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UDNP's Multidimensional poverty index

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Abstract

The measurement of well-being in its multidimensional aspects has become ever more prominent in monitoring development over the last three decades. This note discusses some of the properties of a recent multidimensional index proposed by the UNDP. Unlike most of its predecessors, UNDP's Multidimensional Poverty Index (MPI) succeeds in taking into account both the distribution of deprivation within a number of dimensions as well as the distribution of multiple deprivations across the dimensions. This is important since there are good reasons to think of multidimensional poverty as more than just the sum of poverty across various dimensions of well-being. The MPI suffers, however, from a few unattractive features that need to be better understood (given the recently acquired prominence of the index). The object of this note is to highlight some of them.

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1 Introduction

The measurement and the analysis of multidimensional well-being and poverty have become ever more important in understanding development over the last three decades. Several prominent contributions have been made in that regard. One of the most influential has been the UNDP 's human development index (HDI) (see for instance UNDP 1990-2010, which has popularized the use of three dimensions, life expectancy, educational attainment and living standards, in computing composite indices of development.

It is well known, however, that the HDI fails to account for the *distribution* of individual well-being within each of the HDI's dimensions. It cannot therefore capture inequalities in the distributions of life expectancy, educational attainment and income within societies, and it is therefore difficult to use it to understand poverty, which is a function of the distribution of well-being. This is not the case, however, of the Human Poverty Index (HPI), which was introduced by UNDP in 1997 and which uses indicators of *deprivation* in the same dimensions of longevity, basic education and standards of living.

The HPI fails to account, however, for the distribution and the role of multiple deprivations across the dimensions. This is because the contributions to the HPI of individual deprivations in each of the dimensions is assessed independently of the individual deprivations in the other dimensions. Hence, the HPI index cannot tell whether in some societies the poor suffer more from multiple deprivations than in other societies — where multidimensional poverty could be more equally spread across the population.

This is important since there are good reasons to believe that multidimensional poverty is more than just the sum of dimensional poverty. The interaction between dimensions is important for both normative and positive reasons. From a normative perspective, a society with a greater extent of multiple deprivation would be judged by most analysts as worse, everything else being the same. From a positive perspective, a conjunction of deprivations in multiple dimensions would be expected to cause greater individual and social harm, both in the short term and in the longer term, than if these deprivations were more evenly spread across individuals.

The UNDP has recently implemented a measure that tries to remedy to those shortcomings of the HPI. The remedy takes the form of a multidimensional poverty index (MPI). The MPI aggregates the deprivation of those that are multiply deprived in areas such as education, health outcomes and assets and services. By focusing on multiply deprived individuals, it gives more importance to poverty in multiple dimensions than the HDI, which is neutral to the existence of multiple deprivation.

The MPI is certainly a welcome contribution to the measurement of multidimensional poverty. Although the contribution is to be congratulated, the MPI does suffer from a few unattractive features that have not yet been sufficiently noted in the literature. Given these features, poverty analysts might want to be careful in their use of the MPI, and should contemplate seriously the use of dominance and other techniques to check the reliability of findings derived from the use of the MPI. This note explains why.

2 Difficulties with UNDP's MPI

The difficulties introduced by the MPI are of two sorts. The first sort comes from the discretization that the MPI makes of poverty in the various dimensions. This creates discontinuities in the measurement of poverty that may penalize policies and development processes whose effect is otherwise to equalize well-being across individuals. These discontinuity problems are well understood in the unidimensional literature. It is for instance well-known that the popular (discontinuous) headcount index can *increase* following a redistribution policy that redistributes resources from richer to poorer individuals. This problem is exacerbated in a multidimensional context in which such discontinuities are introduced across several dimensions.

The second sort of problems comes from the fact that an second type of discontinuity is introduced by the MPI. This second discontinuity arises from the introduction of an additional "poverty lines" (in addition to the usual dimensional poverty lines) in the construction of the MPI. This second type of poverty lines (which we denote ζ below) serves to identify those individuals that are deemed to be multiply deprived. ζ sets the number of poverty dimensions that must be equaled or surpassed for individuals to qualify as multiply deprived under the MPI. Whenever a slight change in someone's well-being changes the number of dimensions in which that person is deprived, there is a risk that the person moves suddenly into or out of the set of individuals that are considered multiply deprived. This movement then introduces measurement discontinuities that may again penalize development processes that have the feature of equalizing well-being. Such features also have the effect that a society that seeks to alleviate multiple deprivation may, because of this, see its MPI *increase* over time.

The first sort of problems can be alleviated through the use of less dichotomous (and perhaps more precise) measures of well-being in the various dimensions of interest. For those dimensional indicators that have cardinal value, it can also be useful to replace "counting" poverty measures by more continuous ones. Unfortunately, the second sort of problems cannot generally be avoided with poverty indices of the MPI type, unless ζ is set to 1. Setting ζ to 1, however, makes the MPI an exclusively "union" index (we discuss what this means below) — a feature that would not be desirable since that would prevent the MPI from focusing on those that are multiply deprived, a feature that differentiates it from the HPI.

3 Measuring multidimensional poverty

The construction of multidimensional poverty indices involves a number of different steps. First, the dimensions of interest must be chosen, an exercise which is fraught with difficulties. Second, one must decide whether aggregate dimensional indicators must first be computed (as in the HDI/HPI case), or whether aggregate individual indicators must first be assessed (as in the MPI case). If one wishes to discriminate in the measurement exercise between multiple deprivations for a single individual and single deprivation across multiple individuals, then one must necessarily aggregate indicators at the individual level first.

When constructing composite indicators of poverty at the individual level, it must be decided whether separate poverty lines should be applied to each dimension individually, or whether a single poverty line should be applied to an aggregate individual well-being indicator. The procedure to compute the MPI uses the first option, which means that a number of separate property lines must be specified. The framework of Duclos, Sahn, and Younger (2006) (for instance) follows the second option and allows explicitly for the possibility of substitution across dimensions without the need to identify each level of dimensional poverty separately. This can be useful since choosing values for poverty lines (especially in a multi-dimensional context) is an exercise that is almost always subject to arbitrariness.

Once individual poverty levels have been computed, their aggregation across individuals (which raises questions of whether poverty measurement should be sensitive to inequality across individuals) introduces another source of arbitrariness. The MPI does not explicitly penalize inequality in deprivation across individuals: as we will see below, the poverty contribution of an individual to total poverty is linear in the sum of that individual's deprivation across dimensions.

The issue of whether a dimension can compensate another dimension in producing an overall degree of well-being can be illustrated using Figure 1, which focuses on the case of two dimensions (for expositional simplicity). Figure 1 shows two dimensions of well-being, x_1 and x_2, and those combinations of x_1 and x_2 that produce the same poverty level of overall well-being (those combinations appear on the lines denoted by $\pi(\mathbf{x}) = 0$). Those three lines in Figure 1 illustrate three different general ways of thinking about multidimensional poverty. The first ($\pi_1(\mathbf{x}) = 0$) is the intersection view. Under that first view, someone is poor if and only if he is poor in both dimensions. There is then "more than perfect" substitution of one dimension for another. As soon as the value of well-being in one dimension exceeds the dimensional poverty line, the person is judged overall to be above poverty. The second line ($\pi_2(\mathbf{x}) = 0$) illustrates the union view. This assumes a complete lack of substitution between the dimensions. However large the value of one dimension may be, the person will be considered to be in poverty if he is poor in at least one dimension. The third view (with $\pi_2(\mathbf{x}) = 0$) allows for an intermediate degree of substitution, and is more in line with the traditional way in which economists think about preferences and well-being. Note that having to choose between union, intersection, or intermediate views of multidimensional poverty introduces one more source of arbitrariness.

The two most popular and simplest measures of multidimensional poverty are called the union and the intersection headcount indices. Let again ζ be the minimal number of dimensions in which someone needs to be deprived to be considered as multidimensionally poor. Let $H(\zeta)$ be the multidimensional headcount, and let F_{x_1,x_2} be the bidimensional distribution function. $F_{x_1,x_2} z_1, z_2$ gives the proportion of the population whose levels of dimensional well-being are below both z_1 and z_2. The union form of the headcount is given by

$$H(1) = F_{x_1,x_2}(z_1;\infty) + F_{x_1,x_2}(\infty;z_2) - F_{x_1,x_2}(z_1;z_2), \tag{1}$$

and the intersection form by

$$H(2) = F_{x_1,x_2}(z_1;z_2). \tag{2}$$

$H(1)$ is the proportion of people who are poor in at least one dimension. On Figure 1, this is given by the proportion of those that are to the left of or below $\pi_2(\mathbf{x})$. $H(2)$ is the proportion of people who are poor in the two dimensions. On Figure 1, this is given by the proportion of those that are to the left of or below $\pi_1(\mathbf{x})$.

The MPI index is similarly defined. It also uses ζ to identify those that are multidimensionally poor. Unlike the traditional headcount index, which measures the total number of multidimensionally poor people as a proportion of the total number of people, MPI measures the total number of dimensions in which the multidimensionally poor are poor, as a proportion of the total number of dimensions for which well-being is measured (this is the number of people times the number of dimensions). Let MPI thus be denoted by $M(\zeta)$. It equals:

$$M(1) = .5\left(F_{x_1,x_2}(z_1;\infty) + F_{x_1,x_2}(\infty;z_2)\right) \tag{3}$$

and

$$M(2) = F_{x_1,x_2}(z_1;z_2). \tag{4}$$

Note that the intersection headcount $H(2)$ and $M(2)$ are the same.

Figure 2, 3 and 4 are useful in understanding the distinction between the traditional multidimensional headcount indices and the new MPI. The numbers in the figures show the contribution to total poverty of individuals with different values of x_1 and x_2. Figure 2 indicates that, for the intersection headcount index, only those that are in the lower rectangle count for total poverty. Figure 3 shows why the union headcount is different from the union MPI. The union headcount counts people in poverty; the union MPI counts the poverty dimensions of those in poverty, which, because of the union definition, is proportionately smaller than the people count. Hence, those poor individuals that are poor in fewer dimensions contribute less to MPI poverty than to the union headcount.

We can now understand better how the MPI reacts to changes in levels of well-being. This can be done using changes in well-being involving a single individual, or changes involving several. The way in which poverty measures are expected to react following changes in well-being is traditionally discussed in welfare economics through the use of axioms. Three such axioms of relevant in our current discussion.

The first axiom says that poverty measures should not too sensitive to small changes in individual measures of well-being. This is a continuity axiom that, *inter alia*, protects poverty measurement against the effect of empirical measurement errors and against overly sensitive reactions to small variations in poverty lines and in definitions of measures of well-being. The second axiom says that equalizing well-being across individuals should not increase poverty. The third axiom is designed specifically for multidimensional poverty measurement. It says that reducing the incidence of multiple deprivation, without changing the incidence of dimensional deprivation, should reduce our measure of multidimensional poverty. We look at three axioms in turn.

3.1 Continuity

It is clear from Figures 2 and 3 that headcount-type multidimensional indices cannot obey the continuity axiom. This is because a small change in dimensional well-being or in the dimensional poverty lines can change from 1 to 0, or from 0 to 1, the contribution of any individual to total poverty. When that happens depends on the value set for ζ.

Figure 4 also shows how the union MPI introduces further instances of such discontinuities. This is because this form of MPI can jump whenever the number of poor dimensions that someone experiences changes, even though the person may still be considered to be a multidimensional poor. Although the size of the jumps is quantitatively less important than for the traditional headcount indices (0.5 as opposed to 1 in our Figures), they occur more often with the union MPI than with the union headcount.

To avoid such sensitivity for the (union) $M(1)$, it would be necessary to use a poverty valuation function that is continuous in dimensional well-being. This is done for instance in Alkire and Foster (2011), which presents indices that are a generalization of the MPI. To avoid such sensitivity for the $H(2)$ and $M(2)$ types of indices is not possible, however, because of the role that the ζ parameter plays. All multidimensional poverty indices that are of the intersection type will indeed necessarily jump whenever the number of poverty dimensions of a particular individual moves up or down that ζ parameter.

3.2 Inequality

Figures 5 and 6 show why the MPI can increase following policies or distributional changes that decrease inequality, within the poor or between the poor and the non-poor. Consider first Figure 5, which shows a transfer from (non-poor) person b to (poor) person a. The usual union headcount index increases whenever the richer person falls into poverty following such a transfer. This is a well-known shortcoming of unidimensional headcounts, which naturally extends to the multidimensional headcount. The union MPI ($M(1)$) also displays this property, as shown in Figure 6. Whether this is a serious shortcoming depends on whether equalizing well-being should be considered also to alleviate poverty — which would seem to be a reasonable view.

Figure 6 shows that the union MPI ($M(1)$) has an additional shortcoming, which arises when a transfer is made from a less poor individual to a poorer individual. This happens on Figure 6 when individual d becomes poor in dimension 2 when a transfer is made from him is made to individual c. This is a feature that does not arise with the traditional headcount indices. All in all, Figure 6 shows how the discontinuity of the MPI creates difficulties in reconciling inequality reduction with poverty alleviation, not only when that inequality reduction affects the non-poor and the poor, but also when the effects are limited to the poor.

Figure 7 demonstrates another possible shortcoming of the MPI. In that Figure, individuals c and d are brought closer together through an equalizing transfer. Both individuals, however, see their individual levels of poverty increase, and total poverty therefore also increases.

3.3 Multiple deprivation

As mentioned above, the third axiom is particularly designed for the measurement of multidimensional poverty. To understand it better, consider Figure 8. Figure 8 shows the effect of two transfers. The first transfer moves individual a to position c; the second transfer moves individual d to position b. This is called in the literature a correlation decreasing switch; note that unidimensional poverty is not affected in either dimension. The effect of this switch is generally supposed to decrease total poverty. The reason is that, although poverty in each dimension separately is unaffected, the importance of multiple deprivation has fallen, and multidimensional inequality must therefore have fallen in some general sense.

Figure 9 shows why this is not the case for the MPI using a case of two individuals, three dimensions and $\zeta = 2$. It is assumed that both individuals are poor in the third dimension, with $x_3 < z_3$. The first individual moves from position a to position c, and the second individual goes from position d to position b. Although the correlation of dimensional well-being has fallen across individuals, MPI poverty has increased. This is in fact also true of other discontinuous union indices, such as the usual union headcount index (see Figure 5).

Figure 10 illustrates that the problem can be worse. It now assumes that the first individual is also poor in the third dimension, whereas the second one is not. Hence, the correlation-decreasing switch shown by the arrows benefits clearly the poorest of the two individuals, in addition to decreasing the inequality that exists between them. Here again, however, MPI increases. It would increase even if c were pushed further up.

4 What can be done?

Given the above, a natural question that can be asked is whether we can think of other indices, or modifications of the MPI indices, that would perform better than UNDP's current MPI. As we saw, the problem with the current MPI comes from the dual sources of discontinuity that it introduces, first with respect to the value of the dimensions, second with respect to the cut-off number of dimensions that serve to identify the multidimensional poor. As mentioned above, the first source can be corrected by using the more general MPI proposed by Alkire and Foster (2011). The second source is, unfortunately, a general feature of the methodology that lies behind those general indices.

Other forms of multidimensional poverty indices can be more robust to such shortcomings. The choice of any particular multidimensional poverty index is bound, however, to be arbitrary, because it will impose a particular choice of poverty line(s), a particular choice of indicator-aggregating procedures, and a particular choice of individual-aggregating procedures. All such choices can be contested, which can then undermine the reliability of the findings (including the policy guidance) obtained through the use of such particular poverty indices. And the importance of that problem generally grows with the number of dimensions considered.

Given this, it would seem important to verify that poverty assessments, poverty profiles, and poverty policies are not inadvertently distorted by the possible shortcomings of the MPI or of other multidimensional indices. One effective manner to guard against such distortions is through dominance testing. These have been around for some time in the unidimensional poverty literature; see for instance Atkinson and Bourguignon (1987) and Foster and Shorrocks (1988). They are also now available for multidimensional poverty (Duclos, Sahn, and Younger 2006. Such dominance tests are also easily applied using readily available software (such as DASP, Araar and Duclos 2007). They are also easily understood, since they consist in comparing simple intersection headcounts — the $H(1)$ — not at specific poverty lines, but over ranges of them.

The multidimensional poverty dominance tests do impose conditions on the properties of multidimensional poverty measurement, but these properties are made explicit and are designed to be as widely acceptable as possible. First-order dominance tests (which are the most robust ones) suppose, for instance, that multidimensional poverty indices should decrease with dimensional well-being and with correlation-decreasing switches — which would seem to be reasonable properties in most applied cases. First-order dominance tests do not impose any assumption of cardinality of the dimensional indicators of well-being, which again would seem warranted in many cases. They also suppose that poverty indices should be continuous; this, however, and as we discussed, is a property that would seem *necessary* in order to be able to make robust poverty judgments. They do not require having to choose between union, intersection, or intermediate views of multidimensional poverty: all such views are allowed. They do not force the choice of a particular form of indices: the comparisons are valid for classes of poverty indices. They do not force either the choice of particular values for poverty lines: the comparisons are valid for ranges of such lines. Such dominance procedures would therefore seem to be useful in complementing the information provided by particular multidimensional poverty indices, such as UNDP's MPI.

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Figure 1: Union, intersection and intermediate poverty measurements

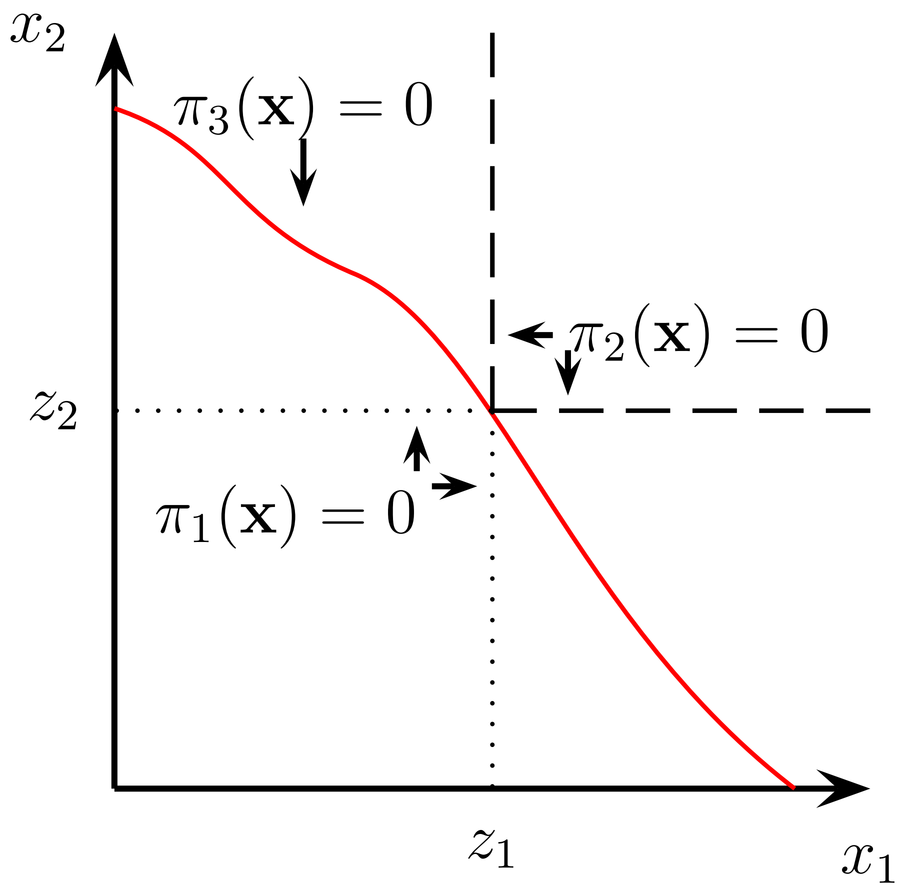


Figure 2: Traditional union headcount: $H(1)$

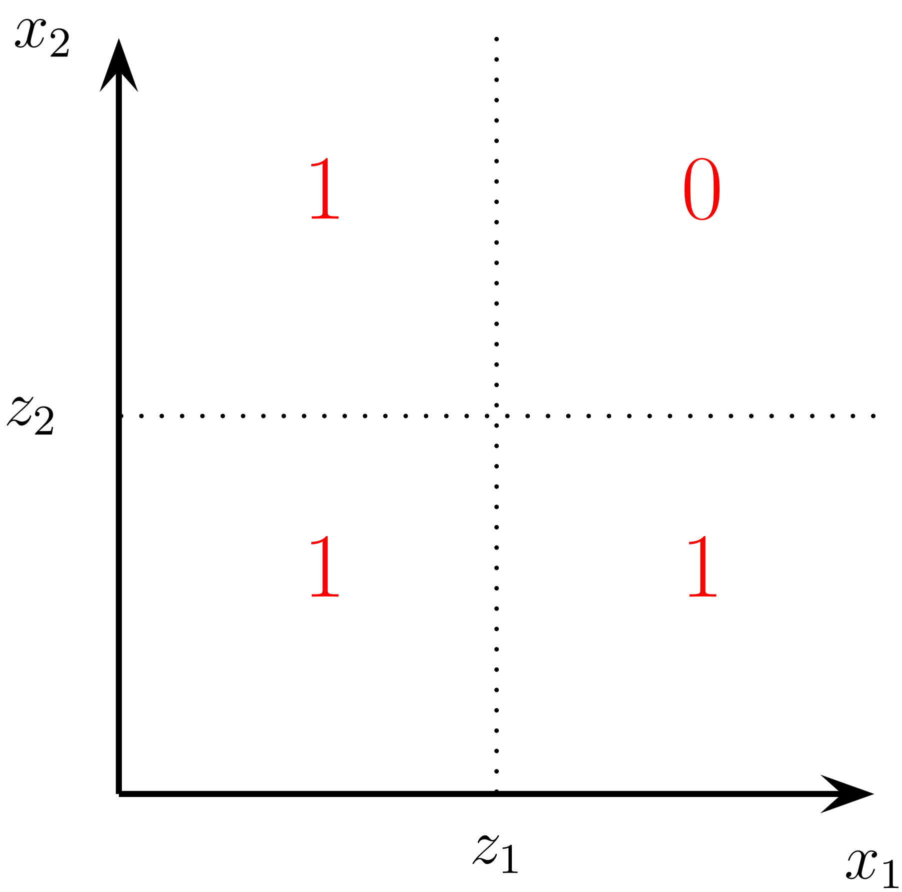


Figure 3: Intersection headcount: $H(2) = M(2)$

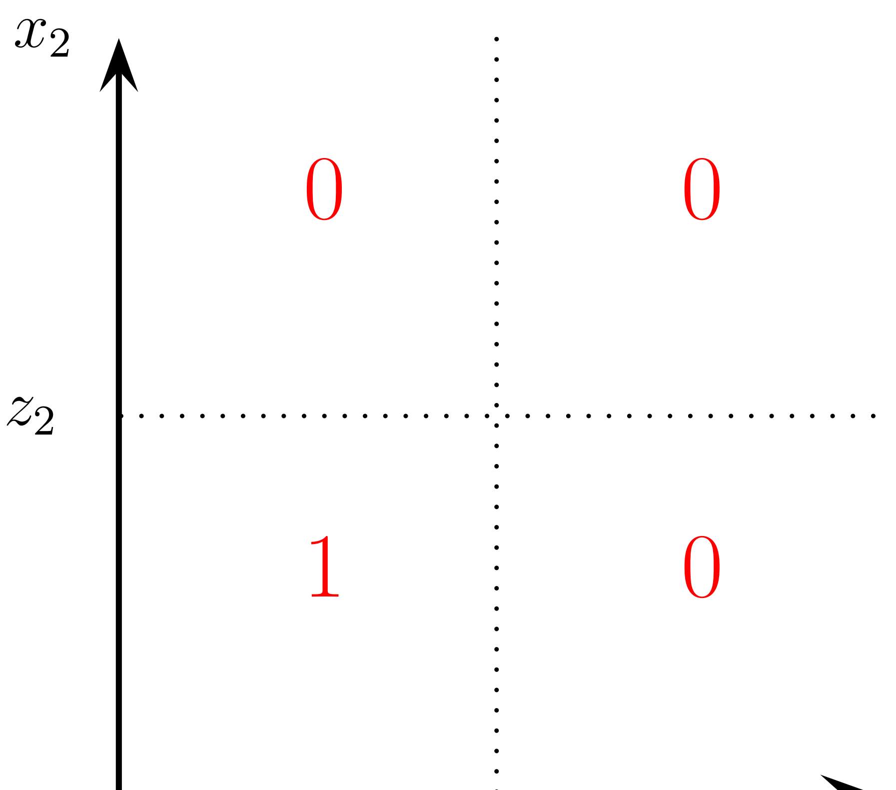


Figure 4: Union MPI: $M(1)$

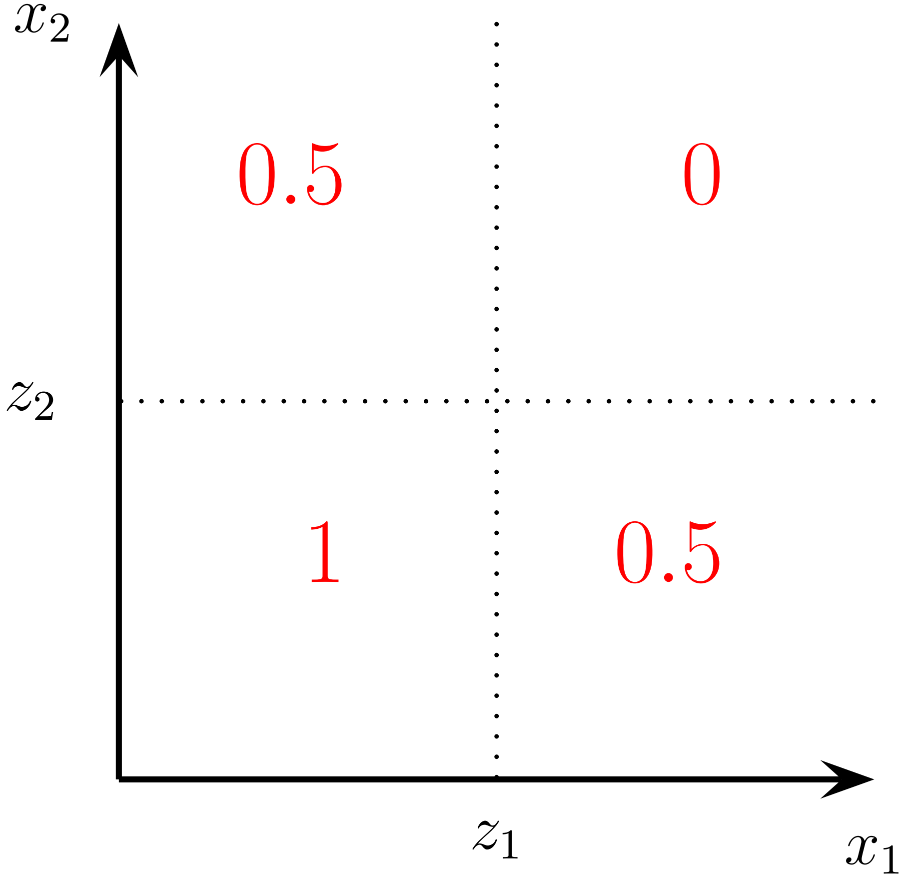


Figure 5: A rich to poor transfer increases the union headcount ($H(1)$)

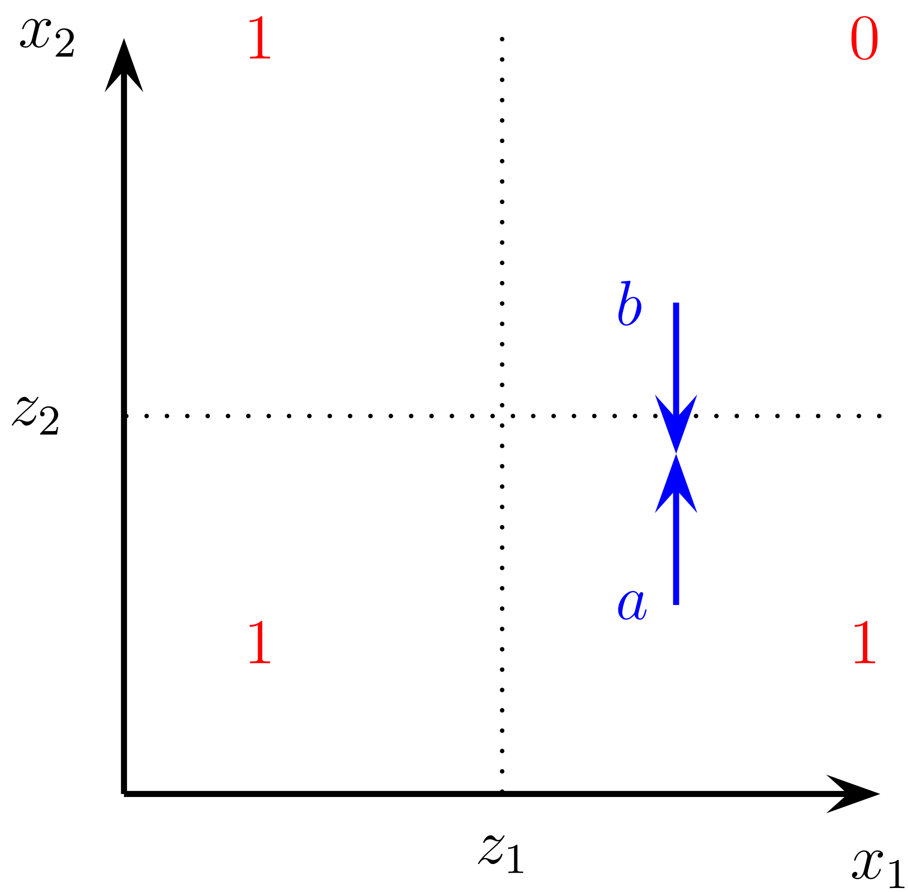


Figure 6: Two rich-to-poor transfers increase the MPI ($M(1)$)

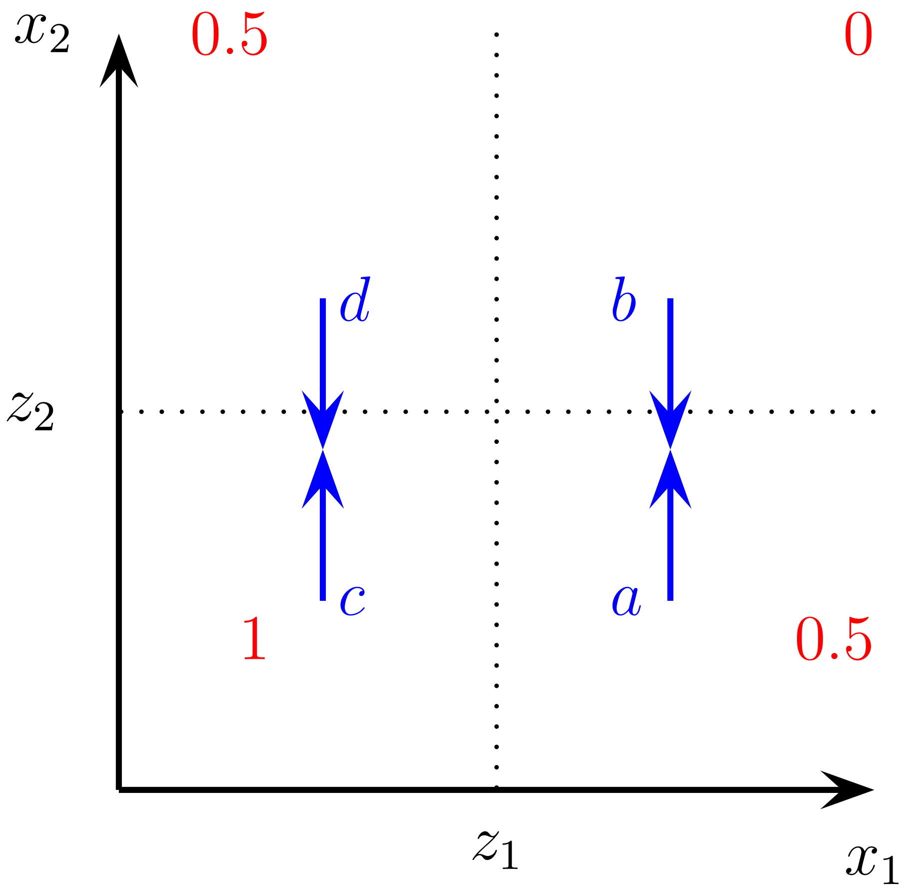


Figure 7: Greater equality increases the MPI ($M(1)$)

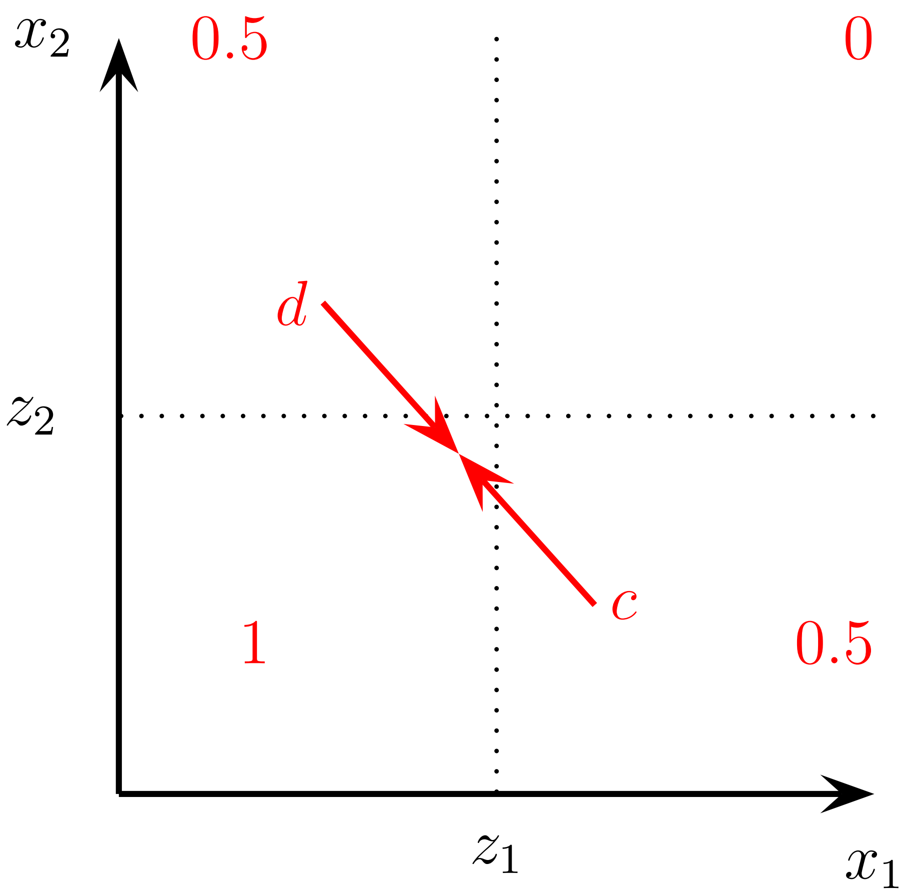


Figure 8: A decrease in correlation — and a fall in multidimensional poverty?

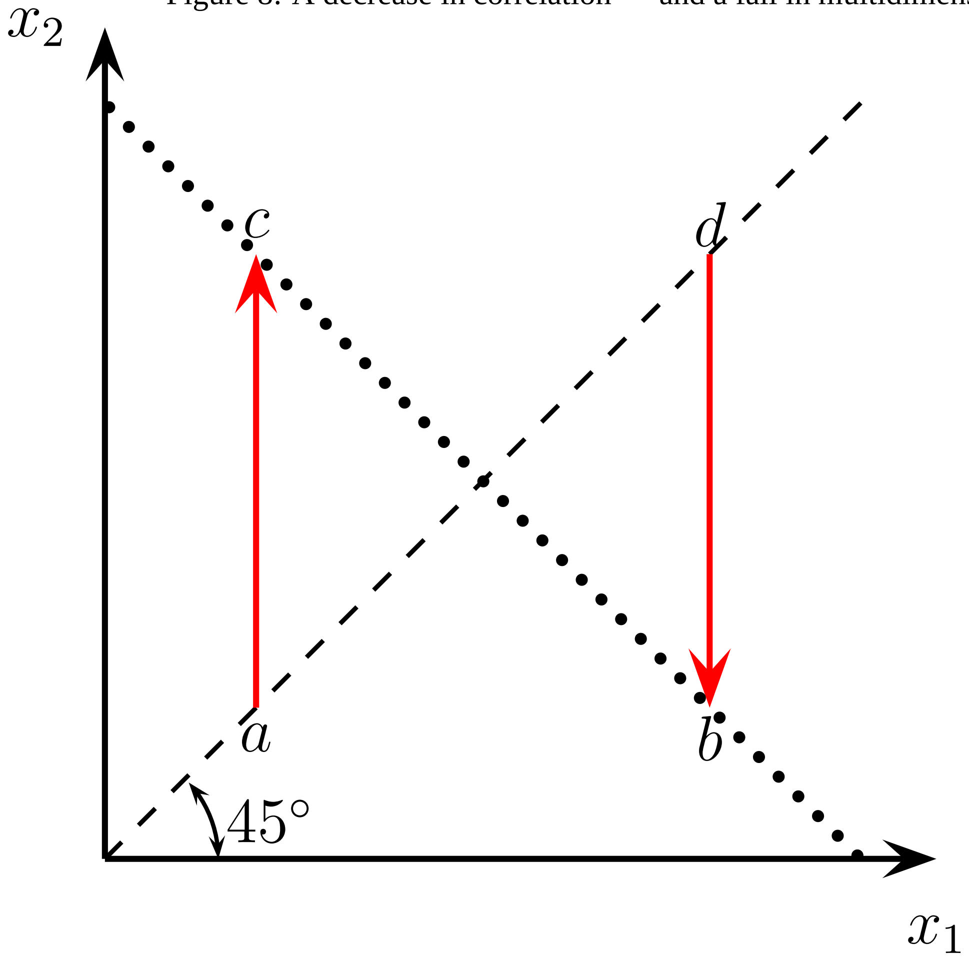


Figure 9: Decreasing correlation increases the MPI ($M(2)$, with 3 dimensions and $x_3 < z_3$)

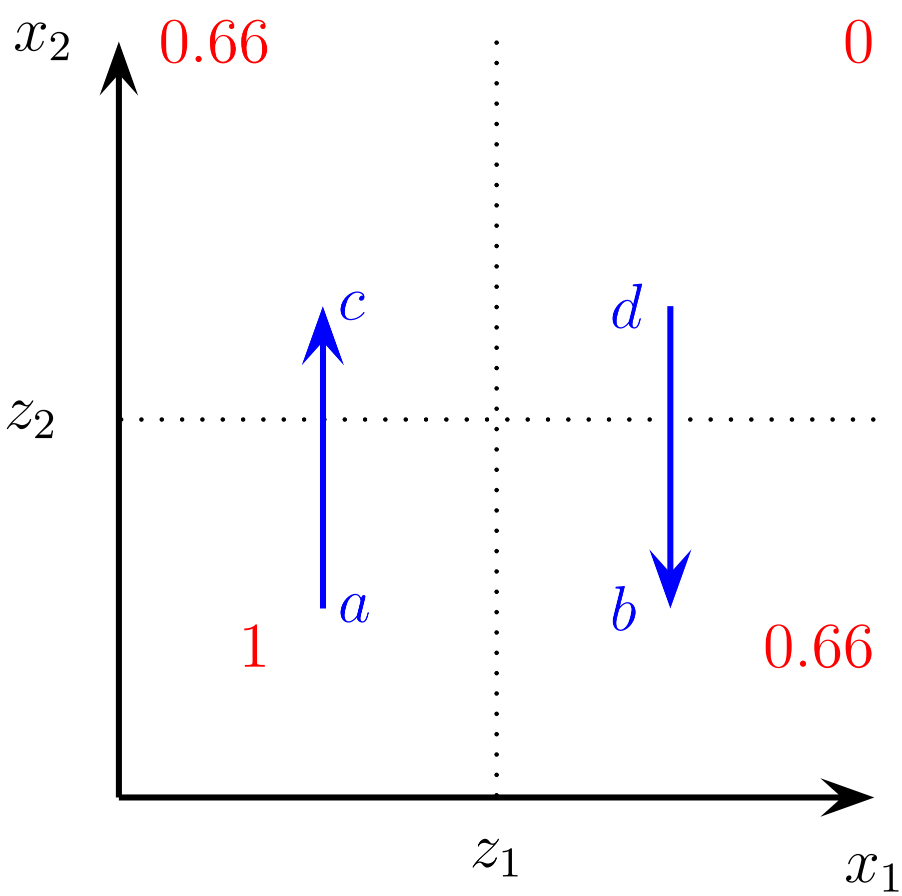
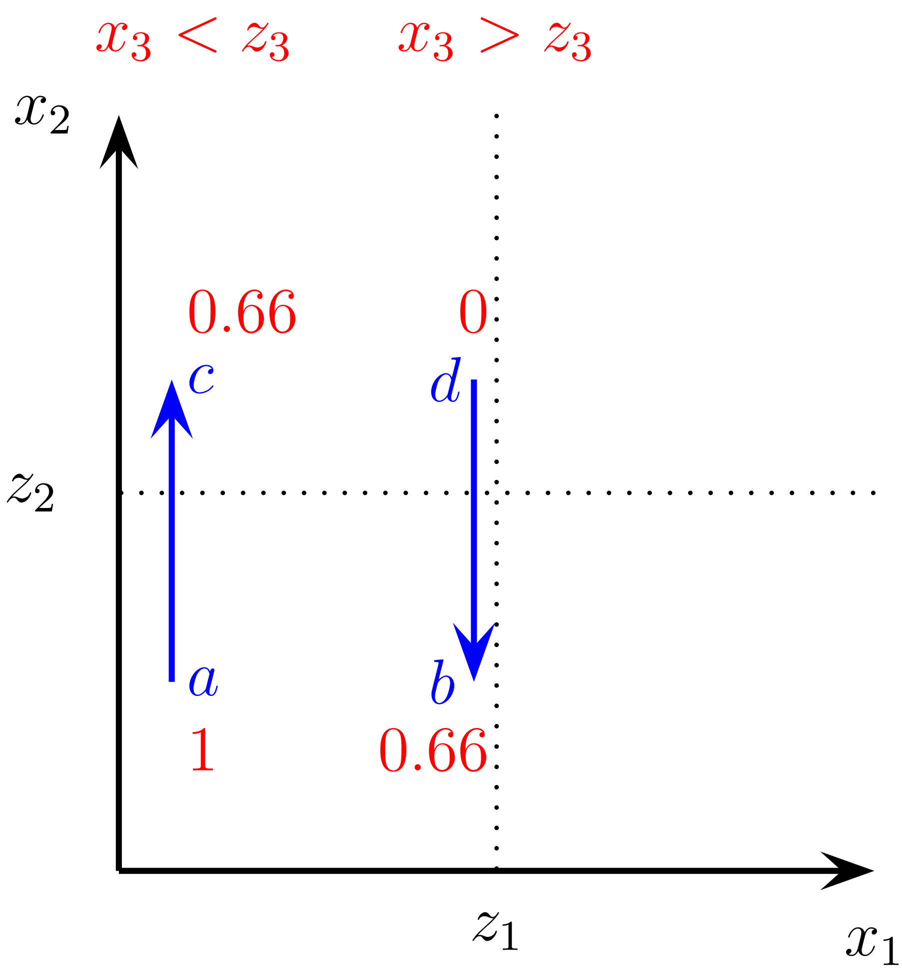


Figure 10: Decreasing correlation and helping the poorest ($x_3 < z_3$) increases poverty



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