement if $\alpha \geq \hat{\alpha}_1$
Angola	2005	1500	$\hat{\alpha}_1 = 389$	Improvement if $\alpha \geq \hat{\alpha}_1$
Armenia	1990	2500	$\hat{\alpha}_1 = 185$	Deterioration if $\alpha \leq \hat{\alpha}_1$
Azerbaijan	2005	3000	?	?
B				
Bangladesh	2005	1000	$\hat{\alpha}_1 = 519$	Improvement if $\alpha \leq \hat{\alpha}_1$
Belarus	1990	4000	?	?
Benin	2005	1000	$\hat{\alpha}_1 = 509$	Improvement if $\alpha \leq \hat{\alpha}_1$
Bhutan	2005	2000	$\hat{\alpha}_1 = 1646$	Improvement if $\alpha \leq \hat{\alpha}_1$
Bolivia	2005	2500	?	?
Bosnia	1990	4500	$\hat{\alpha}^1 = 333$	Improvement if $\alpha \geq \hat{\alpha}^1$
Botswana	2005	3500	$\hat{\alpha}_1 = 2270$	Improvement if $\alpha \leq \hat{\alpha}_1$
Brazil	2005	4000	$\hat{\alpha}_1 = 1352$	Improvement if $\alpha \leq \hat{\alpha}_1$
Bulgaria	1990	9500	$\hat{\alpha}_1 = 7828$	Deterioration if $\alpha \leq \hat{\alpha}_1$
Burkina Faso	2005	1000	$\hat{\alpha}_1 = 408$	Improvement if $\alpha \leq \hat{\alpha}_1$
Burundi	2005	1000	$\hat{\alpha}_1 = 125$	Improvement if $\alpha \leq \hat{\alpha}_1$
C				
Cambodia	2005	1000	$\hat{\alpha}_1 = 598$	Improvement if $\alpha \leq \hat{\alpha}_1$
Cameroon	2005	1500	$\hat{\alpha}_1 = 985$	Improvement if $\alpha \leq \hat{\alpha}_1$
Cape Verde	2005	2000	$\hat{\alpha}_1 = 1550$	Improvement if $\alpha \leq \hat{\alpha}_1$
Central African Rep.	2005	1000	$\hat{\alpha}_1 = 407$	Improvement if $\alpha \leq \hat{\alpha}_1$
Chad	2005	1000	$\hat{\alpha}_1 = 652$	Improvement if $\alpha \leq \hat{\alpha}_1$
Chile	2005	5500	$\hat{\alpha}_1 = 3841$	Improvement if $\alpha \leq \hat{\alpha}_1$
China	2005	3000	$\hat{\alpha}_1 = 2481$	Improvement if $\alpha \leq \hat{\alpha}_1$
Colombia	2005	5000	$\hat{\alpha}^1 = 1481$	Improvement if $\alpha \geq \hat{\alpha}^1$
Comoros	2005	1500	$\hat{\alpha}_1 = 267$	Improvement if $\alpha \leq \hat{\alpha}_1$
Congo	2005	1000	$\hat{\alpha}_1 = 310$	Improvement if $\alpha \leq \hat{\alpha}_1$
Congo Dem. Rep.	2005	1000	$\hat{\alpha}^1 = 556$	Deterioration if $\alpha \geq \hat{\alpha}^1$
Costa Rica	2005	4000	$\hat{\alpha}_1 = 2216$	Improvement if $\alpha \leq \hat{\alpha}_1$
Côte d'Ivoire	2005	1500	?	?
Crotia	1990	9000	$\hat{\alpha}^1 = 4815$	Improvement if $\alpha \geq \hat{\alpha}^1$
Czech Rep.	1990	8000	$\hat{\alpha}_1 = 1856$	Deterioration if $\alpha \leq \hat{\alpha}_1$

Country	Larger population	z^+	Estimated bound	Change in welfare
D, E				
Djibouti	2005	3000	$\hat{\alpha}^1$= 889	Deterioration if $\alpha \geq \hat{\alpha}^1$
Dominican Rep.	2005	5000	$\hat{\alpha}_1$= 2179	Improvement if $\alpha \leq \hat{\alpha}_1$
Ecuador	2005	5000	$\hat{\alpha}_1$= 390	Improvement if $\alpha \leq \hat{\alpha}_1$
Egypt	2005	2000	$\hat{\alpha}_1$= 781	Improvement if $\alpha \leq \hat{\alpha}_1$
El Salvador	2005	4000	$\hat{\alpha}_1$= 687	Improvement if $\alpha \leq \hat{\alpha}_1$
Estonia	1990	7000	$\hat{\alpha}_1$= 5372	Deterioration if $\alpha \leq \hat{\alpha}_1$
Ethiopia	2005	1000	$\hat{\alpha}_1$= 407	Improvement if $\alpha \leq \hat{\alpha}_1$
F, G				
Gabon	2005	2500	$\hat{\alpha}_1$= 799	Improvement if $\alpha \leq \hat{\alpha}_1$
Gambia	2005	1500	$\hat{\alpha}_1$= 702	Improvement if $\alpha \leq \hat{\alpha}_1$
Georgia	1990	2000	$\hat{\alpha}_1$= 211	Deterioration if $\alpha \leq \hat{\alpha}_1$
Ghana	2005	1000	?	?
Guatemala	2005	3000	$\hat{\alpha}_1$= 1393	Improvement if $\alpha \leq \hat{\alpha}_1$
Guinea	2005	1000	$\hat{\alpha}_1$= 399	Improvement if $\alpha \leq \hat{\alpha}_1$
Guinea Bissau	2005	1500	$\hat{\alpha}_1$= 295	Improvement if $\alpha \leq \hat{\alpha}_1$
Guyana	2005	1500	?	?
H, I				
Haiti	2005	1500	?	?
Honduras	2005	3000	?	?
Hungary	1990	9000	$\hat{\alpha}_1$= 7381	Deterioration if $\alpha \leq \hat{\alpha}_1$
India	2005	1500	$\hat{\alpha}_1$= 574	Improvement if $\alpha \leq \hat{\alpha}_1$
Indonesia	2005	1500	$\hat{\alpha}_1$= 945	Improvement if $\alpha \leq \hat{\alpha}_1$
Iran	2005	5000	$\hat{\alpha}_1$= 920	Improvement if $\alpha \leq \hat{\alpha}_1$
J, K, L				
Jaimaca	2005	5000	$\hat{\alpha}_1$= 1201	Improvement if $\alpha \leq \hat{\alpha}_1$
Jordan	2005	4000	$\hat{\alpha}_1$= 1200	Improvement if $\alpha \leq \hat{\alpha}_1$
Kazakhstan	1990	9000	$\hat{\alpha}_1$= 6882	Deterioration if $\alpha \leq \hat{\alpha}_1$
kenya	2005	2000	$\hat{\alpha}_1$= 650	Improvement if $\alpha \leq \hat{\alpha}_1$
Kyrgyzstan	2005	3000	$\hat{\alpha}^1$= 444	Deterioration if $\alpha \geq \hat{\alpha}^1$
Lao republic	2005	1000	$\hat{\alpha}_1$= 602	Improvement if $\alpha \leq \hat{\alpha}_1$
Latvia	1990	10000	$\hat{\alpha}_1$= 8684.97	Deterioration if $\alpha \leq \hat{\alpha}_1$
Lesotho	2005	1500	$\hat{\alpha}_1$= 736.29	Improvement if $\alpha \leq \hat{\alpha}_1$
Liberia	2005	1000	$\hat{\alpha}^1$= 259.25	Deterioration if $\alpha \geq \hat{\alpha}^1$
Lithuania	1990	6000	$\hat{\alpha}^1$= 1259.26	Improvement if $\alpha \geq \hat{\alpha}^1$

Country	Larger population	z^+	Estimated bound	Change in welfare
M, N				
Macedonia	2005	6000	?	?
Madagascar	2005	1000	$\hat{\alpha}_1 = 194$	Improvement if $\alpha \leq \hat{\alpha}_1$
Malawi	2005	1000	$\hat{\alpha}_1 = 354$	Improvement if $\alpha \leq \hat{\alpha}_1$
Malaysia	2005	5000	$\hat{\alpha}_1 = 469$	Improvement if $\alpha \leq \hat{\alpha}_1$
Mali	2005	1000	$\hat{\alpha}^1 = 426$	Deterioration if $\alpha \geq \hat{\alpha}^1$
Mauritania	2005	1500	$\hat{\alpha}_1 = 947$	Improvement if $\alpha \leq \hat{\alpha}_1$
Mexico	2005	8000	$\hat{\alpha}_1 = 2703$	Improvement if $\alpha \leq \hat{\alpha}_1$
Moldova Republic	1990	2000	$\hat{\alpha}^1 = 519$	Improvement if $\alpha \geq \hat{\alpha}^1$
Mongolia	2005	2000	$\hat{\alpha}^1 = 504$	Deterioration if $\alpha \geq \hat{\alpha}^1$
Morocco	2005	4000	?	?
Mozambique	2005	1000	$\hat{\alpha}_1 = 375$	Improvement if $\alpha \leq \hat{\alpha}_1$
Namibia	2005	3500	$\hat{\alpha}_1 = 632$	Improvement if $\alpha \leq \hat{\alpha}_1$
Nepal	2005	1000	$\hat{\alpha}_1 = 489$	Improvement if $\alpha \leq \hat{\alpha}_1$
Nicaragua	2005	4000	$\hat{\alpha}_1 = 1057$	Improvement if $\alpha \leq \hat{\alpha}_1$
Niger	2005	1000	?	?
Nigeria	2005	1500	$\hat{\alpha}_1 = 73$	Improvement if $\alpha \leq \hat{\alpha}_1$
P, Q, R				
Pakistan	2005	1500	$\hat{\alpha}_1 = 716$	Improvement if $\alpha \leq \hat{\alpha}_1$
Panama	2005	7000	$\hat{\alpha}_1 = 2030$	Improvement if $\alpha \leq \hat{\alpha}_1$
Papua New Guinea	2005	2000	$\hat{\alpha}_1 = 578$	Improvement if $\alpha \leq \hat{\alpha}_1$
Paraguay	2005	5000	?	?
Peru	2005	5000	?	?
Philippines	2005	2000	$\hat{\alpha}_1 = 213$	Improvement if $\alpha \leq \hat{\alpha}_1$
Poland	2005	7000	$\hat{\alpha}^1 = 537$	Deterioration if $\alpha \geq \hat{\alpha}^1$
Romania	1990	7000	$\hat{\alpha}_1 = 5874$	Deterioration if $\alpha \leq \hat{\alpha}_1$
Russia	1990	4000	?	?
Rwanda	2005	1000	?	?

Country	Larger population	z^+	Estimated bound	Change in welfare
S				
Senegal	2005	1000	$\hat{\alpha}_1$= 612	Improvement if $\alpha \leq \hat{\alpha}_1$
Sierra Leone	2005	1000	$\hat{\alpha}_1$= 491	Improvement if $\alpha \leq \hat{\alpha}_1$
Slovakia	2005	8000	?	?
Slovenia	2005	12000	?	?
South Africa	2005	5000	$\hat{\alpha}_1$= 157	Improvement if $\alpha \leq \hat{\alpha}_1$
Sri Lanka	2005	2000	$\hat{\alpha}_1$= 1793	Improvement if $\alpha \leq \hat{\alpha}_1$
St. Lucia	2005	3000	$\hat{\alpha}_1$= 1385	Improvement if $\alpha \leq \hat{\alpha}_1$
Suriname	2005	5000	$\hat{\alpha}_1$= 1915	Improvement if $\alpha \leq \hat{\alpha}_1$
Swaziland	2005	1000	$\hat{\alpha}_1$= 600	Improvement if $\alpha \leq \hat{\alpha}_1$
T				
Tajikistan	2005	1500	$\hat{\alpha}_1$= 1067	Improvement if $\alpha \leq \hat{\alpha}_1$
Tanzania	2005	1000	?	?
Thailand	2005	4000	$\hat{\alpha}_1$= 2615	Improvement if $\alpha \leq \hat{\alpha}_1$
Timor-Leste	2005	1500	$\hat{\alpha}_1$= 491	Improvement if $\alpha \leq \hat{\alpha}_1$
Togo	2005	1000	$\hat{\alpha}_1$= 202	Improvement if $\alpha \leq \hat{\alpha}_1$
Trinidad and Tobago	2005	9000	$\hat{\alpha}_1$= 8604	Improvement if $\alpha \leq \hat{\alpha}_1$
Tunisia	2005	9000	$\hat{\alpha}_1$= 2473	Improvement if $\alpha \leq \hat{\alpha}_1$
Turkey	2005	6000	$\hat{\alpha}_1$= 2750	Improvement if $\alpha \leq \hat{\alpha}_1$
Turkmenistan	2005	4000	$\hat{\alpha}_1$= 3369	Improvement if $\alpha \leq \hat{\alpha}_1$
U, V, W, X, Y, Z				
Uganda	2005	1000	$\hat{\alpha}_1$= 271	Improvement if $\alpha \leq \hat{\alpha}_1$
Ukraine	1990	5000	$\hat{\alpha}^1$= 741	Improvement if $\alpha \geq \hat{\alpha}^1$
Uruguay	2005	11000	$\hat{\alpha}_1$= 10550	Improvement if $\alpha \leq \hat{\alpha}_1$
Uzbekistan	2005	3000	$\hat{\alpha}^1$= 365	Improvement if $\alpha \geq \hat{\alpha}^1$
Venezuela	2005	5000	$\hat{\alpha}^1$= 1485	Deterioration if $\alpha \geq \hat{\alpha}^1$
Vietnam	2005	3000	$\hat{\alpha}_1$= 1230	Improvement if $\alpha \leq \hat{\alpha}_1$
Yemen	2005	4000	$\hat{\alpha}^1$= 859	Deterioration if $\alpha \geq \hat{\alpha}^1$
Zambia	2005	1500	$\hat{\alpha}_1$= 196	Improvement if $\alpha \leq \hat{\alpha}_1$

6.2 Developing countries not included in PovcalNet data

East Asia and Pacific

American Samoa	Myanmar
Fiji	Palau
Kiribati	Samoa
Korea Democratic Republic	Solomon Islands
Marshall Islands	Tonga
Micronesia Fed.	Vanuatu

Europe and Central Asia

Kosovo	Serbia
Montenegro	

Latin America and the Caribbean

Argentina	Grenada
Belize	St. Kitts and Nevis
Cuba	St. Vincent and the Grenadines

Middle East and North Africa

Iraq	Syrian Arab Republic
Lebanon	West Bank and Gaza
Libya	

South Asia

Afghanistan
Maldives

Sub-Saharan Africa

Eritrea	Seychelles
Mauritius	Somalia
Mayotte	Sudan
Sao Tomé and Principe	Zimbabwe

6.3 Countries with only one survey

Angola, Benin, Bhutan, Chad, Comoros, Congo Democratic Republic, Gabon, Haiti, Papua New Guinea, Namibia, Saint Lucia, Suriname and Togo.

6.4 High-income countries included in the global population counts

Andorra
Antigua and Barbuda
Aruba
Australia
Austria
Bahamas
Bahrain
Barbados
Bermuda
Brunei Darussalam
Belgium
Canada
Cayman Islands
Channel Islands
Cyprus
Denmark
Equatorial Guinea
Faeroe Islands
Finland
France
French Polynesia
Germany
Greece
Greenland
Guam
Hong Kong, China
Iceland
Ireland
Isle of Man
Israel
Italy
Japan
Korea, Rep.
Kuwait
Liechtenstein
Luxembourg
Macao, China
Malta
Monaco
Netherlands
Netherlands Antilles
New Caledonia
New Zealand
Northern Mariana Islands
Norway
Oman
Portugal
Puerto Rico
Qatar
San Marino
Saudi Arabia
Singapore
Spain
Sweden
Switzerland
United Arab Emirates
United Kingdom
United States
Virgin Islands (U.S.)

6.5 Equivalence results (for online publication)

Consider two income distributions $\mathbf{u} \in \mathbb{R}^M$ and $\mathbf{v} \in \mathbb{R}^N$, with $M < N$. Let z^- and z^+ be appropriate lower and upper bounds of the range of poverty lines and let $z \in [z^-, z^+]$. Let $g^z \in \mathcal{F}^s_{z^-,z^+}$ and α be a given value of the critical level.

To demonstrate the equivalence between $\succsim^{sP}_{z^-,z^+}$ and $\succsim^{sW}_{\alpha,z^-,z^+}$, we redefine FGT and CLGU indices in terms of Stieltjes integrals. Consider a real interval $[a, b]$ such that $u_1, ..., u_M, \alpha$, $v_1, ..., v_N \in (a, b)$. Define:

$$D^1_{\mathbf{u}}(t) = \frac{1}{M}\sum_{i=1}^{M} I(u_i \leq t) \qquad D^s_{\mathbf{u}}(t) = \int_a^t D^{s-1}_{\mathbf{u}}(x)\,dx \text{ for } s \geq 2, \tag{25}$$

$$D^1_{\mathbf{v}}(t) = \frac{1}{N}\sum_{i=1}^{N} I(v_i \leq t) \qquad D^s_{\mathbf{v}}(t) = \int_a^t D^{s-1}_{\mathbf{v}}(x)\,dx \text{ for } s \geq 2 \tag{26}$$

and

$$\theta^1(t) = I(\alpha \leq t) \qquad \theta^s(t) = \int_a^t \theta^{s-1}(x)\,dx \text{ for } s \geq 2. \tag{27}$$

Then, $P^s_{\mathbf{u}_\alpha}(z)$ and $P^s_{\mathbf{v}}(z)$ can be rewritten as

$$P^s_{\mathbf{u}_\alpha}(z) = \frac{M}{N}\int_a^z (z-t)^{s-1} dD^1_{\mathbf{u}}(t) + \frac{(N-M)}{N}\int_a^z (z-t)^{s-1} d\theta^1(t) \tag{28}$$

and

$$P^s_{\mathbf{v}}(z) = \int_a^z (z-t)^{s-1} dD^1_{\mathbf{v}}(t)\,. \tag{29}$$

Let $u_{\alpha_i} = \begin{cases} u_i & \text{if } i = 1, ..., M \\ \alpha & \text{if } i = M+1, ..., N. \end{cases}$ Then, $\sum_{i=1}^{N} g^z(u_{\alpha_i}) = \sum_{i=1}^{M} g^z(u_i) + (N-M)\, g^z(\alpha)$. We can also write

$$\sum_{i=1}^{N} g^z(u_{\alpha_i}) = M\int_a^b g^z(t) dD^1_{\mathbf{u}}(t) + (N-M)\int_a^b g^z(t) d\theta^1(t) \tag{30}$$

and

$$\sum_{i=1}^{N} g^z(v_i) = N\int_a^b g^z(t) dD^1_{\mathbf{v}}(t)\,. \tag{31}$$

We wish to compute $P^s_{\mathbf{v}}(z)$. For $s = 1$,

$$P^1_{\mathbf{v}}(z) = \int_a^z dD^1_{\mathbf{v}}(t) = D^1_{\mathbf{v}}(z) - D^1_{\mathbf{v}}(a)\,.$$

Since by definition, $D^1_{\mathbf{v}}(a) = 0$, we have $P^1_{\mathbf{v}}(z) = D^1_{\mathbf{v}}(z)$. For any integer $s \geq 2$, and using integration by parts, we have:

$$\begin{aligned} P^s_{\mathbf{v}}(z) &= \int_a^z (z-t)^{s-1} dD^1_{\mathbf{v}}(t) \\ &= \left[(z-t)^{s-1} D^1_{\mathbf{v}}(t)\right]_a^z + (s-1)\int_a^z (z-t)^{s-2} D^1_{\mathbf{v}}(t)\,dt \\ &= (s-1)\int_a^z (z-t)^{s-2} D^1_{\mathbf{v}}(t)\,dt. \end{aligned}$$

Applying again $(s-2)$ times integration by parts, we obtain

$$P_{\mathbf{v}}^{s}(z) \;=\; (s-1)!\int_{a}^{z} D_{\mathbf{v}}^{s-1}(t)\,dt = (s-1)!\left[D_{\mathbf{v}}^{s}(z) - D_{\mathbf{v}}^{s}(a)\right].$$

By definition, $D_{\mathbf{v}}^{s}(a) = 0$ and thus $P_{\mathbf{v}}^{s}(z) = (s-1)!D_{\mathbf{v}}^{s}(z)$. Doing the same for $P_{\mathbf{u}_\alpha}^{s}(z)$, we have that

$$P_{\mathbf{u}_\alpha}^{s}(z) = (s-1)!\left[\frac{M}{N}D_{\mathbf{u}}^{s}(z) + \frac{(N-M)}{N}\theta^{s}(z)\right].$$

Therefore, we find

$$P_{\mathbf{u}_\alpha}^{s}(z) - P_{\mathbf{v}}^{s}(z) = (s-1)!\left[\frac{M}{N}D_{\mathbf{u}}^{s}(z) + \frac{(N-M)}{N}\theta^{s}(z) - D_{\mathbf{v}}^{s}(z)\right]. \tag{32}$$

Hence for $s \geq 1$,

$$P_{\mathbf{u}_\alpha}^{s}(z) - P_{\mathbf{v}}^{s}(z) \leq 0, \forall z^{-} \leq z \leq z^{+} \Leftrightarrow \frac{M}{N}D_{\mathbf{u}}^{s}(z) + \frac{(N-M)}{N}\theta^{s}(z) - D_{\mathbf{v}}^{s}(z) \leq 0\,\forall z^{-} \leq z \leq z^{+}.$$

Because $\mathbf{u}_\alpha \succsim_{z^-,z^+}^{sP} \mathbf{v} \Leftrightarrow P_{\mathbf{u}_\alpha}^{s}(z) - P_{\mathbf{v}}^{s}(z) \leq 0, \forall\, z^{-} \leq z \leq z^{+}$, then

$$\mathbf{u}_\alpha \succsim_{z^-,z^+}^{sP} \mathbf{v} \Leftrightarrow \frac{M}{N}D_{\mathbf{u}}^{s}(z) + \frac{(N-M)}{N}\theta^{s}(z) - D_{\mathbf{v}}^{s}(z) \leq 0\,\forall z^{-} \leq z \leq z^{+}. \tag{33}$$

For $s \geq 1$, we also apply successive integration by parts (s times) on $\int_a^b g^z(t)dD_{\mathbf{u}}^{1}(t)$. We obtain that

$$\int_{a}^{b} g^{z}(t)dD_{\mathbf{u}}^{1}(t) \;=\; \sum_{k=1}^{s}\left[(-1)^{k-1}g^{z^{(k-1)}}(t)\,D_{\mathbf{u}}^{k}(t)\right]_{a}^{b} + (-1)^{s}\int_{a}^{b} g^{z^{(s)}}(t)\,D_{\mathbf{u}}^{s}(t)\,dt,$$

where $g^{z^{(0)}}(t) = g^{z}(t)$ and $g^{z^{(k)}}(t) = \frac{d^k g^z(t)}{dt^k}$ for $k \geq 1$. We do the same for $\int_a^b g^z(t)dD_{\mathbf{v}}^{1}(t)$. Thus,

$$\begin{aligned} &M\int_{a}^{b} g^{z}(t)dD_{\mathbf{u}}^{1}(t) + (N-M)\int_{a}^{b} g^{z}(t)d\theta^{1}(t) \geq N\int_{a}^{b} g^{z}(t)dD_{\mathbf{v}}^{1}(t) \;\Leftrightarrow \\ &\sum_{k=1}^{s}\left[(-1)^{k-1}g^{z^{(k-1)}}(t)\left[MD_{\mathbf{u}}^{k}(t) + (N-M)\theta^{k}(t) - ND_{\mathbf{v}}^{k}(t)\right]\right]_{a}^{b} \\ &+ (-1)^{s}\int_a^b g^{z^{(s)}}(t)\left[MD_{\mathbf{u}}^{s}(t) + (N-M)\theta^{s}(t) - ND_{\mathbf{v}}^{s}(t)\right]dt \geq 0\,. \end{aligned} \tag{34}$$

Because of $D_{\mathbf{u}}^{1}(b) = \theta^{1}(b) = D_{\mathbf{v}}^{1}(b) = 1$ and $D_{\mathbf{u}}^{k}(a) = \theta^{k}(a) = D_{\mathbf{v}}^{k}(a) = 0$ for all $k \leq s$, the above inequality reduces to

$$\begin{aligned} &\sum_{k=2}^{s}(-1)^{k-1}g^{z^{(k-1)}}(b)\left[MD_{\mathbf{u}}^{k}(b) + (N-M)\theta^{k}(b) - ND_{\mathbf{v}}^{k}(b)\right] \\ &+ (-1)^{s}\int_a^b g^{z^{(s-1)}}(t)\left[MD_{\mathbf{u}}^{s}(t) + (N-M)\theta^{s}(t) - ND_{\mathbf{v}}^{s}(t)\right]dt \;\geq 0\,. \end{aligned} \tag{35}$$

This expression can be rewritten as

$$\begin{array}{l}\sum_{k=2}^{s}(-1)^{k-1}g^{z(k-1)}(b)\left[MD_{\mathbf{u}}^{k}(b)+(N-M)\theta^{k}(b)-ND_{\mathbf{v}}^{k}(b)\right]\\ +(-1)^{s}\int_{a}^{z}g^{z(s)}(t)\left[MD_{\mathbf{u}}^{s}(t)+(N-M)\theta^{s}(t)-ND_{\mathbf{v}}^{s}(t)\right]dt\\ +\int_{z}^{b}(-1)^{s}g^{z(s)}(t)\left[MD_{\mathbf{u}}^{s}(t)+(N-M)\theta^{s}(t)-ND_{\mathbf{v}}^{s}(t)\right]dt\geq 0\,.\end{array}\tag{36}$$

Given the second line of (8), we have that $g^{z}(t)=g^{z}(z)$ for all $t\geq z$. Hence, $g^{z(k-1)}(t)=0$ for all $t\geq z$ and for all $k\geq 2$. Note that the first and third terms of the above expression vanish. Hence,

$$\begin{array}{l}M\int_{a}^{b}g^{z}(t)dD_{\mathbf{u}}^{1}(t)+(N-M)\int_{a}^{b}g^{z}(t)d\theta^{1}(t)\geq N\int_{a}^{b}g^{z}(t)dD_{\mathbf{v}}^{1}(t)\\ \Leftrightarrow(-1)^{s}\int_{a}^{z}g^{z(s)}(t)\left[MD_{\mathbf{u}}^{s}(t)+(N-M)\theta^{s}(t)-ND_{\mathbf{v}}^{s}(t)\right]dt\geq 0\,.\end{array}$$

Consequently, given the assumption that $(-1)^{s}g^{z(s)}(t)\leq 0\ \forall t\in(z^{-},z^{+})$,

$$M\int_{a}^{b}g^{z}(t)dD_{\mathbf{u}}^{1}(t)+(N-M)\int_{a}^{b}g^{z}(t)d\theta^{1}(t)\geq N\int_{a}^{b}g^{z}(t)dD_{\mathbf{v}}^{1}(t)$$

is equivalent to

$$MD_{\mathbf{u}}^{s}(t)+(N-M)\theta^{s}(t)-ND_{\mathbf{v}}^{s}(t)\leq 0\ \forall z^{-}\leq t\leq z^{+},$$

and this can be rewritten under the form

$$\frac{M}{N}D_{\mathbf{u}}^{s}(t)+\frac{(N-M)}{N}\theta^{s}(t)-D_{\mathbf{v}}^{s}(t)\leq 0\ \forall z^{-}\leq t\leq z^{+}.$$

Because

$$\mathbf{u}\succsim_{\alpha,z^{-},z^{+}}^{sW}\mathbf{v}\Leftrightarrow M\int_{a}^{b}g^{z}(t)dD_{\mathbf{u}}^{1}(t)+(N-M)\int_{a}^{b}g^{z}(t)d\theta^{1}(t)\geq N\int_{a}^{b}g^{z}(t)dD_{\mathbf{v}}^{1}(t)\quad\forall z^{-}\leq t\leq z^{+},$$

we have

$$\mathbf{u}\succsim_{\alpha,z^{-},z^{+}}^{sW}\mathbf{v}\Leftrightarrow\frac{M}{N}D_{\mathbf{u}}^{s}(t)+\frac{(N-M)}{N}\theta^{s}(t)-D_{\mathbf{v}}^{s}(t)\leq 0\ \forall z^{-}\leq t\leq z^{+}.\tag{37}$$

Given (33) and (37), we can conclude that

$$\mathbf{u}\succsim_{\alpha,z^{-},z^{+}}^{sW}\mathbf{v}\Leftrightarrow\mathbf{u}_{\alpha}\succsim_{z^{-},z^{+}}^{sP}\mathbf{v}.\tag{38}$$

It is straightforward to verify that once a lower-order CLGU dominance ranking between two distributions is established, higher-order dominance between these two distributions also holds up for a given value of the critical level. Indeed, for any $s_2 > s_1 \geq 1$,

$$\mathbf{u}_\alpha \succsim^{s_1 P}_{z^-,z^+} \mathbf{v} \Rightarrow \mathbf{u}_\alpha \succsim^{s_2 P}_{z^-,z^+} \mathbf{v}$$

and thus

$$\mathbf{u} \succsim^{s_1 W}_{\alpha,z^-,z^+} \mathbf{v} \Rightarrow \mathbf{u} \succsim^{s_2 W}_{\alpha,z^-,z^+} \mathbf{v}.$$

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