

Economic growth and escaping the poverty trap: how does development aid work?

Ngoc-Sang PHAM, Thi Kim Cuong PHAM

➔ NGOC-SANG PHAM, Montpellier Business School, France | ns.pham@montpellier-bs.com

➔ THI KIM CUONG PHAM, BETA, University of Strasbourg, France | **Corresponding author** : kim.pham@unistra.fr

Abstract

This paper introduces a theoretical framework for studying the effectiveness of aid for a recipient country, receiving aid to finance its public investment. It contributes to the debate on the nexus between aid and economic growth and in particular on the conditionality of aid effects. Focusing on autonomous technology, government effort, corruption in the use of aid, fixed cost and efficiency in public investment, we can distinguish 4 levels of circumstances following which, the same aid flows may have very different effects. Given donor's rules, we determine conditions under which the foreign aid can generate economic growth in the long run for the recipient. We also discuss the conditions leading to an economic take-off and an escape from the poverty trap. Analyses of the dynamics of capital also give conditions for a convergence towards a middle income trap or endogenous fluctuations around it.

Keywords: Aid effectiveness, economic growth, fluctuation, poverty trap, public investment.

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1 Introduction

The most vulnerable and least developed countries, in particular countries under the poverty trap, always need foreign aid which may help them to build up favourable conditions for a generation of economic growth and economic take-off. Since the United Nations Summit in September 2000 at which the Millennium Development Goals (MDGs) were agreed, foreign aid, in particular Official Development Assistance (ODA) has been continually increasing. For example, in 2015, development aid provided by the donors in the OECD Development Assistance Committee (DAC) was 131.6 billion USD, increased by 6.9% in real terms from 2014, and by 83% from 2000. At the same time, bilateral aid, provided by one country to another, rised by 4% in real terms.¹

Many issues are under the debate regarding the effectiveness of aid in terms of economic growth and poverty reduction, and extensive empirical investigations using different data sample show conflicting results. On the one hand, studies such as [Burnside and Dollar \(2000\)](#), [Collier and Dollar \(2001, 2002\)](#), [Chauvet and Guillaumont \(2003, 2009\)](#) show that aid may exert a positive and conditional effect on economic growth. Indeed, the seminal paper [Burnside and Dollar \(2000\)](#) find that foreign aid has a positive effect on growth only in recipient countries having good fiscal, monetary and trade policies. [Collier and Dollar \(2001, 2002\)](#) use the World Bank's Country Policy and Institutional Assessment (CPIA) as a measure of policy quality and show that aid may promote economic growth and reduce the poverty in recipient countries if the quality of their policies is sufficiently high. The findings in [Guillaumont and Chauvet \(2001\)](#), [Chauvet and Guillaumont \(2003, 2009\)](#) indicate that the marginal effect of aid on growth is contingent on the recipient countries' economic vulnerability. While the economic vulnerability is negatively associated with growth, the marginal effect of aid on growth is an increasing function of vulnerability. On the other hand, other studies, do not reject the conditionality of aid effects, show, however, a certain fragility of results and underline a non-linear effect of aid on growth ([Hansen and Tarp, 2001](#); [Easterly et al., 2004](#); [Islam, 2005](#); [Roodman, 2007](#); [Clemens et al., 2012](#); [Guillaumont and Wagner, 2014](#)). For example, the results in [Islam \(2005\)](#) show an aid Laffer curve in recipient countries having a political stability. The effect of aid on growth may be negative at a high level of aid inflows. Concerning [Hansen and Tarp \(2001\)](#), this study finds that the effectiveness of aid is conditional on investment and human capital in recipient countries and aid has no effect on growth when controlling these variables. Their findings shed light on the link between aid, investment and human capital and show that aid increases economic growth through its impact on capital accumulation. Using the same empirical specification as that in [Burnside and Dollar \(2000\)](#), but expanding the sample of data set, [Easterly et al. \(2004\)](#) nuance the claim from that of these authors. The results on aid effectiveness seem to be fragile when varying the sample and the definition of different variables such as aid, growth and good policy ([Easterly, 2003](#)).

¹For more information, see <http://www.oecd.org/development/development-aid-rises-again-in-2015-spending-on-refugees-doubles.htm>

While empirical studies on the aid effectiveness are abundant, there are quite few theoretical analysis on this issue. [Charterjee et al. \(2003\)](#) examine the effects of foreign transfers on economic growth of the recipient country given that foreign transfers are positively proportional to the recipient's GDP, but are not subject to conditions. It is shown that their effects on growth and welfare are different according to the type of transfers, untied or tied to investment in public infrastructures. [Charterjee and Tursnovky \(2007\)](#) follow this issue by underlying the role of endogeneity of labour supply as a crucial transmission mechanism for foreign aid.² Distinguished from the previous studies, [Dalgaard \(2008\)](#) introduces an aid allocation policy rule consistent with empirical literature by considering a flow of aid negatively depending on the recipient's income per capita and on the donor's exogenous degree of inequality aversion. The author considers an OLG growth model applied to the case of a recipient country where government investments are fully financed by aid flows. It is shown that an exogenous increase in foreign aid leads to a higher steady-state income and a higher social welfare in the recipient country. Besides, the degree of inequality aversion of the donor determines the characteristics of the transitional dynamics of income per capita.

The current paper contributes to the debate on the nexus between aid, economic growth and escaping poverty trap. It aims to analyse the effectiveness of development aid for a small recipient country under the poverty trap. Simply put, the main question is to examine how development aid can help a recipient country to escape the poverty trap and then to study the necessary conditions for an economic take-off. To do so, we consider a growth model where public investment, partially financed by aid, may improve the capital productivity. As in [Dalgaard \(2008\)](#), our paper formulates aid flows taking into account the donor's rules and the recipient's need which is represented by a low initial endowment. However, different from that paper, aid flows are limited by an upper threshold. This implies that a country would no longer receive aid if it was rich enough. For the case of a vulnerable country, we also consider the inefficiency in the use of aid and in public investment and examine their impact on the aid effectiveness.

The main results can be summarized as follows: First, if the initial circumstances of the recipient are to a sufficient standard, the country does not need international aid to achieve its development. This result is trivial and corroborates to that of a standard AK model. We analyse then different effects of aid for the case where the recipient economy is under the poverty trap without aid. We conclude that the effects of aid in the long run are complex, non-linear and conditional on recipient country's characteristics. Aid may help the recipient country to reach economic growth, to surpass its poverty trap or to reduce this threshold. This is conditional on the degree of corruption in the use of aid, the autonomous technology, the fixed cost and efficiency of public investment, as well as to the donor's rules. We can distinguish 4 levels of circumstances following which, the same aid flows may have very different effects. Then, our second result shows that if the recipient country has a

²[Chenery and Strout \(1966\)](#) propose a theoretical model where aid would affect growth through investment. Precisely, in a recipient country, investment is function of domestic saving and foreign aid. If we refer to a Solow exogenous model, it is straightforward to find a positive effect of investment, hence positive effect of aid, on economic growth.

high quality of circumstances, then international aid may help it to reach economic growth whatever its initial capital. Consequently, there will exist a period when this economy no longer needs international aid to stimulate its economic development. Third, in the case with a low quality of circumstances where the corruption is high and the government effort in public investment is low, we show that aid does not affect the threshold for an economic take-off. However, if aid is sufficiently generous, the recipient country may surpass this threshold while it is impossible without international intervention.

The most complex results and richest dynamics are found in intermediate qualities of circumstances: high corruption and high government effort in public investment (intermediate circumstances 1), low corruption and low government effort (intermediate circumstances 2). It is hard to conclude which situation is better for aid effectiveness. In the first one, aid reduces the threshold for an economic take-off and increases significantly the probability to escape the poverty trap compared to the low circumstances. In the second one, the probability to escape the poverty trap as well as the probability to collapse is lower. In particular, the economy may converge to a middle-income trap or to fluctuate around it.

Results from our analysis appear to corroborate with empirical findings concerning a conditionality of aid effectiveness. Our paper is also related to the literature on optimal growth with increasing returns ([Jones and Manuelli, 1990](#); [Kamihigashi and Roy, 2007](#); [Bruno et al., 2009](#)). However, different from numerous papers in this literature, we consider a decentralized economy while these authors study centralized economies. Besides, we point out the role of aid which can provide investment for the least developed country, and thanks to this, the recipient country may obtain a positive growth in the long run.

The remainder of the paper is organized as follows: Section 2 characterizes the case of a small recipient country. Section 3 presents the poverty trap without international aid. In Section 4, we emphasize the role of international aid by analyzing the conditions for the effectiveness of aid. Section 5 concludes. Appendices gathering technical proofs are presented in Section A.

2 A small economy with foreign aid

This section will consider an economy with infinitely-lived identical individuals. The population size is constant over time and normalized to unity. Labor is exogenous and inelastic. The representative firm produces a single traded commodity, which can be used for either consumption or investment. The government uses capital tax and international aid to finance public investment which can improve the capital productivity. The waste in spending of aid is considered by the presence of unproductive aid. The latter has no direct effect neither on the household's welfare nor on the production process. The fraction of wasteful aid may reflect the degree of corruption in the recipient government.

2.1 Foreign aid and public investment

Literature on aid conditionalities has a large consensus on the recipient's need as a significant criterion of aid allocation: countries with a high need should receive a high amount of aid (Alesina and Dollar, 2000; Guillaumont and Chauvet, 2001; Berthelemy and Tichit, 2004; Dalgaard, 2008; Carter, 2014). This criterion, among others, is used in several bilateral as well as multilateral aid policies. In this sense, we can consider the following function of aid flows a_t :

$$a_t = (\bar{a} - \phi k_t)^+ \equiv \max\{\bar{a} - \phi k_t, 0\} \quad (1)$$

where $\bar{a} > 0$ is the maximal aid amount that the recipient country can receive. Aid flow is inversely linked to the level of per capita capital k_t in the recipient country. Equation (1) means that the higher the capital k , the lower the country ranks in its need, then the lower the aid received. A similar assumption may be found in Carter (2014) and Dalgaard (2008).³ The form of equation (1) implies that a decrease in ϕ and/or an increase in \bar{a} lead(s) to a higher aid flow. Moreover, from a threshold (\bar{a}/ϕ) of capital, the recipient country no longer receives aid.

Parameter $\phi > 0$, independent of the per capita capital, may be referred to all exogenous rules imposed by the donor. The couple (\bar{a}, ϕ) is taken as given by the recipient country and represents aid conditionalities. Aid may be conditional on the policy performance as underlined in Burnside and Dollar (2000), Collier and Dollar (2001, 2002). Following these authors, a country with a high policy quality is more able to use aid in an efficient way. Guillaumont and Chauvet (2001) focus on a fairness argument when they focus on the recipient's economic vulnerability: more aid should be provided to countries with a high economic vulnerability since in these countries aid would be more efficient. This argument also fits in a philosophy of fairness which proposes that aid should compensate the recipient country for its vulnerable initial situation (in macroeconomic conditions or lack of human capital) so that all countries can obtain the same initial opportunities. McGillivray and Pham (2017), Guillaumont et al. (2017) consider the lack of human capital as a determinant criterion. Other analyses underline the link between aid and political variables (strategic allies, former colonial status, and the ability to use aid effectively), between aid and macroeconomic conditions (trade openness, commercial allies) (Alesina and Dollar, 2000; Berthelemy and Tichit, 2004). All these factors are exogenous for the recipient country as they are chosen by donors and may be considered as different interpretations of the parameter ϕ . Equation (1) may reflect a trade-off between needs (low initial capital)

³Carter (2014) considers that aid flow received by country i is positively correlated with country performance rating as underlined in Collier and Dollar (2001, 2002) (with index Country Policy and Institutional Assessment) and is negatively associated with income per capita. Dalgaard (2008) assumes that per capita flow of aid at time t is also a reversed function of income of per capita at period $(t-1)$, $a_t = \theta y_{t-1}^\lambda$, where $\theta > 0$ and $\lambda < 0$. In this aid function, λ reflects the degree of inequality aversion of the donor. Parameter θ represents exogenous determinants of aid. Although Appendix C.2 in Dalgaard (2008) presents a generalized aid allocation rule, it seems that this rule does not cover the form of (1) because the function $(\bar{a} - \phi x)^+$ is not differentiable and the analyses of effects of threshold (\bar{a}/ϕ) is an added-value of our paper.

and country-selectivity (high ϕ) or a compatibility between needs (low initial capital) and country-selectivity (low ϕ) based on aid performance or other criteria.

The recipient country uses aid and tax on capital to finance public investment, which improves the private capital productivity. As some spending of aid is wasted in most recipient countries, there is a significant part of unproductive activity, noted as a_t^u . This is potentially explained by the corruption, administrative fees, etc. Then, the attribution of aid may be written as:

$$a_t = a_t^i + a_t^u \quad (2)$$

where a_t^i represents the part of aid which contributes to the public investment of the recipient country. If we consider a fixed fraction of aid for each activity, we can rewrite equation (2) as follows:

$$a_t = \alpha_i a_t + \alpha_u a_t \quad (3)$$

with $\alpha_u = 1 - \alpha_i$. Parameter $\alpha_i \in (0, 1)$ reflects the inefficiency or the corruption in the use of aid.

Let us denote B_t the public investment financed by tax on capital and by aid, B_t may be written as:

$$B_t = T_{t-1} + a_t^i \quad (4)$$

where T_{t-1} is the tax at period $t - 1$, $T_{t-1} = \tau K_t$. Since, all capital tax is used to fund public investment, τ may be interpreted as the government effort in financing public investment. The positive effect of foreign aid on public investment is an obvious finding in empirical studies (Khan and Hoshino, 1992; Franco-Rodriguez et al., 1998; Ouattara, 2006; Feeny and McGillivray, 2010). For example, using a sample of recipient countries over the period 1980-2000, Ouattara (2006) shows that aid flows are associated with increases in public investment, but do not reduce tax revenue. Feeny and McGillivray (2010) analyze the interaction between aid flows and different categories of public expenditures and show that for Papua New Guinea, aid flows also increase public investment. However, for this aid recipient, aid flows negatively affect revenue collections.

2.2 Production

At each date, the representative firm maximizes its profit. The production function at date t is given by $F_t(K_t) = A_t K_t$, where K_t represents the capital while A_t represents the total factor productivity. The capital K may be referred to a broad concept of capital including for example human capital. We assume that A_t is endogenous and depends on public investment B_t as follows $A_t := A [1 + (\sigma B_t - b)^+]$.⁴ We remark that parameter $A \in (0, \infty)$

⁴Dalgaard (2008) considers a production function in the spirit of Barro (1990) with public spending as a production factor and entirely financed by international aid. The production function is supposed to

is interpreted as the autonomous productivity. When $B_t \leq b/\sigma$, we have $(\sigma B_t - b)^+ = 0$, and then the production function $F_t(K_t)$ recovers the standard AK model. This means that the positive effect of public investment in technology is observed only from the level b/σ . Parameter $\sigma \in (0, \infty)$ measures the extent to which the public investment translates into technology and then production process. In this sense, σ may reflect the efficiency of public investment and b/σ is considered as a threshold from which the public investment improves the technology. The existence of this threshold is supported by some empirical evidences, for example [Azariadis and Drazen \(1990\)](#).

Combining the threshold for a positive effect of public investment with equation (4), we observe that for a significant effect of aid on the capital productivity and production, aid should verify the following condition:

$$a_t^i \geq \frac{b}{\sigma} - \tau K_t \quad (5)$$

It should be noticed that as in [Charterjee et al. \(2003\)](#), [Charteerjee and Tursnovky \(2007\)](#), [Dalgaard \(2008\)](#), we consider that aid is used to finance public expenditures.⁵ However, we assume that aid should be higher than a critical level to improve the technology, and this is necessary for a positive growth in the long run. This assumption may be referred to the big push concept of aid supported in [Sachs \(2005\)](#) and discussed in [Guillaumont and Guillaumont Jeanneney \(2010\)](#). [Wagner \(2014\)](#) uses a data including 89 recipient countries and identified the existence of a critical level above which aid is effective in terms of economic growth.

At each period t , given public investment B_t , the representative firm maximizes its profit:

$$P_{ft} : \quad \pi_t \equiv \max_{K_t \geq 0} \left(F_t(K_t) - r_t K_t \right) \quad (6)$$

It is straightforward to obtain r_t and π_t for a competitive economy:

$$r_t = A \left[1 + (\sigma B_t - b)^+ \right] \text{ and } \pi_t = 0. \quad (7)$$

be homogeneous of degree 1 with respect to private capital and public spending. Given that aid flow is decreasing with income, it converges to a null value in the long run and the economy converges to a steady state where income per capita is constant. [Dalgaard \(2008\)](#) shows that the donors' rule affect only the transitional dynamics of the economy and the steady state income.

⁵[Dalgaard \(2008\)](#) assumes that the first dollars received from donors have a positive effect on the recipient's production. This author considers a production function in the spirit of [Barro \(1990\)](#) with public spending as a production factor and entirely financed by international aid. The production function is supposed to be homogeneous of degree 1 with respect to private capital and public spending. Given that aid flow is decreasing with income, it converges to a null value in the long run and the economy converges to a steady state where income per capita is constant. [Dalgaard \(2008\)](#) shows that the donors' rule affect only the transitional dynamics of the economy and the steady state income.

2.3 Consumption

Let us consider the representative consumer's optimization problem. She maximizes her intertemporal utility by choosing consumption and capital sequences (c_t, k_t) :

$$P_c : \max_{(c_t, k_{t+1})_{t=0}^{+\infty}} \sum_{t=0}^{+\infty} \beta^t U(c_t) \quad (8)$$

$$\text{s.t: } c_t + k_{t+1} + T_t \leq (1 - \delta)k_t + r_t k_t + \pi_t \quad (9)$$

where $k_0 > 0$ is given, β is the rate of time preference and $u(\cdot)$ is the consumer's instantaneous utility function. T_t is the tax, r_t is the capital return while π_t is the firm's profit at date t .

For the sake of tractability, we assume that the consumer knows that $T_t = \tau k_{t+1}$ and instantaneous utility function is logarithmic, $U(c_t) = \ln(c_t)$. According to Lemma 6 in Appendix A.1, we establish the relationship between k_{t+1} and k_t

$$k_{t+1} = \beta \frac{1 - \delta + r_t}{1 + \tau} k_t. \quad (10)$$

By the concavity of the utility function, this solution is unique.

2.4 Intertemporal equilibrium

Definition 1. (*Intertemporal equilibrium*) Given capital tax rate τ , a list $(r_t, c_t, k_t, K_t, a_t)$ is an intertemporal equilibrium if

1. (c_t, k_t) is a solution of the problem P_c , given a_t^i, r_t, π_t .
2. (K_t) is a solution of the problem P_{f_t} , given B_t and r_t .
3. Market clearing conditions are satisfied:

$$K_t = k_t \quad (11)$$

$$c_t + k_{t+1} + T_t = (1 - \delta)k_t + Y_t. \quad (12)$$

4. The government budget is balanced: $T_t = \tau k_{t+1}$.

5. $a_t = \max\{\bar{a} - \phi k_t, 0\}$ and $a_t^i = \alpha_i a_t$.

Combined with (7), the dynamics of capital stock (equation (10)) may be rewritten as follows:

$$k_{t+1} = G(k_t) \equiv f(k_t)k_t \quad (13)$$

$$\text{where } f(k_t) \equiv \beta \frac{1 - \delta + A \left[1 + (\sigma(\tau k_t + \alpha_i(\bar{a} - \phi k_t)^+) - b)^+ \right]}{1 + \tau} \quad (14)$$

This dynamic system is non-linear and non-monotonic. The next sections analyze the dynamics of capital stocks (k_t) over time and the effects of international aid on the economy's economic growth. Before doing this, it should be useful to introduce some notions of growth and collapse.

Definition 2. (*Growth and collapse*)

1. The economy collapses if $\lim_{t \rightarrow \infty} k_t = 0$. It grows without bounds if $\lim_{t \rightarrow \infty} k_t = \infty$.
2. A value k is called a poverty trap if for any $k_0 < k$, we have $\lim_{t \rightarrow \infty} k_t = 0$ and for any $k_0 > k$, we have $\lim_{t \rightarrow \infty} k_t = \infty$.

3 Poverty trap or growth without foreign aid

This section considers an economy which does not receive foreign aid, public investment B_t is entirely financed by tax revenue. We will analyze the dynamics of capital in the long run. From equation (13), we have:

$$k_{t+1} = f_b(k_t)k_t \quad (15)$$

$$\text{where } f_b(k_t) \equiv \beta \frac{1 - \delta + A \left[1 + (\sigma\tau k_t - b)^+ \right]}{1 + \tau} \quad (16)$$

Let us denote:

$$r_a \equiv \beta \frac{1 - \delta + A}{1 + \tau}. \quad (17)$$

We observe $f_b(k_t) \geq r_a$ for any t . Therefore, we have

Proposition 1 (Role of technology). *Consider an economy without aid. If $r_a > 1$, the economy will grow without bounds.*⁶

Condition $r_a > 1$ is equivalent to $A > \frac{1+\tau}{\beta} + \delta - 1$. Our result indicates that when the autonomous technology A is sufficiently high, it may generate growth whatever the levels of other factors such as: initial capital, efficiency of public investment. In this case, the country is not eligible to receive aid. Since our purpose is to look at the impacts of public investment and foreign aid, from now on, we will work under the following assumption.

Assumption 1 (Assumption for the rest of the paper). $r_a < 1$ (or equivalently, $A < \frac{1+\tau}{\beta} + \delta - 1$).⁷

Under this assumption, the economy would never reach economic growth in the long run without public investment B_t (in infrastructure, in R&D program, etc.). Public investment

⁶In this case, $f(k_t) \geq r_a > 1$, then $k_{t+1} > k_t$ for any t .

⁷We ignore the case $r_a = 1$ because this case is not generic.

B_t is then required to improve technology, and this is necessary for a positive economic growth in the long run.

According to equation (15) and the fact that $f_b(k_t)$ is an increasing function, we get the following analysis concerning the dynamics of capital stock.

Proposition 2 (Poverty trap and growth). *Consider an economy with a low level of autonomous technology (Assumption 1 holds), and without foreign aid. The public investment in technology is entirely financed by tax revenue and the dynamics of capital is characterized by (15). There exists a steady state:*

$$k^{**} = \frac{b + D}{\tau\sigma} \quad (18)$$

$$\text{where } D \equiv \frac{1}{A} \left(\frac{1 + \tau}{\beta} + \delta - 1 \right) - 1 > 0. \quad (19)$$

We have then three cases:

1. If $f_b(k_0) > 1$, i.e., $\sigma\tau k_0 > b + D$, then (k_t) increases and the economy grows without bounds.
2. If $f_b(k_0) < 1$, i.e., $\sigma\tau k_0 < b + D$, then (k_t) decreases and the economy collapses.
3. If $f_b(k_0) = 1$, i.e., $\sigma\tau k_0 = b + D$, then $k_t = k_0$ for any t .

It should be noticed that $\frac{b+D}{\sigma}$ may be interpreted as the threshold from which public investment τk_0 generates economic growth. Our result indicates that if the public investment in technology (without aid) is high enough in the sense that $\tau k_0 > \frac{b+D}{\sigma}$, the economy will grow without bounds.

In another way, we may consider b as a fixed cost of public investment. If the return of public investment ($\sigma B_t \equiv \sigma\tau k_0$) is less than $b + D$, public investment τk_0 does not make any change on the total factor productivity. Following this interpretation, $b + D$ can be viewed as the threshold so that if the return of public investment in R&D (σB_t) is less than this level, there is no growth of capital stock, i.e. $k_{t+1} < k_t$ for all t .

[Figure 1 here]

Figure 1 illustrates Proposition 2. The point of interaction between the convex curve and the first bisector corresponds to the unstable steady state k^{**} which is considered as a poverty trap for this economy. For all initial capital k_0 higher than k^{**} (corresponding to $\sigma\tau k_0 > b + D$), the economy will grow without bounds while it collapses if the initial capital is lower than k^{**} . It should be noticed that k^{**} is decreasing in A , σ while it is increasing in b . This means that an economy having a high autonomous technology A , high efficiency σ and low fixed cost b in public investment, obtains a higher probability to surpass its poverty trap as the condition $\sigma\tau k_0 > b + D$ is more likely to be satisfied.

4 Role of foreign aid: how does aid work?

Point 2 of Proposition 2 shows that the economy collapses without international aid if the initial capital and the return of public investment in technology ($\sigma\tau k_0$) are low. Since we want to investigate the effectiveness of aid, we will work under the following assumption in Section 4.

Assumption 2 (Assumption for the whole Section 4).

$$\sigma\tau k_0 < D + b \quad (20)$$

where D is defined by (19)

Given this pessimist initial situation of the recipient country, we examine how international aid could generate positive perspectives in the long run. Let us recall that $k_{t+1} = G(k_t)$ where

$$G(k) \equiv f(k)k = \beta \frac{1 - \delta + A \left[1 + (\sigma(\tau k + \alpha_i(\bar{a} - \phi k)^+) - b)^+ \right]}{1 + \tau} k \quad (21)$$

Before providing the dynamics of capital stock, it is useful to underline some properties of function $f(k)$ and $G(k)$. Notice that function G is non linear and may not be monotonic.

4.1 Properties of function $G(k)$

First, we provide some properties of the function $f(\cdot)$.

Lemma 1 (Properties of f).

1. The function $f_1(k) \equiv (k - a)^+$ is increasing in k .
2. The function $f_2(k) \equiv \tau k + \alpha_i(\bar{a} - \phi k)^+$ is increasing on $[0, \infty]$ if $\tau \geq \alpha_i\phi$. When $\tau < \alpha_i\phi$, the function f_2 is decreasing on $[0, \bar{a}/\phi]$ and increasing on $[\bar{a}/\phi, \infty]$.
3. $f_2(k) \equiv \tau k + \alpha_i(\bar{a} - \phi k)^+ \geq \bar{a} \min(\alpha_i, \tau/\phi)$.
4. $f(k_t) \geq \frac{\beta}{1 + \tau} \left[1 - \delta + A \left(1 + (\sigma \bar{a} \min(\alpha_i, \tau/\phi) - b)^+ \right) \right]$.

We now study the monotonicity of function G . Before doing this, we introduce some notations

$$x_1 \equiv \bar{a}/\phi \quad (22)$$

$$x_2 \equiv \frac{\sigma\alpha_i\bar{a} - b}{\sigma(\alpha_i\phi - \tau)} \quad \text{i.e. } x_2 \text{ such that } \sigma(\tau k + \alpha_i(\bar{a} - \phi k)) - b = 0 \quad (23)$$

$$x_3 \equiv \frac{1 - \delta + A(1 + \sigma\alpha_i\bar{a} - b)}{2A\sigma(\alpha_i\phi - \tau)}. \quad (24)$$

Let us explain the meaning of x_1, x_2, x_3 . x_1 is the maximum level of capital stock that the recipient country does not receive international aid. When the country receives aid, x_2 is the critical threshold from which public investment B_t (financed by aid and tax revenue) has positive impact on productivity.

When the country receives aid ($\bar{a} - \phi k > 0$) and public investment has positive impact on productivity ($\sigma(\tau k + \alpha_i(\bar{a} - \phi k)) - b > 0$), x_3 is a local-maximum of the function G (because $f'_3(x_3) = 0$) where

$$f_3(x) \equiv \beta \frac{1 - \delta + A \left[1 + (\sigma(\tau x + \alpha_i(\bar{a} - \phi x)) - b) \right]}{1 + \tau} x. \quad (25)$$

Lemma 2 (Monotonicity of G). *The function G is increasing on $[0, \infty)$ if one of the following conditions is satisfied.*

1. $\tau \geq \alpha_i \phi$.
2. $\tau < \alpha_i \phi$ and $\sigma \alpha_i \bar{a} < b$ (this implies that $x_2 < 0$).
3. $\tau < \alpha_i \phi$, $\sigma \alpha_i \bar{a} > b$ and $x_3 > \min(x_1, x_2)$.

Lemma 3 (Non-monotonicity of G). *Assume that $\tau < \alpha_i \phi$, $\sigma \alpha_i \bar{a} > b$, and $x_3 < \min(x_1, x_2)$. Then G is increasing on $[0, x_3]$, decreasing on $[x_3, \min(x_1, x_2)]$, and increasing on $[\min(x_1, x_2), \infty)$.*

Proofs of Lemmas 1, 2, and 3 are presented in Appendix A.2.

We end this section by computing no-trivial fixed points (steady states), i.e. strictly positive solutions of the equation $G(k) = k$. So, we have to find k such that

$$f_2(k) \equiv \tau k + \alpha_i(\bar{a} - \phi k)^+ = \frac{D + b}{\sigma}.$$

The following result is obtained using Lemmas 2 and 3.

Lemma 4 (Steady states).

1. If $\sigma \bar{a} \min(\alpha_i, \tau/\phi) > D + b$, then there is no fixed point.
2. Consider the case where $\sigma \bar{a} \min(\alpha_i, \tau/\phi) \leq D + b$.
 - (a) If $\tau > \alpha_i \phi$, which implies $\sigma \bar{a} \alpha_i \leq D + b$, then
 - i. the unique fixed point is $k^* \equiv \frac{D+b - \bar{a}\alpha_i}{\tau - \alpha_i\phi} \in (0, \bar{a}/\phi)$ when $\sigma \bar{a} \tau/\phi > D + b$.⁸
 - ii. the unique fixed point is $k^{**} \equiv \frac{D+b}{\tau\sigma} \in (\bar{a}/\phi, \infty)$ when $\sigma \bar{a} \tau/\phi < D + b$.⁹
 - (b) If $\tau < \alpha_i \phi$, which implies $\sigma \bar{a} \tau/\phi \leq D + b$, then
 - i. If $\sigma \bar{a} \alpha_i < D + b$, then the unique fixed point is $k^{**} \equiv \frac{D+b}{\tau\sigma} \in (\bar{a}/\phi, \infty)$.

⁸This condition guaranties that $k^* < \bar{a}/\phi$

⁹This condition guaranties that $k^* > \bar{a}/\phi$

- ii. If $\sigma\bar{a}\alpha_i > D + b$, then there are two fixed points $k^* \equiv \frac{\bar{a}\alpha_i - \frac{D+b}{\sigma}}{\alpha_i\phi - \tau} \in (0, \bar{a}/\phi)$ and $k^{**} \equiv \frac{D+b}{\tau\sigma} \in (\bar{a}/\phi, \infty)$.¹⁰

Proof. See Appendix A.2. □

4.2 Growth under a high quality of circumstances

This section presents different effects of aid on recipient perspectives in the long run. The effects of aid may depend on the initial situation in the recipient as well as on the generosity of donors. The following proposition characterizes the first scenario when the recipient country has a high quality of circumstances in terms of corruption degree, fixed cost and efficiency in public investment, autonomous technology, etc.

Proposition 3 (Growth). *Considering an aid recipient under poverty trap without aid, characterized by condition (20). The dynamics of capital with foreign aid is characterized by (13).*

If

$$r_d \equiv \frac{\beta}{1+\tau} \left[1 - \delta + A \left(1 + (\sigma\bar{a} \min(\alpha_i, \tau/\phi) - b)^+ \right) \right] > 1 \quad (26)$$

$$\text{or equivalently, } \sigma\bar{a} \min(\alpha_i, \tau/\phi) > D + b, \quad (27)$$

then we have that,

1. the economy will grow without bounds for any level of initial capital k_0 ,
2. international aid $a_t = (\bar{a} - \phi k_t)^+$ decreases in t . Consequently, there exists a time T such that aid flows $a_t = 0$ for any $t \geq T$.

Proposition 3 can be easily proved by using point 4 of Lemma 1 and point 1 of Lemma 4. In this case, there is no fixed point for the dynamics of capital and $f(k_t)$ and $G(k_t)$ are increasing in k_t for all k_t . Condition (27) may be written as follows

$$\sigma\bar{a}\frac{\tau}{\phi} > D + b \quad \text{and} \quad \sigma\alpha_i\bar{a} > D + b, \quad (28)$$

where D is given by equation (19). Two conditions in (28) mean that the foreign aid is generous (high \bar{a} and low ϕ) and/or the recipient country has a high quality of circumstances with a high efficiency σ and low fixed cost b in public investment, and/or a high level of autonomous technology A . In particular, the first condition in (28) may be associated to a high government effort (high τ) in financing public investment while the second condition may be associated to a low corruption in the use of aid (high α_i). In other words, given aid flows and the donor's rules characterized by the couple (\bar{a}, ϕ) , condition (28) is more likely to be satisfied if the recipient country has a high quality of circumstances, decisive for the effectiveness of aid.

¹⁰Condition $\sigma\bar{a}\alpha_i > D + b$ is to ensure that $k^* > 0$.

Proposition 3 presents the best and ideal scenario since whatever the initial capital, generous aid combined with high quality of circumstances could help the recipient country to grow without bounds in the long run. Figures 2 and 3 illustrate this Proposition under condition (27) (or (28)). Figure 2 corresponds to the case $\alpha_i < \tau/\phi$ and Figure 3 to the case $\alpha_i > \tau/\phi$. We observe that, without exogenous aid (corresponding to $\bar{a} = 0$), the dynamics of capital correspond to that in Figure 1 and there is one poverty trap. Thanks to development aid, the dynamics of capital change and are represented by the curve above the first bisector.

Experiences in some recipient countries such as South Korea, Thailand, Tunisia, Indonesia may illustrate this scenario. In particular, South Korea offers an exceptional case since this country, being a recipient country during the period of 1960-1990 (after the Korean War 1950-1953), figures today among the developed countries in the world, and becomes a member of the OECD-DAC (since 2010). The average aid flows have decreased during the period of 1960-1980, from 6.3% to 0.1% of GDP. It became negative at the beginning of the 1990s.¹¹ For the case of Tunisia, aid flows have also decreased during the 1960-2003, from 8.1% of GDP in the 1960s to 1.5% during 1990-2003.¹²

[Figure 2 here]

[Figure 3 here]

We are now interested in the case where condition (27) (or (28)) is not satisfied: recipient countries do not have a high quality of circumstances and/or aid flows, subject to conditionalities, are bounded due to the budget constraint from the donors. In the next sections, we consider the following condition:

$$\sigma\bar{a} \min(\alpha_i, \tau/\phi) < D + b. \quad (29)$$

From (29), we can identify three possibilities:

$$\text{Low circumstances: } \frac{\sigma\tau}{\phi} < \frac{D+b}{\bar{a}} \text{ and } \sigma\alpha_i < \frac{D+b}{\bar{a}} \quad (30)$$

$$\text{Intermediate circumstances 1: } \sigma\alpha_i < \frac{D+b}{\bar{a}} < \frac{\sigma\tau}{\phi} \quad (31)$$

$$\text{Intermediate circumstances 2: } \frac{\sigma\tau}{\phi} < \frac{D+b}{\bar{a}} < \sigma\alpha_i \quad (32)$$

Condition (30) characterizes a situation that is contrary to that presented by condition (28), i.e. it represents a low quality of circumstances with a high degree of corruption (low α_i) and a low government effort (low τ), as well as a high fixed cost and/or a low efficiency in public investment and/or a low level of autonomous technology. If we focus on the degree of corruption in the use of aid (α_i) and the government effort in financing public investment (τ) considering constant other parameters, condition (31) characterizes a high degree of corruption (low α_i) and a high government effort (high τ) while condition (32)

¹¹See also Marx and Soares (2013) and Guillaumont and Guillaumont Jeanneney (2010).

¹²See also Guillaumont and Guillaumont Jeanneney (2010).

characterizes a lower degree of corruption (high α_i) and a lower government effort (low τ). Both these situations are represented as intermediate circumstances compared to the low circumstances.

4.3 Poverty trap: growth or collapse?

According to Lemma 2, we have the following result.

Proposition 4 (Poverty trap). *Consider an aid recipient under poverty trap without aid, characterized by condition (20). The dynamics of capital with foreign aid is characterized by (13). Assume that one of three conditions in Lemma 2 holds, then (k_t) is monotonic in t . Given aid rules (\bar{a}, ϕ) , we have two cases:*

1. (Low circumstances) *If the recipient country has a low quality of circumstances so that condition (30) holds, then there exists one poverty trap k^{**}*

$$k^{**} = \frac{D + b}{\tau\sigma}$$

2. (Intermediate circumstances 1) *If the recipient country has an intermediate quality of circumstances so that condition (31) holds, then there exists another poverty trap k^* ,*

$$k^* = \frac{\bar{a}\alpha_i - \frac{D+b}{\sigma}}{\alpha_i\phi - \tau}.$$

and $k^* < k^{**}$.

In both cases, we have:

- *If $f(k_0) > 1$, i.e., $(\sigma(\tau k_0 + \alpha_i(\bar{a} - \phi k_0)^+) - b)^+ > D$, then (k_t) increases and the economy grows without bounds. Consequently, there exists a time T such that aid flows $a_t = 0$ for any $t \geq T$.*
- *If $f(k_0) < 1$, i.e., $(\sigma(\tau k_0 + \alpha_i(\bar{a} - \phi k_0)^+) - b)^+ < D$, then (k_t) decreases and the economy collapses. Consequently, there exists a time T_1 such that aid flows $a_t > 0$ for any $t \geq T_1$.*
- *If $f(k_0) = 1$, then $k_t = k_0$ for any t .*

Point 1 in Proposition 4 is formulated from Lemma 4, points 2.a.ii and 2.b.i while point 2 in this proposition is from Lemma 4, point 2.a.i. This proposition underlines that given the donor' rules, the effectiveness of aid is conditional on the initial situation in the recipient country. Following the circumstances in the recipient country, the same flow of aid may be more or less effective. Indeed, point 1 in this proposition is associated to condition (30) reflecting a bad situation which is opposite to that given by condition (27) in Proposition 3, i.e. the recipient country should suffer a high corruption (low α_i) and a low

government effort in financing public investment, other characteristics such as autonomous technology, efficiency in public investment, or fixed cost b may be identical or worse than in the high circumstances. We remark that the poverty trap is always k^{**} , like in the case without international aid. However, this result does not mean that development aid does not exert any effect on the recipient country. Indeed, it should be noticed that we are under condition $\sigma(\tau k_0 - b)^+ < D$, i.e. the country is under the poverty trap without development aid. With the same poverty trap, development aid could impede the collapse and help the recipient country to escape poverty if the aid flow is sufficiently high so that $(\sigma(\tau k_0 + \alpha_i(\bar{a} - \phi k_0)^+) - b)^+ > D$. In other words, the development aid might help the recipient to surpass its poverty trap while this is impossible without foreign assistance. This result which suggests that low-income and vulnerable countries need a large scaling-up of aid to help them to get out of the poverty trap, may be considered as a theoretical illustration for the argument evoked in [Kraay and Raddatz \(2007\)](#) using a Solow model.¹³

If we compare both circumstances (low and intermediate 1), then the intermediate circumstances 1 with a higher government effort give a better result since its steady state k^* is lower than k^{**} in the low circumstances. Other characteristics may be maintained unchanged, or take the values so that condition (31) holds. Precisely, with a higher government effort, the same aid flows and corruption degree in the use of aid may improve the recipient country's probability of escaping its poverty trap and obtaining an economic take-off.

Our result is related to the literature on optimal growth with increasing returns ([Jones and Manuelli, 1990](#); [Kamihigashi and Roy, 2007](#); [Bruno et al., 2009](#)). Our added-value is twofold. First, we consider a decentralized economy while these authors study centralized economies. Second, we point out the role of aid which can provide investment for the recipient country, and thanks to this, the recipient country may obtain a positive growth in the long run. Besides, since $\alpha_i(\bar{a} - \phi k_0)^+$ is increasing in α_i and \bar{a} , decreasing in ϕ , Proposition 4 indicates that: the more generous the donors are (high value of \bar{a} and/or low value of ϕ) or/and the lower level of corruption (high value of α_i), the more likely that the recipient country escapes the poverty trap.¹⁴ In this case, it is more likely to satisfy conditions $k_0 > k^*$ or $k_0 > k^{**}$ for getting out of the poverty trap.¹⁵

¹³In a Solow model with two exogenous saving rates, there are two steady states which are locally stable. [Kraay and Raddatz \(2007\)](#) indicate that in such a model, if the saving rate is low, foreign aid could help the recipient to accumulate capital. Saving rate might jump to the higher level, and then, the economy would converge to the high steady state.

¹⁴As indicated previously, the donor's rules are exogenous and represented by (ϕ, \bar{a}) in function of aid (1). These donor's rules representing aid conditionalities may be determined by macroeconomics conditions of the recipient. For example, referring to [Guillaumont and Chauvet \(2001\)](#), [Chauvet and Guillaumont \(2003\)](#), we may interpret ϕ as the initial situation in recipient country in terms of economic vulnerability. A low value of ϕ may be associated to a high economic vulnerability. Therefore, country with low ϕ will receive more aid given all others variables including initial poverty (low k_0). [Guillaumont and Chauvet \(2001\)](#), [Chauvet and Guillaumont \(2003\)](#) recommend that countries with high economic vulnerability should receive more aid than others as aid is more efficient in these countries. This implies that the more generous the donors are, the more likely that the recipient country escapes the poverty trap as the threshold for economic take-off becomes lower.

¹⁵Notice that there is an upper threshold for capital above which the recipient does no longer receive aid:

4.4 Middle-income trap: stability, fluctuations or take-off?

We have so far analyzed three circumstances in which the capital path (k_t) is monotonic. The recipient's economy may or may not fully exploit the same aid flows following its initial situation. In particular, with a high quality of circumstances, the recipient's economy may grow without bounds while in the opposite circumstances, its initial poverty trap remains unchanged. The latter situation needs a scaling-up of aid.

Other results may be observed as we have shown in Lemma 4. When the capital path (k_t) is not monotonic, there may be two steady states under the following assumption.

Assumption 3 (Assumptions for the whole Section 4.4).

1. $\sigma\tau/\phi < \frac{D+b}{\bar{a}} < \sigma\alpha_i$ (condition (32))
2. $0 < x_3 < \min(x_1, x_2)$ where x_1, x_2, x_3 are given by (22), (23) and (24).¹⁶

The first point in this assumption refers to condition (32) characterized as intermediate circumstances 2 with a lower government effort as well as lower degree of corruption in the use of aid, compared to the intermediate circumstances 1 (condition (31)), other characteristics being unchanged. The second point in this assumption implies that its potential level of capital stock is quite low so that the country still needs international aid ($x_3 < x_1$), and its public investment without aid does not have impact on the productivity ($x_3 < x_2$).

Under this assumption, G is increasing on $[0, x_3]$, decreasing on $[x_3, \min(x_1, x_2)]$, and increasing on $[\min(x_1, x_2), \infty)$. By combining Assumption 3, Lemma 3 and point (2.b.i) of Lemma 4, there exist two steady states k^* and k^{**} with $k^* < k^{**}$

$$\begin{aligned} \text{low steady state: } k^* &= \frac{\bar{a}\alpha_i - \frac{D+b}{\sigma}}{\alpha_i\phi - \tau} \in (0, \bar{a}/\phi) \\ \text{high steady state: } k^{**} &= \frac{D+b}{\tau\sigma} \in (\bar{a}/\phi, \infty). \end{aligned}$$

The low steady state k^* (stable or unstable) is interpreted as *middle-income trap* while the high steady state k^{**} is unstable and takes the same value as that in the case without aid. Let us investigate now whether capital stock converges to the middle income trap or fluctuates around it? If none of them is the case, is there an opportunity for the recipient contry to encompass this middle-income trap and attain an economic take-off?

4.4.1 Stability of the middle-income trap

Proposition 5 (Stability of low steady state). *Under Assumption 3:*

1. *Considering the case where $\sigma\bar{a}\alpha_i < D + b + \frac{1}{A} \left(\frac{1+\tau}{\beta} \right)$, or equivalently $x_3 > k^*$. We have that if $k_0 \in (0, k^*)$, then $k_t \in (0, k^*)$ for any t and $\lim_{t \rightarrow \infty} k_t = k^*$.*

$k_t < \frac{\bar{a}}{\phi}$ (cf. equation (1)).

¹⁶This condition implies the non-monotonicity of transitional function G .

2. Considering the case where $\sigma\bar{\alpha}\alpha_i > D + b + \frac{1}{A} \left(\frac{1+\tau}{\beta} \right)$, or equivalently $x_3 < k^*$. The steady state k^* is locally stable¹⁷ if and only if

$$\sigma\bar{\alpha}\alpha_i < D + b + \frac{2}{A} \left(\frac{1+\tau}{\beta} \right) \quad (33)$$

Proof. See Appendix A.3. □

[Figure 4 here]

[Figure 5 here]

As underlined in previous section, the initial conditions in the recipient country are decisive for the effectiveness of aid. Comparing 3 circumstances, low, intermediate 1 and intermediate 2, we can give two observations. On the one hand, given the same flow of aid, the intermediate circumstances 2 described by condition (32), lead to an aid effect more satisfying than the low circumstances do. Indeed, in the intermediate circumstances 2, for all initial capital lower than k^{**} , the economy no longer collapses, it may converge to the middle-income trap k^* if this one is stable as shown in Proposition 5. Aid may not help to generate growth, but may conduce the economy to a stable steady state where income per capita is constant. Figure 4 illustrates the global stability of the low steady state k^* while Figure 5 illustrates the local stability.

On the other hand, outcomes from two intermediate circumstances are very different. We recall that given all other factors, the intermediate circumstances 1 give a low steady state k^* while the intermediate circumstances 2 give two steady states, with $k^* < k^{**}$. Hence, in the latter situation k^* may be stable and considered as a middle-income trap as indicated in Proposition 5. Hence, for all initial capital lower than k^{**} , if the initial situation verifying (32) with low corruption and low government effort, there may not be the risk of collapse as the economy may converge to a middle-income.

4.4.2 Endogenous fluctuation

It should be noticed that when the low steady state is not locally stable (condition (33) is not satisfied), there will be other possibilities for this economy. In this section and the next one, we will focus on this case. It is shown that fluctuations around this steady state may occur. Besides, there is also a possibility to obtain a “lucky growth”.

Our finding about the fluctuation of capital paths is based on the following intermediate result.

Lemma 5. *Assume conditions in Assumption 3 hold and $x_3 < k^*$. Assume also that*

$$\sigma\bar{\alpha}\alpha_i > D + b + \frac{2}{A} \left(\frac{1+\tau}{\beta} \right). \quad (34)$$

¹⁷It means that there exists $\epsilon > 0$ such that $\lim_{t \rightarrow \infty} k_t = k^*$ for any $k_0 \in (k^* - \epsilon, k^* + \epsilon)$.

Then, there exist $y_1 \in (x_3, k^*)$ and $y_2 > 0$ in $(0, x_2)$ such that

$$y_1 \neq y_2, \quad f_3(y_1) = y_2, \quad f_3(y_2) = y_1. \quad (35)$$

Moreover, if we add assumption that $G(y_1) < x_2$, then the above y_1, y_2 satisfy

$$y_1 \neq y_2, \quad G(y_1) = y_2, \quad G(y_2) = y_1. \quad (36)$$

Proof. See Appendix A.4. □

Considering y_1, y_2 determined in (36) of Lemma 5, let us denote

$$\mathcal{F}_0 \equiv \{y_1, y_2\}, \quad \mathcal{F}_{t+1} \equiv G^{-1}(\mathcal{F}_t) \quad \forall t \geq 0, \quad \mathcal{F} \equiv \cup_{t \geq 0} \mathcal{F}_t.$$

The following result is a direct consequence of Lemma 5 and definition of \mathcal{F} .

Proposition 6 (Fluctuations around the low steady state). *Under Assumption 3 and conditions in Lemma 5, we have that: if $k_0 \in \mathcal{F}$, then there exists t_0 such that $k_{2t} = y_1$, $k_{2t+1} = y_2$ for any $t \geq t_0$.*

[Figure 6 here]

Figure 6 illustrates the fluctuation of the recipient economy around the low steady state following the description in Lemma 5. There exists a subset \mathcal{F} of \mathbb{R}^+ such that for all initial capital belonging to this subset, there is neither possibility for the recipient country to converge to the middle-income trap, nor the possibility to reach an economic take-off. The key for obtaining Proposition 6 is condition (34) which is equivalent to $3\frac{1+\tau}{\beta} - (1-\delta) < A(1 + \sigma\alpha_i\bar{a} - b)$. This holds if and only if the two following conditions are satisfied:

$$1 + \sigma\alpha_i\bar{a} > b \quad (37)$$

$$A > \frac{3\frac{1+\tau}{\beta} - (1-\delta)}{1 + \sigma\alpha_i\bar{a} - b} \quad (38)$$

Results show that the economy fluctuates around the middle-income trap under condition (34) (Figure 6) while it converges to it under condition (33) (Figure 4). It should be noticed that the fluctuation around the middle income trap is not necessarily worse than the convergence towards this middle trap. It occurs under the initial condition which may be slightly better than the condition for a convergence. For example, we take $\alpha_i = 0.7$, $A = 0.4$, $b = 3$, $\sigma = 1$ in Figure 5 for convergence and $\alpha_i = 0.8$, $A = 0.5$, $b = 2$, $\sigma = 1.2$ in Figure 4 for fluctuations.¹⁸

¹⁸Observe that these characteristics always verify condition (32) for the existence of a low and a high steady state. However, they are not sufficiently good to verify condition (28) for a growth without bounds as shown in Proposition 3.

4.4.3 Lucky growth vs middle-income trap: role of aid

When $x_3 < k^{**}$, we can consider two subcases: $G(x_3) \leq k^{**} = G(k^{**})$ corresponding to a lower dynamics of capital and $G(x_3) > k^{**} = G(k^{**})$ a strong dynamics of capital.

Let us denote

$$U_0(k^{**}) \equiv \{x \in [0, k^{**}] : G(x) > k^{**}\}, \quad U_{t+1}(k^{**}) \equiv G^{-1}(U_t(k^{**})), \quad \forall t \geq 0$$

$$U(k^{**}) \equiv \cup_{t \geq 0} U_t(k^{**}).$$

Note that $k^* \notin U(k^{**})$ and $k^* > x_3$. Here, k^{**} is the high steady state. It is easy to see that k_t tends to infinity if $k_0 > k^{**}$. The following result shows the asymptotic property of equilibrium capital path (k_t) for the case $k_0 < k^{**}$.

Proposition 7 (Lucky growth vs middle-income trap). *Assume that conditions in Assumption 3 hold.*

1. If $G(x_3) \leq k^{**}$, then $k_t \leq k^{**}$ for any $k_0 \leq k^{**}$.
2. If $G(x_3) > k^{**}$, then we have: $U(k^{**}) \neq \emptyset$, and $\lim_{t \rightarrow \infty} k_t = \infty$ for any $k_0 \in U(k^{**})$.

Proof. See Appendix A.5. □

[Figure 7 here]

We remark that condition $G(x_3) > k^{**}$ is equivalent to

$$\frac{\beta}{1 + \tau} \frac{\left(1 - \delta + A(1 + \sigma\alpha_i\bar{a} - b)\right)^2}{4A(\alpha_i\phi - \tau)} > \frac{D + b}{\tau}. \quad (39)$$

The right hand side depends neither on (\bar{a}, ϕ) nor on (σ, α_i) . Under Assumption 3, the left hand side increases in $\bar{a}, \sigma, \alpha_i$ but decreasing in ϕ .¹⁹ It means that when aid flows and the efficiency in public investment are sufficiently high, and corruption is low, the dynamics of capital are strong, $G(x_3) > k^{**}$. There always exist some “lucky values” of initial capital k_0 so that $G(k_0) > k^{**}$, foreign aid may help the economy to surpass the poverty trap k^{**} and reach an economic take-off (high curve in Figure 7).²⁰ Otherwise, the recipient country may converge to the middle-income trap with a null growth or to fluctuate around it (low curve in Figure 7).

Notice that these characteristics in degree of corruption, government effort, autonomous technology and efficiency of public investment, for a “lucky growth” do not verify condition (28) for a growth without condition on the initial value of capital. This means that there exist only the neighborhood $U_0(k^{**})$ of x_3 so that for all k_0 belonging to this set, the stock

¹⁹It is easy to see that the left hand side increases in \bar{a}, σ but decreasing in ϕ . It is increasing in α_i because $x_3 < x_1$.

²⁰The term “lucky values” of initial capital reflects its random character as there is no regular rule for initial capital. For $G(x_3) > k^{**}$, there exist certainly other values of k_0 so that $G(k_0) < k^{**}$, then the economy converges to the low steady state even with its strong dynamics of capital (corresponding to the high curve).

of capital at the next period, i.e., $k_1 = G(k_0)$ will be very high (thanks to development aid) and surpasses the poverty trap k^{**} , then the recipient economy may reach the lucky growth.

5 Conclusions

Least developed and developing countries always need international assistance to generate positive and autonomous growth in the long run, in particular to attain targets of millennium development and sustainable development. It is then legitimate to ask whether development aid from donors is effective in recipient countries. This paper aims to contribute to this debate by proposing a theoretical framework for examining the effectiveness of aid. The recipient country has initial conditions unfavourable to generate economic growth and uses aid to finance its investment in technology which allows it to improve the capital productivity. Adopting a positive approach which cares about the aid effects in the recipient country, we consider the donors' rules as exogenous.

Considering the situation in which the recipient is under the poverty trap if there is no aid, we show that given the same aid flows, their effects are very different following the initial characteristics. Focusing on the circumstances in terms of autonomous technology, government effort in financing public investment, fixed cost and efficiency of public investment, corruption in the use of aid, the paper can distinguish 4 qualities of circumstances, ranked from low to high quality. It is shown that the effectiveness of aid is conditional on the recipient's initial situation. Aid effects are not linear and complex as underlined in numerous empirical studies. Certain initial situations need a scaling-up of aid, others do not. Our first result shows that if the recipient has a relatively high quality of circumstances, the development aid may help it to reach economic growth whatever the initial capital. Consequently, there will exist a period when this economy no longer needs international aid to stimulate its economic development. Our second result concerns the low quality of circumstances in which the recipient country would escape the poverty trap only if aid flows are sufficiently high. This result might justify a scaling-up of aid for countries suffering initial disadvantages which are not in favour of generating economic growth.

For the intermediate circumstances compared to the low circumstances, aid may lead the recipient to a better perspective in the long run. Aid may help the recipient to reduce its threshold for an economic take-off. This implies that the probability that the recipient escapes the poverty trap is higher. In another scenario, our analysis shows that the recipient's economy may converge to a middle-income trap or to fluctuate around it. It should be noticed that the endogenous fluctuation is not necessarily worse than the convergence towards the middle income trap where the income per capita is constant. We show that the endogenous fluctuation occurs under the initial conditions slightly better than the conditions for a convergence. Finally, a particular result deserves to be emphasized. It concerns the possibility to reach a lucky growth rather to converge to the middle income trap. This may occur when the initial capital belongs to a neighbourhood of the middle income trap and its dynamics is sufficiently strong. In this case, aid may help the recipient country to

jump through the poverty trap and to have an economic take-off.

This paper contributes to the debate regarding the effectiveness of aid in terms of economic growth, comprising numerous empirical investigations. The evaluation of aid effects is necessary for the determination of an efficient allocation of aid to recipient countries from donors. This paper only focuses on the effectiveness of aid following the recipients' initial conditions and considers as exogenous the degree of corruption in the use of aid and the government effort. However, it should be noticed that the effectiveness of aid also depends strongly on the manners in which aid is used in recipient countries and on the absorptive capacity of these countries. Is aid used to finance public spending promoting economic growth and poverty reduction or is it misappropriated by corrupt governments? Does aid reduce recipient governments' effort in financing public investment? If so, what is the impact of the crowding-out effect on economic growth and different targets of sustainable development? These questions would deserve an in-depth analysis in future researches and their answers would have important implications for the choice of efficient and fair allocation of aid.

A Appendix

A.1 The solution of the consumer's problem in Section 2

Lemma 6. *Consider the optimal growth problem*

$$\max_{(c_t, s_t)_t} \sum_{t=0}^{\infty} \beta^t \ln(c_t) \quad (40)$$

$$c_t + s_{t+1} \leq A_t s_t, \quad c_t, s_t \geq 0. \quad (41)$$

The unique solution of this problem is given by $s_{t+1} = A_t s_t$ for any $t \geq 0$.

Proof. Indeed, the Euler condition $c_{t+1} = \beta A_{t+1} c_t$ jointly with the budget constraint becomes $s_{t+2} - \beta A_{t+1} s_{t+1} = A_{t+1}(s_{t+1} - \beta A_t s_t)$. Thus, a solution is given by $s_{t+1} = \beta A_t s_t$. It is easy to check the transversality condition $\lim_{t \rightarrow \infty} \beta^t u'(c_t) s_{t+1} = 0$.

By the concavity of the utility function, the solution is unique. □

A.2 Properties of function f and G in Section 4.1

Proof of Lemma 1. The three first points are obvious. Let us prove the last point. We consider 2 cases. If $k \geq \bar{a}/\phi$, it is easy to see that $f_2(k) \geq \tau k \geq \tau \bar{a}/\phi \geq \bar{a} \min(\alpha_i, \tau/\phi)$.

If $k \leq \bar{a}/\phi$, then $f_2(k) = \alpha_i \bar{a} + (\tau - \alpha_i \phi)k$.

When $\tau - \alpha_i \phi \geq 0$, we have $f_2(k) \geq \alpha_i \bar{a}$.

When $\tau - \alpha_i \phi \leq 0$, we have $f_2(k) \geq \alpha_i \bar{a} + (\tau - \alpha_i \phi) \bar{a}/\phi = \alpha_i \bar{a}/\phi$. □

Proof of Lemma 2. To prove Lemma 2, we need the following result.

Claim 1. 1. G is increasing on $[x_1, \infty)$.

2. Assume that $x_2 > 0$. We have G increasing on $[x_2, \infty)$.

Consequently, G is increasing on $[\min(x_1, x_2), \infty)$.

Proof of Claim 1. 1. G is increasing on $[x_1, \infty)$ because when $x \geq x_1$, we have

$$G(x) = \beta \frac{1 - \delta + A(1 + (\sigma\tau x - b)^+)}{1 + \tau} x.$$

2. If $x_1 < x_2$, it is trivial that G is increasing on $[x_2, \infty)$ because it is increasing on $[x_1, \infty)$.

We now consider the case where $x_1 > x_2$. Let x and y such that $x \geq y \geq x_2$. We have to prove that $G(x) \geq G(y)$. It is easy to see that $G(x) \geq G(y)$ when $x, y \in [x_2, x_1]$ or $x, y \in [x_1, \infty)$. We now assume that $x \geq x_1 \geq y$. In this case, we have

$$\begin{aligned} G(x) &= \beta \frac{1 - \delta + A(1 + (\sigma\tau x - b)^+)}{1 + \tau} x \geq \beta \frac{1 - \delta + A}{1 + \tau} x \\ G(y) &= \beta \frac{1 - \delta + A(1 + (\sigma\alpha_i \bar{a} - b - \sigma(\alpha_i \phi - \tau)y)^+)}{1 + \tau} y = \beta \frac{1 - \delta + A}{1 + \tau} y \end{aligned}$$

where the last equality is from the fact that $y \geq x_2 \equiv \frac{\sigma\alpha_i \bar{a} - b}{\sigma(\alpha_i \phi - \tau)}$. So, it is clear that $G(x) \geq G(y)$. □

We now prove Lemma 2.

1. When $\tau \geq \alpha_i \phi$, by using point 3 of Lemma 1, we get that G is increasing on $[0, \infty)$.

2. When $\tau < \alpha_i \phi$ and $x_2 < 0$. We consider two cases.

If $x \leq \bar{a}/\phi$, then $(\sigma(\tau x + \alpha_i(\bar{a} - \phi x)^+) - b)^+ = (\sigma\alpha_i \bar{a} - b - \sigma(\alpha_i \phi - \tau)x)^+ = 0$ (because $\sigma\alpha_i \bar{a} - b < 0$). So, in this case, we have $G(x) = \beta \frac{1 - \delta + A}{1 + \tau} x$.

If $x \geq \bar{a}/\phi$, we have

$$G(x) = \beta \frac{1 - \delta + A(1 + (\sigma\tau x - b)^+)}{1 + \tau} x.$$

It is easy to see that G is increasing on $[0, \infty)$.

3. We now consider the last case where $\tau < \alpha_i \phi$ and $x_2 > 0$, and $x_3 > \min(x_1, x_2)$.

First, according to Lemma 1, we observe that G is increasing on $[\min(x_1, x_2), \infty)$.

Second, we also see that G is increasing on $(0, x_3)$. Since $x_3 > \min(x_1, x_2)$, we obtain that G is increasing on $[0, \infty)$.

□

Proof of Lemma 3. According to Lemma 1, we have that G is increasing on $[\min(x_1, x_2), \infty)$.

We now consider G on $[0, \min(x_1, x_2)]$. Let $x \in [0, \min(x_1, x_2)]$. We have

$$G(x) = f_3(x) = \beta \frac{1 - \delta + A(1 + \sigma\alpha_i\bar{a} - b - \sigma(\alpha_i\phi - \tau)x)}{1 + \tau}. \quad (42)$$

By definition of x_3 , we have $f'_3(x_3) \geq 0$ if and only if $x \leq x_3$. Therefore, G is increasing on $[0, x_3]$, decreasing on $[x_3, \min(x_1, x_2)]$. □

A.3 Proof Proposition 5

Part 1. First, we need the following result.

Claim 2. Assume that $\bar{a}\alpha_i > \frac{D+b}{\sigma} > \bar{a}\tau/\phi$ and $x_3 < x_2$.

If $x_3 > k^*$, then $G(x_3) < x_3$. And therefore, $G(x_3) < x_3 < x_2 < k^* = G(k^*)$. In this case, we have $G(x) < k^*$ for any $x < k^*$.

Proof of Claim 2. It is easy to see that if $x_3 > k^*$, then $G(x_3) < x_3 < x_2 < k^* = G(k^*)$.

If $x < k^*$, then we have $G(x) \leq \max_{x \leq k^*} G(x) \leq G(x_3) < x_3 \leq k^*$. □

We now come back to the proof of Proposition 5.

If $k_0 < k^*$, according to Claim 2, we have $k_1 = G(k_0) < k^*$. By induction, we have $k_t < k^*$ for any t .

We now prove that $\lim_{t \rightarrow \infty} k_t = k^*$ for any $k_0 \in (0, k^*)$.

Case 1: $k_0 \in (0, x_3]$. Since G is increasing on $[0, x_3]$, we have $\lim_{t \rightarrow \infty} k_t = k^{**}$ for any $k_0 \in (0, x_3]$.

Case 2: $k_0 \in (x_3, x_2]$. We see that $k_1 = G(k_0) \leq \max_{x \in [0, x_2]} G(x) = G(x_3) < x_3$. Therefore $k_1 < x_3$, and so $\lim_{t \rightarrow \infty} k_t = k^{**}$.

Case 3: $k_0 \in [x_2, \bar{a}/\phi]$, we have $k_1 = G(k_0) = \frac{\beta(1-\delta+A)}{1+\tau}k_0$. Since $\frac{\beta(1-\delta+A)}{1+\tau} < 1$, there exists t_0 such that $k_{t_0} < x_2$. Thus $\lim_{t \rightarrow \infty} k_t = k^{**}$.

Case 4: $k_0 \in [\bar{a}/\phi, k^*]$, we have $G(k_0) < k_0$ which means that $f(k_0) < 1$. Combining with $k_1 = f(k_0)k_0$, there exists t_1 such that $k_1 < \bar{a}/\phi$. This implies that $\lim_{t \rightarrow \infty} k_t = k^{**}$.

Part 2. Recall that

$$G(k) = f_3(k) \equiv \frac{\beta}{1 + \tau} \left[1 - \delta + A \left(1 + \sigma\alpha_i\bar{a} - \sigma(\alpha_i\phi - \tau)k - b \right) \right] k \quad (43)$$

$$= \frac{\beta}{1 + \tau} \left[1 - \delta + A(1 + \sigma\alpha_i\bar{a} - b) - A\sigma(\alpha_i\phi - \tau)k \right] k \quad (44)$$

$$G'(k) = f'_3(k) = \frac{\beta}{1 + \tau} \left[1 - \delta + A(1 + \sigma\alpha_i\bar{a} - b) - 2A\sigma(\alpha_i\phi - \tau)k \right]. \quad (45)$$

It is easy to compute that

$$G'(k^*) = \frac{\beta}{1 + \tau} \left[1 - \delta + A(1 + b + 2B - \sigma\bar{a}\alpha_i) \right]. \quad (46)$$

According to [Bosi and Ragot \(2011\)](#), k^* is locally stable if and only if $\|G'(k^*)\| < 1$. Since $x_3 < k^*$, have have $G'(k) < 0$. So, k^* is locally stable if and only if $G'(k) > -1$ which is equivalent to

$$3\frac{1+\tau}{\beta} - (1-\delta) + A(b-1-\sigma\alpha_i\bar{a}) > 0. \quad (47)$$

A.4 Proof of Lemma 5

We will find $y_1, y_2 > 0$ such that (35). Let us denote $n = 1 - \delta + A(1 + \sigma\alpha_i\bar{a} - b)$ and $m = A\sigma(\alpha_i\phi - \tau)$. y_1, y_2 must satisfy

$$\frac{\beta}{1+\tau}(n - my_1)y_1 = y_2, \quad \frac{\beta}{1+\tau}(n - my_2)y_2 = y_1. \quad (48)$$

Since $y_1 \neq y_2$, we have $\frac{\beta}{1+\tau}(n - m(y_1 + y_2)) = -1$. So, we obtain

$$H(y_1) \equiv \frac{\beta}{1+\tau}(n - my_1)y_1 + y_1 - \frac{1}{m}\left(n + \frac{1+\tau}{\beta}\right) = 0 \quad (49)$$

We have $H(y_1) < 0$. We also see that $H(k^*) > 0$ if condition (34) is satisfied.

Under condition (34), there exists y_1 such that $H(y_1) = 0$. Therefore, y_1 and $y_2 = f_3(y_1)$ satisfy (35).

A.5 Proof of Proposition 7

Point (1). Since conditions in Assumption 3 hold, Lemma 3 implies that G is increasing on $[0, x_3]$, decreasing on $[x_3, \min(x_1, x_2)]$, and increasing on $[\min(x_1, x_2), \infty)$. So, $\max_{x \leq k^{**}} G(x) \leq \text{Max}(G(x_3), G(k^{**})) \leq k^{**}$. Therefore $k_t = G(k_{t-1}) \leq k^{**}$ for any $k_0 \leq k^{**}$.

Point (2). If $G(x_3) > k^{**}$, then $x_3 \in U_0(k^{**}) \subset U(k^{**})$. So, $U(k^{**}) \neq \emptyset$. Now, let $k_0 \in U(k^{**})$, then there exists t_0 such that $G^{t_0}(k_0) > k^{**}$, where $G^1 \equiv G$ and $G^{s+1} \equiv G(G^s)$ for any $s \geq 1$. So, $k_{t_0} = G^{t_0}(k_0) > k^{**}$. This implies that $(k_t)_{t \geq t_0}$ is an increasing sequence and $\lim_{t \rightarrow \infty} k_t = \infty$.

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Figures

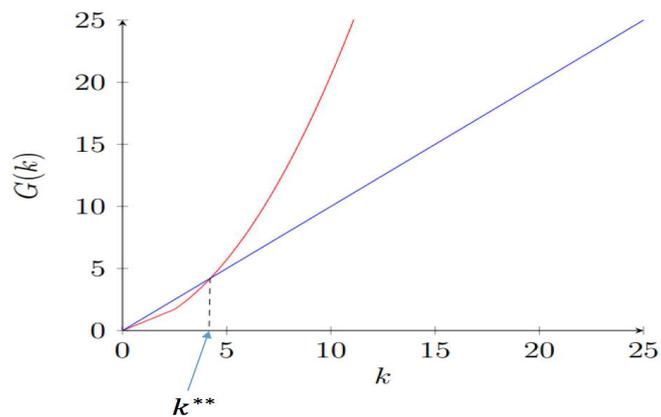


Figure 1: *Poverty trap without foreign aid.* Parameters in function $G(k)$ are $\beta = 0.8$; $\delta = 0.2$; $A = 0.5$; $\tau = 0.4$; $\sigma = 2$; $\bar{a} = 0$; $b = 2$; verifying condition $r_a < 1$.

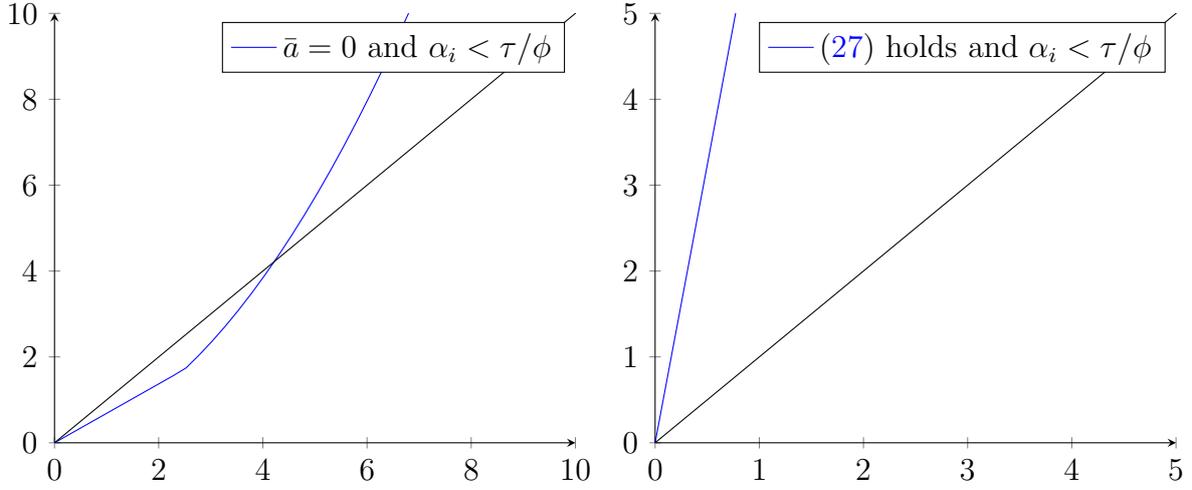


Figure 2: *Growth without bounds.* Parameters in function $G(k)$ are $\beta = 0.8; \tau = 0.4; \delta = 0.2; A = 0.4; \sigma = 2; \alpha_i = 0.8; b = 2, \phi = 0.4$ verifying conditions $r_a < 1$ and $\alpha_i < \tau/\phi$. On the left: $\bar{a} = 0$, and condition (27) does not hold. On the right: $\bar{a} = 17$, and condition (27) holds.

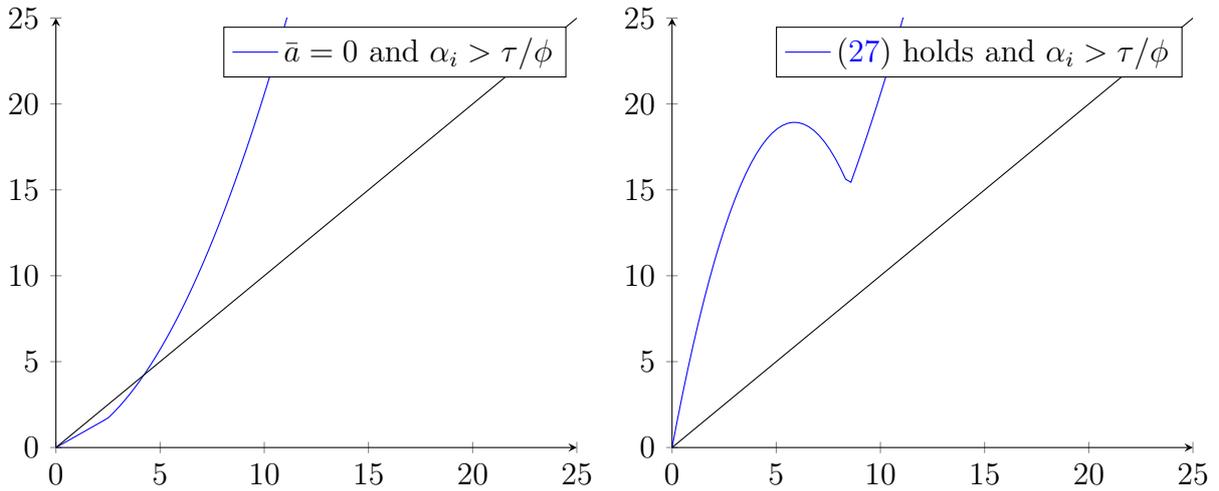


Figure 3: *Growth without bounds.* Parameters in function $G(k)$ are $\beta = 0.8; \tau = 0.4; \delta = 0.2; A = 0.4; \sigma = 2; \alpha_i = 0.8; \bar{a} = 17, b = 2, \phi = 2$ verifying conditions $r_a < 1$ and $\alpha_i > \tau/\phi$. On the left: $\bar{a} = 0$, and condition (27) does not hold. On the right: $\bar{a} = 17$, and condition (27) holds.

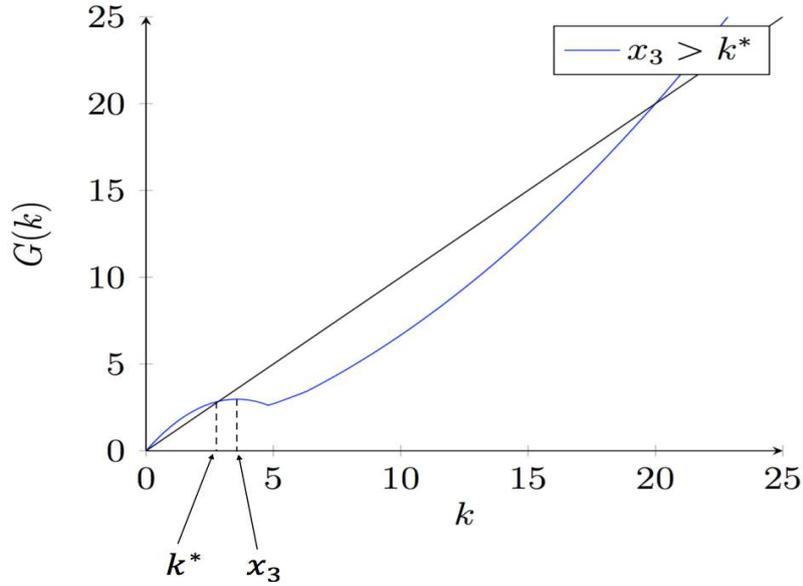


Figure 4: *Global stability of low steady state.* Parameters in function $G(k)$ are $\beta = 0.5, \tau = 0.2; \delta = 0.8, A = 0.5, \sigma = 0.8, \alpha_i = 0.8; \bar{a} = 10, b = 1, \phi = 2$, verifying condition (32) and $x_3 > k^*$. We have $\lim_{t \rightarrow \infty} k_t = k^*$ for any $k_0 \in (0, k^*)$.

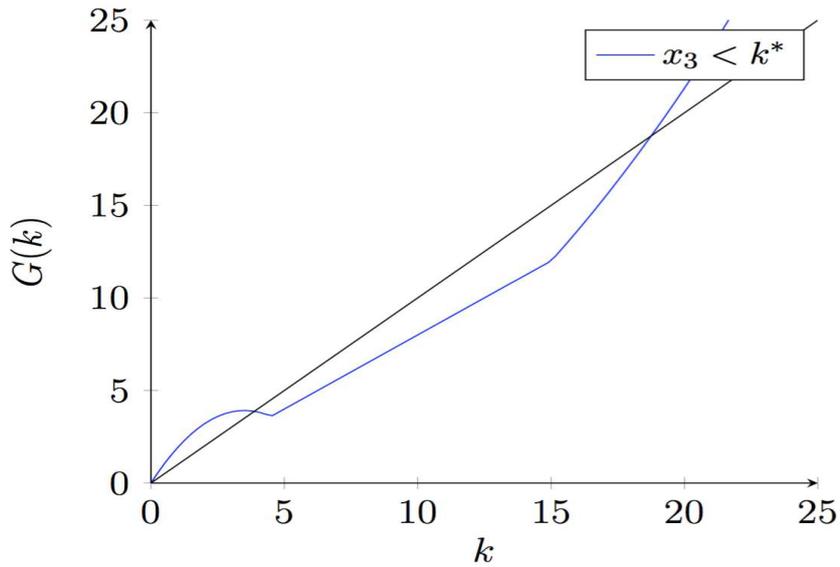


Figure 5: *Local stability of low steady state.* Parameters in function $G(k)$ are $\beta = 0.8, \tau = 0.2; \delta = 0.8, A = 0.4, \sigma = 1, \alpha_i = 0.7; \bar{a} = 12, b = 3, \phi = 2$, verifying conditions (32) and (33), $x_3 < k^*$.

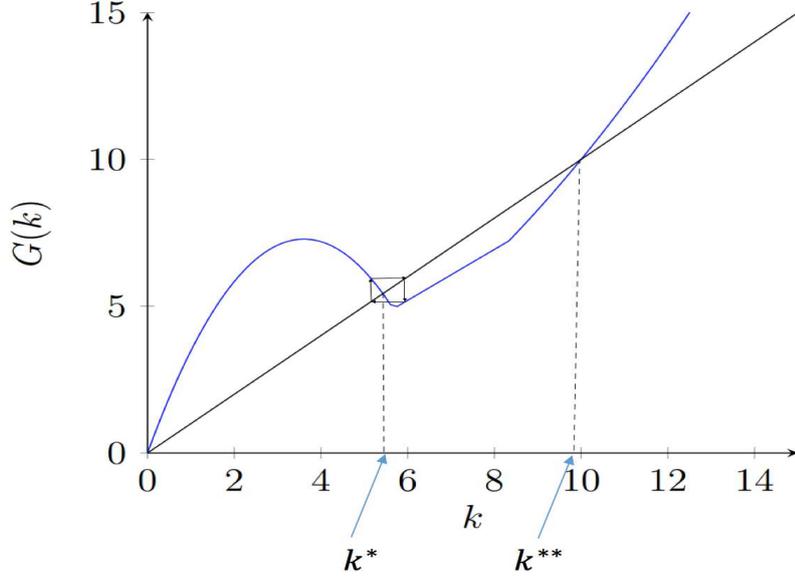


Figure 6: *Fluctuation around the low steady state.* Parameters in function $G(k)$ are $\beta = 0.8, \tau = 0.2; \delta = 0.2, A = 0.5, \sigma = 1.2, \alpha_i = 0.8; \bar{a} = 12, b = 2, \phi = 2$, verifying condition (34) and $x_3 < k^*$.

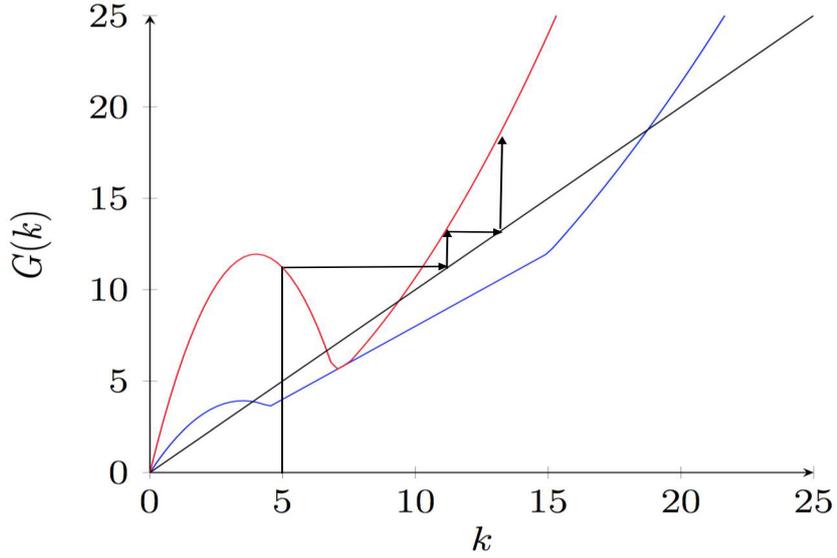


Figure 7: *Lucky growth vs. middle-income trap.* Low curve corresponding to $G(x_3) < G(k^{**})$, parameters in function G are $\beta = 0.8, \tau = 0.2; \delta = 0.2, A = 0.4, \sigma = 1, \alpha_i = 0.7; \bar{a} = 12, b = 3, \phi = 2$. High curve corresponding to $G(x_3) > G(k^{**})$, with $\bar{a} = 14, \alpha_i = 0.8; \sigma = 2$, other parameters unchanged.

“Sur quoi la fondera-t-il l'économie du monde qu'il veut gouverner? Sera-ce sur le caprice de chaque particulier? Quelle confusion! Sera-ce sur la justice? Il l'ignore.”

Pascal



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 **Contact**

www.ferdi.fr

contact@ferdi.fr

+33 (0)4 73 17 75 30