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Commodity market instability and asymmetries in developing countries:

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Les asymétries et l'instabilité du marché des matières premières dans les

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Farm storage and asymmetric price volatility

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Abstract

We analyze the role of farm storage on price volatility. Whereas speculative behaviours by stockholders are known to reduce price volatility, seasonal liquidity constraints on farmers behaviours with respect to stock management modify this general result. Like any stockholders, most farmers sell grain if they expect a price drop in the near future, but unlike stockholders, they often also sell grain for liquidity reasons even if they expect a price increase in the next period. Therefore, farm stock management is not as price stabilizing by nature as speculative stock management, notably because it fails to mitigate price drops. Under certain conditions that favor carry-over, farm stock management increases the occurrence of unexpected price drops. We merge historical maize price and household-level storage data in Burkina Faso in order to examine the latter through the use of a dynamic panel analysis over the 2004-2014 period.

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1 Introduction

The impact of food price volatility on farmers decisions is well documented, but the impact of farmers' stock management decisions on price volatility is not so well documented. The main existing explanations for price volatility on rural markets include international changes in demand and supply (FAO, IFAD, IMF, OECD, UNCTAD, WFP, BANK, WTO, IFPRI, and UNHLTF, 2011; Prakash, 2011) and domestic factors such as production shocks, public stock or public interventions (Hazell, Shields, and Shields, 2005; Demeke, Pangrazio, and Maetz, 2008; Tschirley and Jayne, 2010). The role of stockholder decisions on price dynamics has also been comprehensively analyzed, and it is generally accepted that speculative stock management has a smoothing effect on price volatility (Williams and Wright, 1991; Deaton and Laroque, 1992; Serra and Gil, 2013). A key feature of this literature rests on competitive storage theory, based on the "buy low, sell high" principle by Gustafson (1958). Stock release and storage tend to compensate for price changes. However, there is an asymmetry in this compensation, as price drops are better mitigated than price increases. Indeed, if stockholders have sold their entire stock, they cannot release more stock and thus cannot mitigate further price increases (Deaton and Laroque, 1992).

The competitive storage model describes stock management by professional stockholders, not by farmers. In contrast to professional stockholders, many farmers in Africa have a strong liquidity constraint, and this constraint fluctuates seasonally with the harvest. Limited cash liquidity and limited access to credit seriously hinder the "buy low, sell high" principle at the heart of speculative stock management. In many countries including Burkina Faso, where this study takes place, grain prices are often at their lowest level during the time of the harvest. A speculative behaviour model would predict that low grain prices make the harvest period an ideal time to purchase grain. However, many farmers have no liquidity during this period and consequently cannot purchase grain at this time. If they wish to consume non-grain goods or services such as educational tuition fees, they are forced to sell their current stock of grain at low prices. On the contrary, high prices during the lean season present a good opportunity for farmers to sell their stock. However, most of them have already sold out their stock by this time (Burke, 2014). In this context, farm stock management may instead correspond to a "sell low, buy high" principle, and, far from smoothing prices, this type of activity is more likely to amplify price shocks.

Another consequence of the liquidity constraint is the maximisation time-frame. In the context of a competitive storage model, decisions at time t depend only on the expected price at time $t + 1$, so that events further in the future do not matter when liquidity is available. The decision frequency can be years as in Deaton and Laroque (1992), months, or days. Because a farmers liquidity constraint typically follows a seasonal pattern, decisions regarding stock management depend on the entire spectrum of expected prices from the harvest to the end of the lean season. Economic decisions made directly after the harvest differ from decisions made during the lean season not solely because expected prices are different, but also because liquidity constraints differ. Therefore, at

harvest time, the farmer has to plan the use of his harvest over the whole year, which implies considering both today's price as well as the series of expected future prices until the next harvest. To model this decision process, we consider a simplified representation of the yearly price pattern, in which farmers' expected prices increase from the harvest until the lean season and then decrease following the price peak. If at month t a farmer believes the price of grain has reached its peak, i.e. if he expects a price decrease ¹ at month $t + 1$, he sells all of his grain surplus. This selling out is similar to speculative behaviour. If this occurs, there is no carry-over at the end of the farming year. But if the farmer misses the price peak, i.e. if prices drop at month $t + 1$ when the farmer had expected an increase, he may opt not to sell his surplus grain. The opportunity to sell the surplus is renewed at month $t + 1$ if he expects a further price drop at month $t + 2$. Otherwise, the farmer continues to hold his stock. Carry-over in our framework occurs when the farmer does not sell when prices are highest due to having misestimated the timing of the price peak. One or several unexpected price drops (i.e. an episode of negative volatility) occurring during the lean season produces conditions that favor a carry-over.

The model that we develop highlights the fact that if prices move as expected, farm stock management creates no price volatility. However, if expected prices differ from actual prices for some farmers, these farmers may re-evaluate their selling plan during the year. When farmers miss the price peak, this can result in carry-over, which in turn is a source of negative price volatility in future months. If other farmers are unaware that some farmers continue to hold surplus grain from a carry-over, this is likely to create a sharp unexpected price drop in the next harvest period. We test the empirical relevance of these theoretical predictions using maize price and carry-over data from Burkina Faso. We show how farm storage behavior, combined with price anticipation errors, explain why farm stock management does not mitigate price drops, and can even create or amplify them.

Price volatility is defined in this paper as the unpredictable component of price shifts. The empirical assessment of the statement above rests on the empirical determination of what is predictable, which is a crucial issue we address. To do this, we utilize an autoregressive estimation of price at month t where the right hand side of the equation is a linear combination of variables measured at month $t - 1$. The deterministic part of the right hand side is the predicted price for month t as predicted at month $t - 1$, and we use this value as the expected price at $t - 1$. Each monthly value of the error term is the unexpected increase or drop in price. The variance of these unpredicted shifts over a certain period of time represents price volatility, which is in general not constant. We thus distinguish between negative volatility, or the variance of unpredicted price drops, and positive volatility, or the variance of unpredicted price increases.

The next section reviews the economic literature on the impact of storage decisions on price volatility. Section 3 models the dynamics of farm stock management and illustrates the impact of a late price-anticipation error on the occurrence of carry-over and on negative price volatility. Section 4 presents the Burkina Faso case and describes the

¹or an increase by less than his discount rate

empirical strategy for testing these features, which builds upon an ARCH model and a dynamic panel analysis to establish the relationship between farm storage and price volatility. Section 5 provides empirical evidence that carry-overs increase unexpected price drops.

2 The literature on the effect of storage on price volatility

2.1 Measuring price volatility

Several definitions of food price volatility have been published that differentiate price variation from price volatility and isolate regular patterns of price variation (price trend and price seasonality in particular) from irregular ones (Huchet-Bourdon, 2011; Piot-Lepetit and M'Barek, 2011; Prakash, 2011). In line with these works, we refer to price volatility as the unpredictable component of price variations, assuming that what is irregular is less predictable. It is not our purpose to discuss whether unpredictable price variations are more or less harmful than predictable variations (FAO, IFAD, IMF, OECD, UNCTAD, WFP, BANK, WTO, IFPRI, and UNHLTF, 2011; Prakash, 2011). Our main purpose in differentiating between these two components of price variation is to avoid improperly including the regular fluctuation of prices, which do not surprise economic agents, into the measure of price volatility.

The empirical measurement of unpredictable price variability requires several assumptions regarding the ability of agents to anticipate prices. Notably, we assume that a price series is less volatile for well-informed agents than for poorly informed agents. For instance, the same seasonal price fluctuations in West Africa that may be perceived as volatile by an outsider, may be regarded as perfectly predictable by local farmers. We must therefore choose a benchmark level of information intended to reflect the amount of information that is available to farmers, and a specific price forecast model to represent their ability to anticipate prices. The series of differences between actual prices and predicted prices can be interpreted as unpredicted price shifts, and it is the variance of these residuals that provides a measure of volatility. After the seminal work of Engle (1982), several authors have used the conditional variance of prices as an indicator of price volatility through the use of Auto-Regressive Conditional Heteroskedastic (ARCH) models (Shively, 1996; Barrett, 1997; Yang, Haigh, and Leatham, 2001; Karanja, Kuyvenhoven, and Moll, 2003; Nyange and Wobst, 2005; Maître d'Hôtel, Le Cotty, and Jayne, 2013). The use of ARCH-family models is adapted to the measurement of volatility because the variance of the residuals is not required to be constant, and as a consequence, the measure of volatility can be a series of values, the different levels of which can be interpreted.

A few studies on volatility distinguish between negative and positive price moves. Most deal with asymmetric price transmission, finding that price spikes in a central market are transmitted to peripheral markets more rapidly and with a greater magnitude than price drops (Abdulai, 2000; Meyer and Cramon-Taubadel, 2004). The measurement of asymmetries in volatility can be accomplished through the analysis of price distribution

(Deaton and Laroque, 1992), or by the analysis of the residuals in autoregressive models, which we develop below.

2.2 The competitive storage model

The competitive storage model explains the role of storage in price volatility, and was originally applied to commodities that may be stored from one year to another and are subject to random production shocks (Gustafson, 1958; Cafiero, Bobenrieth, Bobenrieth, and Wright, 2011). In this model, the demand for storage depends on consumption, yields, storage costs, interest rates, and expected prices in the next year. In general, competitive storage models smooth price variations. Nevertheless, this model is more efficient for handling price drops than price spikes. Since storage cannot be negative, stocks "cannot be borrowed from the future" (Wright, 2011), and thus, in the event of a shortfall, stocks are inefficient in mitigating price increases. Indirect estimations of this storage effect on price volatility have been carried out without stock data (Deaton and Laroque, 1992), and more direct estimations remain difficult because of the scarcity of data series on stocks (Bobenrieth, Wright, and Zeng, 2013; Serra and Gil, 2013). Nevertheless, the competitive storage model provides a theoretically useful framework to understand the main links between storage and volatility.

2.3 Empirical studies on storage and price

The empirical literature confirms that periods with low stocks correspond to price spikes (Wright, 2011). At the international level, this has been true for each of the last three episodes of low grain stocks: the early 1970s, mid 1990s, and late 2000s. When stocks are low, a small production or consumption shock can have large price impacts because adjustments are characterized by greater inelasticity (Williams and Wright, 1991; Gilbert and Morgan, 2010). Working on nine commodities at the international level, Balcombe (2009) estimates a panel model with annual data and shows that stocks significantly reduce price volatility.

At the national level, a majority of empirical studies come to the conclusion that price volatility increases as public inventory declines (Barrett, 1997; Nyange and Wobst, 2005; Jayne, Myers, and Nyoro, 2008). Simulation approaches have been used to compare a no-storage regime to a storage regime and, when applied to soybean prices in USA, these approaches have shown that storage greatly reduces the variance of annual prices (Helmberger and Akinyosoye, 1984). However, many empirical studies question the effect of storage on volatility. In Madagascar, Barrett (1997) used an ARCH model to analyze price volatility in the rice market and found evidence that storage had no significant effect on price volatility (Barrett, 1997). The application of another ARCH model applied to maize prices in Tanzania revealed that storage policies had no effect on price volatility (Nyange and Wobst, 2005). At the international level, Roache (2010) also found that storage did not have a significant impact on price variability. Given these mixed results,

the empirical study of the effect of storage on price volatility still deserves attention.

2.4 Is farm storage different from speculative storage ?

The drivers of on-farm storage decisions differ from those of speculative storage decisions. Prices in Africa generally follow a cyclical pattern every year. They are at their lowest level just after harvest and then increase until the lean season before falling again at the time of the next harvest. The predictability of this price dynamic creates incentives to store (Abbott, 2010). The monthly counterpart of the yearly competitive model implies that agents purchase grain at the harvest season and store it until the lean season. The "buy low, sell high" principle that applies in the competitive storage model for professional traders should also apply for farmers, as well. In reality, however, this decision rule does not fit with farmers storage dynamics. After the lean season, farmers have a strong liquidity constraint, and typically cannot purchase grain, as explicitly included in Fackler and Livingston (2002)'s model. In Africa, however, farmers are not only restricted in their purchasing power, but they are also forced to sell at harvest time in order to consume non-grain goods and services, unless they have alternative sources of income. Therefore, speculative grain sales are also restricted, and sales also occur at the time of harvest, when prices are low. When the lean season arrives, prices increase again, and many farmers in Africa find it difficult to sell because they have already sold most of their harvest. This pattern has been described in many African contexts (Saha and Stroud, 1994; Kazianga and Udry, 2006; Burke, 2014).

Moreover, even though producers have an incentive to sell when prices are high, they may prefer to keep their grain stock to ensure family consumption until the next harvest, especially when production and consumption decisions are closely linked. In addition to consumption purposes, on-farm storage management is governed by profit purposes. In this vein, the competitive storage literature also singles out price arbitrage as an important determinant of storage decisions. However, price arbitrage may only constitute a partial reason for farmers to store, as they must also meet the needs of their households consumption (Saha and Stroud, 1994). Producers are thus jointly maximizing profits and minimizing consumption risks, as possessing an adequate stock of grain prevents them from buying food when prices are high (Park, 2009). That means that on-farm storage can be considered as an insurance for producers against starvation until next harvest (Saha and Stroud, 1994).

Because of the rationale behind on farm storage, the impact of on-farm storage on price volatility cannot be accounted for through the competitive storage model, and this assertion has not yet been documented. The purpose of the present paper is to describe farmers storage behavior in developing countries, including the cyclical aspects of production, price, and on-farm storage, and the consequences in terms of positive and negative volatility. To do so, we develop a conceptual framework concerning the dynamics of stocks and we illustrate the impact of a late price-anticipation error on carry-overs and on the occurrence of negative price volatility. We then empirically measure the main elements of these features, building upon historical price and household data in Burkina

Faso.

3 The seasonal dynamics of storage

At the time of the harvest, the farmer must anticipate the pattern of prices over the next 12 months and decide on a sales plan for his grain surplus based on the expected difference between grain production and the consumption needs of the family. The sales plan consists in deciding on the amount of grain the farmer intends to sell in each coming month, and can be revised every month after observing actual prices and updating expectations about future prices. The production surplus y^j of year j is observed after the harvest and is not subject to revision during the year. We denote $E_t P_i$ as the price expectation formulated at month t for expected price at month i .

At the time of the harvest, arbitrarily denoted month 1, the farmer observes present price P_1 and formulates price expectations for each subsequent month, so that his sales plan is based on the price series $\{P_1, E_1 P_2, \dots, E_1 P_{12}, E_1 P_{13}, \dots, E_1 P_{12(k-1)+12}\}$. The index k is the last year that the farmer considers in his plan, which can be the year of his retirement or sooner. The initial sales plan, established at month 1 of the present year is written $\{x_1, \tilde{x}_2^1, \dots, \tilde{x}_{12}^1, \tilde{x}_{13}^1, \dots, \tilde{x}_{12(k-1)+12}^1\}$, where x_1 is the actual sales at month 1, and \tilde{x}_t^1 is what the farmer, at month 1, plans to sell at month t given his price expectations at month 1. In the same way, we denote $\{c_1, \tilde{c}_2^1, \dots, \tilde{c}_{12}^1, \tilde{c}_{13}^1, \dots, \tilde{c}_{12(k-1)+12}^1\}$ as the farmer's consumption plan in terms of a generic good purchased at a constant price arbitrarily set to unity.

The rationale for each selling decision in the speculative behaviour model is similar to that found in the competitive storage model. If a farmer expects the price of grain to decrease between the current month and the following month, or if he expects next months discounted price to be inferior to the present price, he sells some surplus, just as a stockholder (Deaton and Laroque, 1992) or speculative producer Fackler and Livingston (2002) would do. If he is price taker, he sells all of his remaining surplus. If the price of grain is expected to decrease, selling is his best option, whether the profit is destined for non-grain purchases or for savings. Nevertheless, when prices are expected to increase, contrary to stockholders, who purchase grain in this situation, farmers sell grain due to a liquidity constraint, as this allows them to purchase non-grain goods (we assume that credit does not exist). Therefore, throughout the post harvest season and until the price peak in the lean season, i.e. when prices are generally increasing and monetary savings are zero, every month farmers sell the exact amount of grain needed to support their families monthly non-grain consumption.

In the case of a binding cash constraint, a farmer's selling decisions depend simultaneously on present prices as well as on the entire sequence of expected future prices up until the expected price peak, which is the point at which they plan to sell their remaining surplus. When the farmer believes the price peak has been reached, he sells out his stock, and reserves some of the money from this sale in order to buy non-grain food until the next harvest. The existence of an expected price peak is defined by following

conditions

$$\begin{aligned} \exists T \in [2; 12] : \quad & \forall t \in [1; T[, \quad E_t P_{t+1} > (1 + \delta)P_t \\ & E_T P_{T+1} < (1 + \delta)P_T \end{aligned}$$

The price peak does not need to be unique in order for the farmer to sell his surplus. To keep the model simple, each farmer is assumed to expect prices to increase each month from the time of the harvest until the expected price peak, and to decrease immediately following the peak (see figure 2). This price drop frequently occurs before the next harvest, either for exogenous reasons (like public stock release or grain imports), or endogenous reasons (for example, if farmers expect a price drop in October, they will tend to sell their remaining surplus in September). Because some farmers are price makers, this can create a price drop in September, in which case the wisest farmers would sell their surplus in August. Since every year is different, farmers may have to reconsider their sales plan every month in light of new information regarding prices, or the likely date of the next harvest. Since expectations based on this information are subjective, stock releases may produce price volatility. The model presented here analyses how changes in expectations about future prices impact positive and negative price volatility. An interesting case of mis-anticipation is the case of carry-over. If prices drop before a farmer expects, he has some remaining stock to sell at this time. If he believes that prices will increase again, however, he refrains from selling this stock and this decision has the potential to result in carry-over, which occurs when the next seasons yield is harvested before the remaining stock from the previous harvest is sold. Thus, unexpected price drops (negative price volatility) are likely to induce carry-over, which in turn is likely to produce further unexpected price drops. For this reason, in contrast to the competitive storage model, farm stock management is likely to mitigate price peaks more easily than price drops.

3.1 The initial sales plan

The farmer's initial sales plan at the time of the harvest is subject to a yearly resource constraint to which we attribute the Lagrange multiplier μ_j for year j and to monthly budget constraints, to which we attribute Lagrange multipliers λ_i for month i of current year, and λ_{ji} for month i of year j . Taking as our starting point a situation in which there is no carry-over, the sum of the amounts of grain that the farmer expects to sell each month for the first year is inferior or equal to the grain surplus on hand at the beginning of the year.

$$\mu_1, \quad y^1 - x_1 - \tilde{x}_2^1 - \dots - \tilde{x}_T^1 \geq 0 \quad (1)$$

Recall that the farmer plans to sell out his surplus at the expected price peak T , $\tilde{x}_{T+1}^1 = \dots = \tilde{x}_{12}^1 = 0$.

Similarly, the sum of the stock that is expected to be sold in the second year is inferior or equal to the expected production surplus in the second year, since at month 1 (of the first year) the farmer does not intend to have any carry-over at the end of the year.

To keep our analysis simple, our model does not account for those farmers who could engage in speculative purchases at this time.

$$\mu_2, \quad \tilde{y}^2 - \tilde{x}_{13}^1 - \dots - \tilde{x}_{12+T}^1 \geq 0$$

We note \tilde{y}^2 as the production surplus of year 2 when it is unknown and y^2 as the production surplus of year 2 when it is known. Identically, the resource constraint for year k as it is perceived at month 1 in year 1 is :

$$\mu_k, \quad \tilde{y}^k - \sum_{i=1}^T \tilde{x}_{12(k-1)+i}^1 \geq 0$$

The monthly consumption of the non-grain good is additionally limited by the monthly budget, which depends on monthly grain sales and eventually, savings. As the last sale of the year is planned to occur at month T , there are T planned budget constraints per year, leading to a maximum of $12k$ budget constraints in the intertemporal maximization model. If we assume that the farmer expects the price peak to occur at the same time every year, there are only Tk budget constraints. In this section we consider the case in which cash savings between one year and the next is impossible. In the Appendix we treat the case in which cash may be saved from one year to another and show that this savings activity eliminates the seasonal nature of our problem. Constraints on cash liquidity is thus the root of the problem we observe, and the most convenient way to study it is to suppose that saving is not possible from year to year.

$$\lambda_1 \quad x_1 P_1 - c_1 \geq 0 \quad (2)$$

The budget constraint for the second month is :

$$\lambda_2 \quad x_1 P_1 - c_1 + \tilde{x}_2^1 E_1 P_2 - \tilde{c}_2^1 \geq 0 \quad (3)$$

and this constraint holds for each month until the month T during which the farmers expects the price of grain to reach its peak. So $\forall t < T$,

$$\lambda_t \quad x_1 P_1 - c_1 + \sum_{i=2}^t (\tilde{x}_i^1 E_1 P_i - \tilde{c}_i^1) \geq 0 \quad (4)$$

At month T , the farmer sells all of his remaining stock and uses the proceeds from this sale for consumption in months T to 12, before the next harvest. That is

$$\lambda_T \quad x_1 P_1 - c_1 + \sum_{i=2}^T \tilde{x}_i^1 E_1 P_i - \sum_{i=2}^{12} \tilde{c}_i^1 \geq 0 \quad (5)$$

The budget constraint for the first month of year 2 is

$$\lambda_{13} \quad x_{13} E_1 P_{13} - c_{13} \geq 0$$

And the budget constraint for year k , month T is

$$\lambda_{kT} \quad \sum_{i=1}^T \tilde{x}_{12(k-1)+i}^1 E_1 P_{12(k-1)+i} - \sum_{i=1}^{12} \tilde{c}_{12(j-1)+i}^1 \geq 0 \quad (6)$$

Subject to these constraints, the farmer chooses a current consumption level c_1 and amount of sales x_1 , as well as future levels of consumption and sales. Assuming a constant relative risk aversion expected utility function, the farmer's decision is written as

$$EU = \max_{c_1, \tilde{c}_2^1, \dots, \tilde{c}_{12(k-1)+12}^1, x_1, \tilde{x}_2^1, \dots, \tilde{x}_{12(k-1)+T}^1} \frac{c_1^{1-r}}{1-r} + \sum_{j=1}^k \sum_{i=1}^{12} \frac{1}{(1+\delta)^{12(j-1)+i-1}} \frac{(\tilde{c}_{12(j-1)+i}^1)^{1-r}}{1-r} \quad (7)$$

Where δ is the monthly individual discount rate. The derivation of the farmer's objective function with respect to endogenous variables leads to a solution denominated in terms of planned consumption and sales in month t , since the farmer formulates the plan at month 1 (see details in the Appendix).

$$\begin{aligned} \forall t \in [1, T[\quad \tilde{c}_t^1 &= \left(\frac{E_1 P_t}{\mu_1 (1+\delta)^{t-1}} \right)^{\frac{1}{r}} \\ \forall t \in [T, 12] \quad \tilde{c}_t^1 &= \left(\frac{E_1 P_T}{\mu_1 (1+\delta)^{t-1}} \right)^{\frac{1}{r}} \end{aligned}$$

The planned consumption of non-grain goods at month t increases with $E_1 P_t$ and decreases with δ . Since $E_1 P_t$ increases with t until T , planned consumption can either increase with time or decrease with time between month 1 and month T . Using equation (1), we eliminate μ_1 to get

$$\tilde{c}_t^1 = \frac{y^1 E_1 P_t^{\frac{1}{r}-1}}{P_1^{\frac{1}{r}-1} (1+\delta)^{\frac{t-1}{r}} + \sum_{i=2}^{T-1} (E_1 P_i)^{\frac{1}{r}-1} (1+\delta)^{\frac{t-i}{r}} + (E_1 P_T)^{\frac{1}{r}-1} \sum_{i=T}^{12} (1+\delta)^{\frac{t-i}{r}}}$$

Furthermore, using budget constraint equations (42) to (5), we obtain

$$\tilde{x}_t^1 = \begin{cases} \frac{\tilde{c}_t^1}{E_1 P_t} & \text{if } t < T, \\ y^1 - \sum_{i=1}^{T-1} \frac{\tilde{c}_i^1}{E_1 P_i} & \text{if } t = T \\ 0 & \text{if } t > T \end{cases} \quad (8)$$

3.2 The revised sales plan

If all farmers correctly anticipate prices from the time of the harvest until the end of the cropping year and maintain these expectations, actual consumption at month t is as calculated above: $c_t = \tilde{c}_t^1$. The first months sales plan perfectly predicts the sales and price pattern for the coming year, and is not revised either at the second month or at any month until the next harvest. At month T all farmers anticipate that prices will drop at month $T + 1$, that is $E_T P_{T+1} < P_T(1 + \delta)$, and they sell all of their remaining surplus grain at T . Note that this surplus is not large enough to produce a price drop at month T , which would invalidate their anticipation made at $T - 1$: $E_{T-1} P_T > P_{T-1}(1 + \delta)$. This holds only if the surplus left at T is small enough with respect to demand at T so that the price of grain does not drop as a result of the sales made. At month $T + 1$, the price of grain falls due to farmers anticipations of the imminent harvest. Farmers may anticipate the arrival of the next harvest when grain from the harvest begins to appear on the market or when grain from neighboring countries that begin their harvest earlier than Burkina Faso start to become available on the market. This scenario describes the perfect anticipation case. We keep this situation in mind as the benchmark price pattern, in which all farmers correctly anticipate yearly price patterns including the price peak. Because these patterns are correctly anticipated by every farmer, they do not represent volatility.

However, if at $T - 1$, some farmers expect a price drop at T instead of at $T + 1$, they will sell all of their remaining surplus at $T - 1$, which can either lead to a fall in the price of grain at $T - 1$, or to an attenuation of a price increase at $T - 1$. This action also reduces their own grain stock at T , which eventually increases the peak price at T if these farmers are among those who influence the price of grain. This early sale of surplus grain by a few produces a price anticipation error for the rest of the community, who did not expect a price drop at $T - 1$. The rest of the community may have sold more grain at $T - 2$ if they had anticipated the price drop. If many farmers make the same error and sell their surplus grain at $T - 1$ instead of T , the price of grain may actually be lower at $T - 1$ than at $T - 2$, and higher at T than at $T - 1$. This activity feeds negative price volatility, and because the price of grain is potentially higher in T than people originally expected, it also feeds positive price volatility, as well. Calling c_i the actual consumption at time i , we proceed to a similar maximisation as above for farmers choices at time t given all past choices, and derive the following optimal consumption:

$$c_t = \frac{(y^1 - \sum_{i=1}^{t-1} \frac{c_i}{P_i}) P_t^{\frac{1}{r}}}{\sum_{i=t}^{T-1} E_t P_i^{\frac{1}{r}-1} (1 + \delta)^{\frac{t-i}{r}} + E_t P_T^{\frac{1}{r}-1} \sum_{i=T}^{12} (1 + \delta)^{\frac{t-i}{r}}} \quad (9)$$

This expression is equal to the above expression of \tilde{c}_t^1 if and only if there are no price anticipation errors until month t , i.e. $\forall i, l \in [1, t - 1]$, with $i < l$, $E_i P_l = P_l$. In the case of anticipation errors, equation (9) holds.

3.3 The case for carry-over

We have explained above that, before month T of the expected price peak, a rational farmer does not plan to carry-over any stock because he plans to sell all remaining surplus at T . However, if he misses the price peak, i.e. $P_T < (1 + \delta)P_{T-1}$ and $E_{T-1}P_T > (1 + \delta)P_{T-1}$, he does not sell all of his surplus at $T - 1$. He may then either sell his surplus at T if he expects a further price decrease in $T + 1$, or continue to wait to sell if he expects another price increase in $T + 1$. If at $t = 11$ he expects price increase at $t = 12$, he keeps his surplus as carry-over.

The case for carry-over requires two conditions : the first is that the farmer misses the price peak, i.e. prices fell before he expected, and the second is that he expects a price increase following the price drop and before the next harvest.

A necessary and sufficient condition for carry-over is that

$$\forall t \in [1, 12], E_t P_{t+1} > (1 + \delta)P_t$$

Since prices generally drop at least once a year, discounted prices decrease at least once a year, so that the above pattern of expectations is not likely to occur unless the farmer has been surprised by an unanticipated price drop. This is why missing the price peak is a key condition for the carry-over, i.e.

$$E_{T-1}P_T > (1 + \delta)P_{T-1} > P_T$$

An unexpected price drop is necessary to produce carry-over since expected price drops are anticipated by selling out any remaining surplus grain. According to this framework, if all price drops occurred at the same time every year, carry-over would never occur.

The following function describes a farmers choice at month 12 after having missed the price peak and thus retaining a surplus of grain.

$$EU = \max_{c_{12}, \tilde{c}_{13}^{12}, \dots, \tilde{c}_{12(k-1)+12}^{12}, x_{12}, \tilde{x}_{13}^{12}, \tilde{x}_{12(k-1)+12}^{12}} \frac{c_{12}^{1-r}}{1-r} + \sum_{j=2}^k \sum_{i=1}^{12} \frac{1}{(1+\delta)^{12(j-2)+i}} \frac{(\tilde{c}_{12(j-1)+i}^1)^{1-r}}{1-r}$$

In order to model choices relating to carry-over, we integrate into the resource constraint sales made at any time in the current year, including at $t = T + 1, t = T + 2, \dots, t = 12$, that is

$$\mu_1, \quad y^1 - \sum_{i=1}^{12} x_i \geq 0$$

For simplicity, we retain the same symbols for Lagrange multipliers as in the previous section, although the associated constraints have changed. Expected sales for the second year end at month $12 + T$, which is the expected price peak month for year 2, and we assume that the price peak occurs during the same month every year.

$$\mu_2, \quad y^1 - \sum_{i=1}^{12} x_i + y^2 - \sum_{i=13}^{12+T} \tilde{x}_i^{12} \geq 0 \quad (10)$$

and identically until the year k :

$$\mu_k, \quad \sum_{j=1}^k y^j - \sum_{i=1}^{12} x_i - \sum_{j=2}^k \sum_{i=12(j-1)+1}^{12(j-1)+T} \tilde{x}_i^{12} \geq 0$$

The budget constraint for the 12th month of the current year is:

$$\lambda_{12} \quad x_{12}P_{12} - c_{12} \geq 0$$

for the first month of the second year

$$\lambda_{13} \quad x_{12}P_{12} - c_{12} + \tilde{x}_{13}^{12}E_{12}P_{13} - \tilde{c}_{13}^{12} \geq 0$$

For year k , month $t < T$:

$$\begin{aligned} \lambda_{kt} \quad x_{12}P_{12} - c_{12} + \sum_{j=2}^{k-1} \left(\sum_{i=1}^T \tilde{x}_{12(j-1)+i}^{12} E_{12}P_{12(j-1)+i} - \sum_{i=1}^{12} \tilde{c}_{12(j-1)+i}^{12} \right) \\ + \sum_{i=1}^t \tilde{x}_{12(k-1)+i}^{12} E_{12}P_{12(k-1)+i} - \sum_{i=1}^t \tilde{c}_{12(k-1)+i}^{12} \geq 0 \end{aligned}$$

and identically until the T th month of year k :

$$\lambda_{kT} \quad x_{12}P_{12} - c_{12} + \sum_{j=2}^k \left(\sum_{i=1}^T \tilde{x}_{12(j-1)+i}^{12} E_{12}P_{12(j-1)+i} - \sum_{i=1}^{12} \tilde{c}_{12(j-1)+i}^{12} \right) \geq 0$$

The solution to this objective function gives us the general term for consumption in the second year as the farmer plans it in month 12 of the first year:

$$\tilde{c}_{12+t}^{12} = \frac{1}{(1+\delta)^{\frac{t-1}{r}}} \left(\frac{E_{12}P_{12+t}}{\sum_{j=2}^k \mu_j} \right)^{\frac{1}{r}} \quad (11)$$

and planned sales for the second year are as follows;

$$\begin{aligned} \forall t < T, \quad \tilde{x}_{12+t}^{12} &= \frac{\tilde{c}_{12+t}^{12}}{E_{12}P_{12+t}} \\ t = T \quad \tilde{x}_{12+T}^{12} &= y^1 - \sum_{i=1}^{12} x_i + y^2 - \sum_{i=1}^T \frac{\tilde{c}_{12+i}^{12}}{E_{12}P_{12+i}} \\ \forall t > T, \quad \tilde{x}_{12+t}^{12} &= 0 \end{aligned}$$

The carry-over at the end of year 1 is denoted χ^1 and defined by $\chi^1 = y^1 - \sum_{i=1}^{12} x_i$ is given by the equation from inequation (10) where $\mu_2 > 0$, i.e. $\chi^1 = \sum_{i=13}^{12+T} \tilde{x}_i^{12} - \tilde{y}^2$ To

summarize, by replacing the value of sales we obtain the following expression, in which it is clear that high expected prices in year 2 increase carry-over at the end of year 1 and high prices in year 1 decrease carry-over into year 2.

$$\chi_1 = \begin{cases} \frac{\sum_{i=1}^T \frac{(E_{12}P_{12+i})^{\frac{1}{r}-1}}{(1+\delta)^{\frac{i}{r}}} + \sum_{i=T+1}^{12} \frac{(E_{12}P_{12+i})^{\frac{1}{r}-1}}{(1+\delta)^{\frac{i}{r}}}}{\sum_{i=1}^{12} \frac{(P_i)^{\frac{1}{r}-1}}{(1+\delta)^{\frac{i-12}{r}}}} y^1 - y^2 & \text{if } \forall t \in [1, 12], E_t P_{t+1} > (1 + \delta) P_t \\ 0 & \text{if } \exists T \in [1, 12], E_T P_{T+1} < (1 + \delta) P_T \end{cases} \quad (12)$$

In particular, we observe that anticipation errors that overestimate future prices after the price peak increase the likelihood of carry-over. If $P_{T+1} < (1 + \delta) P_T$, which occurs at least once a year in a seasonal pattern, there is no carry-over if there is no anticipation error. Thus, the likeliness of carry-over increases with anticipation error after a price peak

$$\frac{\partial \text{Prob}(\chi_1 > 0)}{\partial (E_T P_{T+1} - P_{T+1})} > 0 \quad (13)$$

Proposition 1. *Unexpected price drops -i.e. negative price volatility- during the lean season increase the likelihood of carry-over*

3.4 The equilibrium price

Demand is assumed to be generated by monthly income I_t , whereas supply is generated by yearly income y so that the utility function of food consumption is not discounted on a monthly basis. In the interest of simplicity, we choose a CRRA utility function for the consumer which can be described as follows:

$$U_t = \max_{d_t} \frac{d_t^{1-\rho}}{1-\rho} + \gamma(I_t - P_t d_t)$$

Where I_t stands for the consumers monthly budget, d_t is the demand for grain at month t , ρ is a parameter representing the consumers risk aversion, and γ is the Lagrange multiplier associated with the budget constraint. The derivation of this utility function yields the demand for grain:

$$d_t = (\gamma p_t)^{-1/\rho}$$

Market clearing results from the demand and supply functions, above, of the farmer who imperfectly anticipates the price peak, x_t . Since we wish to understand the impact of carry-over on volatility, we examine the market price of grain in the year following the carry-over. To do this, we denote P_{12+t}^* as the market-clearing price of grain at time t

in the second year.

$$\forall t < T, \quad x_{12+t}(\chi^1, y^2, P_{13}, \dots, P_{12+t}, E_{12+t}P_{12+t+1}, \dots, E_{12+t}P_{12+12}) = d_{12+t}(P_{12+t}) \quad (14)$$

where y is known by everyone, but χ is unknown by everyone except for the farmer who retains this carry-over. Rearranging, we derive an expression of the price in year 2, month t :

$$P_{12+t}^{\frac{1}{r} + \frac{1}{\rho} - 1} = \frac{\sum_{i=12+t}^{12+T} E_i P_t^{\frac{1}{r} - 1} (1 + \delta)^{\frac{t-i}{r}} + E_t P_T^{\frac{1}{r} - 1} \sum_{i=12+T+1}^{24} (1 + \delta)^{\frac{t-i}{r}}}{(y^2 + \chi^1 - \sum_{i=13}^{12+t-1} \frac{c_i}{P_i}) \gamma^{1/\rho}}$$

from which we observe that

$$\frac{\partial P_{12+t}}{\partial \chi^1} < 0 \quad (15)$$

Since the carry-over is unknown by most farmers, it does not impact their expectations about the price of grain, so that $\frac{\partial E_{12+t-1} P_{12+t}}{\partial \chi^1} = 0$, thus

$$\frac{\partial (P_{12+t} - E_{12+t-1} P_{12+t})}{\partial \chi^1} < 0 \quad (16)$$

which means that, as long as it is unknown by other farmers, a carry-over generates episodes of unexpected price drops. We can show that this impact decreases with time since $\frac{\partial |\frac{\partial (P_{12+t} - E_{12+t-1} P_{12+t})}{\partial \chi^1}|}{\partial t} < 0$, meaning that unexpected price drops generated by carry-overs occur especially during the first month after harvest and to a lesser extent during the following months.

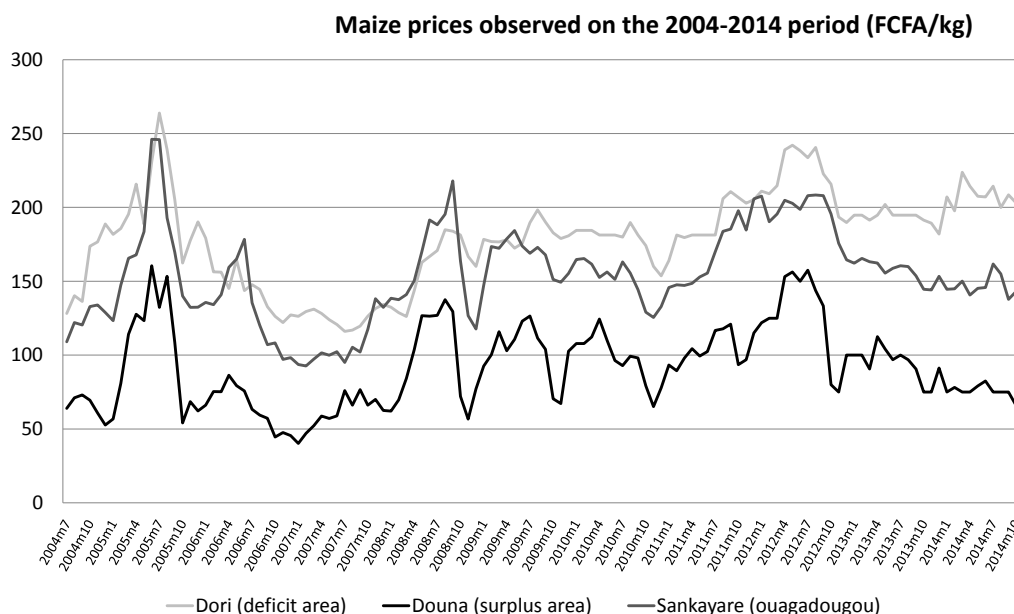
Proposition 2 *carry-over generates negative price volatility -i.e. unexpected price drops- after harvest*

4 Empirical strategy

4.1 Maize prices and storage in Burkina Faso

Maize production is a strategic activity with respect to food security and agricultural development in Burkina Faso. Maize is appreciated and consumed by Burkinabe people, both in rural and urban areas, and the production of maize has significantly increased in the last decade, rising at a faster pace than other cereals (millet and sorghum), notably in the western and southern parts of the country. Furthermore, its production is increasingly market oriented. Surveys we conducted in the Boucle du Mouhoun and Hauts Bassins regions established that 47% of the grain harvested was sold, and the remaining 53% was consumed by households. Maize is mainly traded within the country, flowing from surplus to deficit regions, and is occasionally exported to Niger and Mali, as well as imported from the Ivory Coast and Ghana.

Figure 1: Real maize prices in Burkina Faso, 3 markets, 10 years



SONAGESS (Societe Nationale de Gestion du Stock de Securite) regularly collects data on agricultural prices in 48 markets throughout the country. The data are collected on a weekly basis and published on a monthly basis. We use a subset of 33 series of maize prices collected during 2004-2014. Monthly prices have been deflated using the Burkina Consumer Price Index obtained from INSD (Institut National des Statistiques Demographiques). The evolution of maize producer real prices is represented in Figure 1 for three markets: one market that is located in a surplus area, another that is located in a deficit area, and a final market that located somewhere in-between and close to the capital city of Ouagadougou. While grain prices are higher in deficit areas than in surplus areas, they follow a similar dynamic from one market to another. This dynamic is characterized by a seasonal pattern in which prices are high from July to September, corresponding to the lean season in Burkina Faso, and low between October and December, corresponding to the harvest season. In each of the three markets studied, price rises and drops were quite pronounced in 2005, 2008, 2009 and 2012. In these years, price fluctuations were mainly associated with poor harvests, related to events such as insect infestations (2005), episodes of drought (2009 and 2012), and international price spikes (2008 and 2012).

The Burkinabe Ministry of Agriculture has been collecting data on agricultural produc-

tion since 1992 through the implementation of a detailed annual rural household survey. Once a year, an average of 4500 rural households are interviewed and their agricultural production measured. The survey relies upon a stratification method by administrative levels and a randomization method within administrative levels. Interviewed households are therefore assumed to be representative of Burkinabe rural households. To combine household data with price data, we use a subset of available household data that corresponds to the 33 different provinces for which we have maize price data. From this subset, we use data on annual maize production as well as maize carry-over, which is defined as the amount of on-farm maize stock remaining when the next harvest arrives following the end of the lean season. Individual data have been aggregated at the province level and we use an average of individuals market activity data for each one of the 33 markets analyzed.

Descriptive statistics on maize price, storage, and production in each of the 33 markets are given in Table 1.

4.2 Identifying negative and positive volatility

Our purpose in this section is to obtain and compare the series of the unpredictable components of price changes for each village. In order to facilitate these comparisons, we must establish a common specification for each price series. We do this by estimating a price formation model for each market and each year based on a unique ARCH structure. The conditional variance of the error term of the mean equation at any month t is a standard measure of price volatility as long as the deterministic portion of the mean equation is an acceptable measure of the predictable part of price. The ARCH model structure is as follows.

$$P_{mt} = \beta_0 + \beta_1 P_{mt-1} + \sum_{i=1}^{11} \beta_i D_i + \varepsilon_{mt} \quad \varepsilon_{mt} \sim \mathcal{N}(0, h_{mt}) \quad (17)$$

$$h_{mt} = \alpha_0 + \alpha_1 \varepsilon_{mt-1}^2 + \nu_{mt} \quad \nu_{mt} \sim \mathcal{N}(0, \sigma) \quad (18)$$

where the subscripts m in this section stands for the location of the market.

Equation (17) is the mean equation that determines the deflated producer price of maize as an autoregressive process in one period. D_i is a monthly dummy variable taking the value 1 for month i . The one period lag for price autoregression was selected after testing the number of significant periods in each individual market. While introducing P_{t-2} and P_{t-3} in the model is significant for some markets, we opted to keep the same simple model structure for each village in order to avoid altering the information included in the predicted price across markets. A trend variable was tested and rejected due to low statistical significance. Equation (18) determines the conditional variance of the error term ε_{mt} as a function of the shock in the last period and confirms the significant ARCH nature of the price process in 20 out of the 33 villages. In the 13 remaining villages, the price process is autoregressive with homoscedastic variance.

Table 1: Descriptive statistics on maize price, storage, and harvest per household, 33 markets, 8 years

	Price Mean	Price Std Dev	Stock Mean	Stock Std Dev	Harvest Mean	Harvest Std Dev
Banfora	120	32	411	885	1733	1156
Batie	148	41	97	147	1076	831
Dedougou	131	32	171	357	1046	986
Diapaga	125	38	50	84	470	521
Dibougou	128	35	121	277	490	336
Djibo	174	35	1	1	91	75
Dori	179	32	1	2	25	17
Douna	91	28	698	1242	2489	748
Fada	142	34	70	148	483	277
Fara	106	28	265	380	1834	695
Faramana	97	31	366	488	2033	872
Founzan	104	27	478	859	2242	907
Gaoua	147	27	75	151	500	356
Gourcy	157	30	7	11	80	63
Guelwongo	155	33	53	66	259	157
Hamele	124	33	67	120	555	505
Kaya	155	32	17	31	266	556
Kompienga	111	25	133	139	1229	707
Kongoussi	150	35	5	11	141	107
Koudougou	143	33	14	10	35	22
Leo	130	32	300	652	2618	2761
Manga	147	34	51	73	353	145
Ndorola	87	25	1649	2129	5440	3723
Ouargaye	115	27	77	121	668	508
Pouytenga	151	28	14	18	95	53
Sankaryare	154	31	24	41	125	75
Sapouy	126	34	146	270	1038	566
Solenzo	109	33	468	707	2141	1433
Tenkodogo	139	27	43	64	297	92
Tougan	154	33	62	87	738	1038
Yako	152	30	4	7	37	17
Zabre	142	30	40	66	260	80
Ziniare	157	32	33	49	106	62

We conducted the estimations above for each of the 33 markets so as to obtain 33 series of price volatility. Next, we segregated these series into two groups: those characterized by positive volatility (indicating unexpected price increases) and those characterized by negative volatility (indicating unexpected price drops). We calculated the average volatility for each series over a period of time varying from one month to 12 months in order to examine the robustness of the relationship between volatility and carry-over. Positive volatility for market m , for year j between month τ_0 to month τ_1 is calculated as follows:

$$h_{mj\tau_0\tau_1}^+ = \frac{1}{\tau_1 - \tau_0} \sum_{t=\tau_0}^{\tau_1} \hat{h}_{mt}^+ = \hat{\alpha}_0 + \frac{\hat{\alpha}_1}{\tau_1 - \tau_0} \sum_{\substack{t=\tau_0 \\ \varepsilon_{mt-1} > 0}}^{\tau_1} \varepsilon_{mt-1}^2 \quad (19)$$

A similar calculation is made for $h_{mj\tau_0\tau_1}^-$, the negative volatility in market m for year j between month τ_0 and month τ_1 . The final panel database is made up of 33 markets for which we have yearly carry-over data over 8 years (2005 to 2012), and price data over 10 years (2004-2013).

4.3 Estimating the effect of lean season unexpected price drops on carry-over

The empirical counterpart of equation (12) is

$$\chi_{mj} = \gamma_0 + \gamma_1 \chi_{mj-1} + \gamma_2 h_{mj\tau_0\tau_1}^- + \gamma_3 y_{mj} + \theta_{mj} \quad (20)$$

Where χ_{mj} is the average amount of carry-over for region m at the end of farming year j and y_{mj} is the production of grain at the beginning of farming year j . Equation (12) predicts that carry-over increases with negative volatility at the time of the expected price peak of the same year, that is $\gamma_2 > 0$ for τ_0 varying around May (expected to be before the price peak) and τ_1 varying around October (expected to be after the price peak).

4.4 Estimating the effect of carry-over on post-harvest unexpected price drops

The empirical counterpart of equation (16) is

$$h_{mj\tau_0\tau_1}^- = \rho_0 + \rho_1 h_{mj-1\tau_0\tau_1}^- + \rho_2 \chi_{mj-1} + \rho_3 y_{mj-1} + \eta_{mj} \quad (21)$$

Equation (16) predicts that negative volatility around the harvest season increases with the amount of carry-over that occurred during the previous farming year, that is, $\rho_3 > 0$

for τ_0 varying between September and November and τ_1 varying between October and March.

Both panel equations are estimated using the generalized moments method following the Arellano and Bover/ Blundell and Bond procedure with predetermined variables (Arellano and Bover, 1995; Blundell and Bond, 1998). The Arellano-Bond dynamic panel procedure generates moment conditions using lagged values of the dependent variable and the predetermined variables with first-differences of the disturbances (Arellano and Bond, 1991). Because the autoregressive process is persistent, we must obtain additional moment conditions in which lagged differences of the dependent variable are used as instruments (Arellano and Bover, 1995; Blundell and Bond, 1998). Lagged production and lagged prices are used as predetermined variables, and the dummy variables of fixed market effects are used as exogenous variables. Table 2 describes the volatility variables for the 33 markets we analyse.

5 Results

5.1 Price volatility

The mean equation in the ARCH model shows that prices follow an autoregressive process with a large and significant monthly autocorrelation parameter, and that pre-harvest prices (during the lean season) are significantly higher than prices during the rest of the year, while post harvest prices significantly lower. These results are consistent with those of Shively (1996); Barrett (1997); Karanja, Kuyvenhoven, and Moll (2003). For a deflated price index with a mean of approximately 100 (depending on the markets) the seasonal average difference between high and low season prices is ten, which is inferior to the seasonal effect that we obtain by defining the season for each market. The ARCH1 term is significant in 20 of the 33 markets, meaning that the autoregressive process has a heteroskedastic error term in 20 markets and a homoskedastic error term in 13 markets.

Figure 2 depicts the annual evolution of average prices and average negative and positive price volatilities during the year. Month 1 stands for January, 2 for February, and so on until month 12, which stands for December. This evolution illustrates that, even after price series are deseasonalized, the frequency of large positive price shocks is not the same throughout the year. Prices are on average higher between June and August and unexpected prices shocks occur most frequently in July. Conversely, negative prices shocks occur mainly in October, when prices drop. Note that the harvest period (October-November) is both a period of positive volatility and negative volatility, meaning that there is on average more unexpected drops and more unexpected peaks in those two months than during other periods. Unexpected peaks could occur due to low harvests, generating earlier price increases than usual, whereas unexpected price drops could occur due to unobserved carry-overs.

Figure 2 also illustrates why carry-over that is measured before the harvest in September may impact negative volatility more strongly than positive volatility. Whereas the

Figure 2: Unexpected price drops and spikes throughout the year in Burkina Faso, 33 markets, 10 years

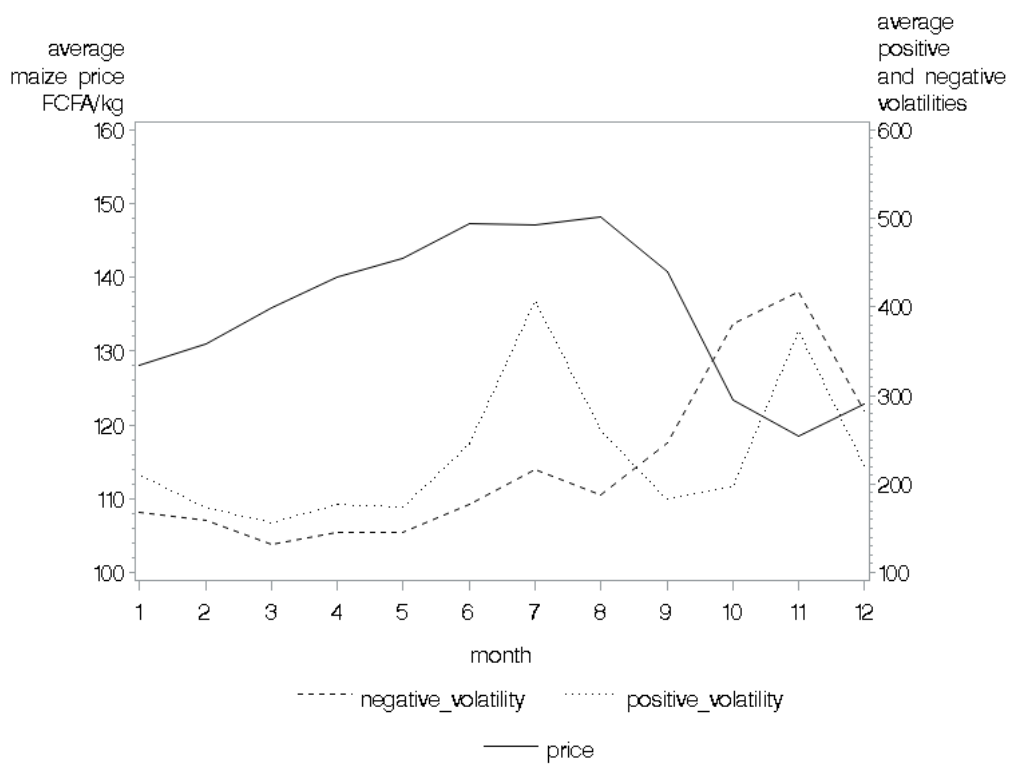
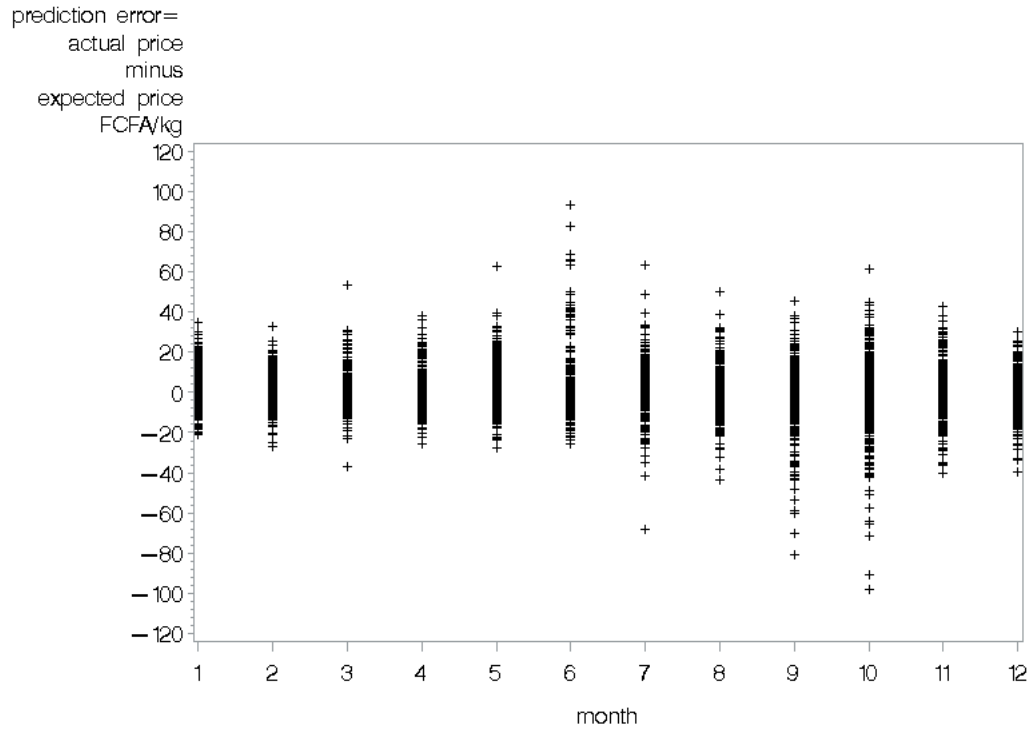


Figure 3: Unexpected price drops and spikes throughout the year in Burkina Faso, 33 market places, 10 years



peak of volatility in June and July may be smoother in a year following one in which significant carry-over occurred, carry-overs that occur in September may on the contrary increase the price drop in October as long as these carry-overs increase sales and reduce the price peak.

Descriptive statistics on average volatilities for the 33 studied markets are given in Table 2 below. The occurrence of negative and positive volatility episodes throughout the year is depicted in Figure 3 below.

5.2 Effect of volatility on carry-over

The model predicts that carry-over at the end of the farming year j should be zero if volatility has been low in year j , and that the amount of carry-over should be large if negative volatility has been significant in year j around the price peak (proposition 1). This is confirmed by the estimation presented in Table 3. Negative volatility episodes during the lean season increase the level of stocks remaining at the end of the farming year prior to a new harvest.

The Arellano and Bover/ Blundell and Bond procedure here relies on past carry-over, harvest at the beginning of the farming year, and negative and positive volatility ob-

Table 2: Descriptive statistics for volatility on the 33 analyzed market places

Market	Volatility		Negative volatility		Positive volatility	
	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev
Banfora	136,9	90,6	141,8	106,2	132,4	74,3
Batie	461,0	868,8	559,4	1152,6	365,7	439,3
Dedougou	530,9	1827,1	556,4	1150,0	516,9	2115,9
Diapaga	279,5	0,0	279,5	0,0	279,5	0,0
Diebougou	733,2	1742,6	583,7	1215,1	864,5	2100,9
Djibo	122,0	23,8	121,7	23,0	122,3	24,9
Dori	207,2	342,4	187,3	255,0	225,4	407,0
Douna	135,8	28,0	138,9	32,6	132,5	21,9
Fada	252,1	434,8	252,5	337,3	251,8	503,7
Fara	171,8	227,3	180,3	256,8	160,5	182,2
Faramana	165,3	18,0	165,8	21,0	164,9	14,4
Founzan	111,0	0,0	111,0	0,0	111,0	0,0
Gaoua	187,2	302,5	197,3	367,6	177,0	221,7
Gourcy	103,8	8,5	103,0	6,6	104,7	10,1
Guelwongo	347,3	626,2	286,7	417,1	407,8	780,5
Hamele	138,8	69,3	130,7	32,7	153,2	105,7
Kaya	150,6	8,1	149,9	5,9	151,6	10,2
Kompienga	207,7	633,5	273,4	929,1	162,7	299,8
Kongoussi	181,4	271,2	174,8	268,5	188,2	276,1
Koudougou	261,1	529,5	207,3	431,6	306,8	599,9
Leo	339,2	468,8	415,5	597,3	257,8	253,6
Manga	167,8	45,7	169,0	49,9	166,4	40,7
Ndorola	91,1	43,2	92,4	45,3	89,0	40,1
Ouargaye	120,9	9,5	120,8	9,7	121,1	9,3
Pouytenga	244,2	491,9	270,4	615,9	220,4	346,4
Sankaryare	268,3	537,9	271,9	447,9	265,8	597,6
Sapouy	218,3	0,0	218,3	0,0	218,3	0,0
Solenzo	135,2	0,0	135,2	0,0	135,2	0,0
Tenkodogo	136,1	67,9	142,8	80,3	128,5	49,8
Tougan	197,2	279,0	198,8	295,7	195,2	258,5
Yako	292,1	577,3	282,2	616,0	305,0	528,3
Zabre	165,2	197,3	161,5	227,0	168,6	166,4
Ziniare	212,2	227,7	221,1	181,8	195,5	297,4

served throughout the farming year. Since stocks depend not only on last year’s stock, but also on stocks from previous years, we assume the autoregressive process to be persistent. These results hold when negative volatility is defined as an annual average (from November of the previous year to October of the present year), as an observation in September, as an average between August and September, and finally, as an average between July and September.

When significant, stocks are positively correlated with past stocks and the level of harvest

Table 3: Effect of lean season negative volatility on carry-over

	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
Lagged carry-over	0,19 ***	0,42 ***	0,15 ***	0,19 ns	0,10 ***	0,09 ns	0,10 ns	0,26 ns
Negative volatility	0,28 **	0,38 ns	0,57 ns	1,13 **	0,33 ns	0,96 **	1,33 *	-0,02 ns
Harvest	0,13 ***	0,06 *	0,22 *	0,06 **	0,10 **	0,19 ***	0,23 ***	0,06 ns
Const	-36,68 ns	-60,43 ns	-214,66 ns	113,88 *	123,39 ns	-192,05 *	-279,28 ns	10,85 ns
Obs	226	109	148	177	105	149	103	132
Period for volatility								
$\tau_0 - \tau_1$	Nov-Oct	Jul	Jul-Aug	Jul-Sept	Aug	Aug-Sept	Sept	Oct

at the beginning of the farming year, which is as expected. Negative volatility episodes occurring during the lean season (July-September) increase the amount of carry-over at the 5% level, as is predicted in the model (Proposition 1). We confirm this effect empirically. Indeed, this effect holds for average annual negative volatility (specification [1] in Table 3 and is even more pronounced for the negative volatility observed during the lean season, i.e. the July-September period (specifications [4], [6] and [7]). If we exclude September from the volatility period, the result does not hold, indicating that unexpected price drops in September are critical in producing carry-over (specifications [2], [3] and [5] in Table 3).

By contrast, the effect of positive volatility on carry-over is zero. The events of unexpected price increases during either the year before the next harvest or during the months directly leading up to the next harvest (July to September) (Table 4) have no effect on the amount of stocks held by farmers at the end of the year. This supports the theoretical assumption that carry-over is not a planned activity (see table 4). If it were, positive price shocks would lead farmers to alter their plans by selling more, hence reducing the amount of carry-over, which is not what we observe here.

5.3 Effect of carry-over on volatility

The effect of carry-over on negative price volatility is estimated with the Arrellano-Bover/ Blundel-Bond procedure, which accounts for persistence in autoregressive processes. The results of this procedure are presented in Table 5 and Table 6 below, where

Table 4: Effect of pre-harvest positive volatility on carry-over

	[1]	[2]	[3]	[4]
Lagged stock	0,193 ns	0,111 ns	-0,005 ns	0,224 ns
Lagged positive volatility	0,056 ns	0,049 ns	-0,052 ns	0,112 ns
Lagged harvest	0,128 **	0,103 **	0,005 ns	0,016 ns
Const	28,943 ns	81,445 *	206,075 ***	69,001 ns
Obs	221	192	168	124
Period for lagged volatility	Nov-Oct	Jul-Sept	Aug-Sept	Sept

different specifications correspond to the different time periods used to measure negative volatility. Carry-over increases the occurrence of negative volatility episodes throughout the year (specification [1] in 5) and this effect is even stronger when considering the period following the harvest, from November to March (specifications [3] to [7] in Table 5). This confirms the predictions made by the model (Proposition 2). However, this effect does not hold if we consider months that are more distant from harvest time. As we can observe in Table 6, carry-over generates negative price volatility after the harvest, but this effect tends to smooth out in the following months.

By contrast, carry-over apparently has no significant effect on positive price volatility,

Table 5: Effect of carry-over on post-harvest negative volatility

	[1]	[2]	[3]	[4]	[5]	[6]	[7]
Lagged volatility	0,13 s	-0,12 ns	-0,14 **	-0,10 **	-0,09 *	0,00 ns	0,07 ns
Stock	0,02 **	0,09 ns	0,12 ***	0,13 *	0,11 **	0,11 **	0,06 *
Harvest	-0,03 *	0,01 ns	-0,12 **	-0,07 **	-0,05 ns	-0,04 ns	-0,03 ns
Const	235,38 ***	273,63 ns	588,70 ***	430,85 ***	368,17 ***	304,92 ***	269,24 ***
Obs	224	46	143	183	204	217	219
Period for volatility	Nov-Oct	Nov	Nov-Dec	Nov-Jan	Nov-Fev	Nov-Mars	Nov-Avr

price volatility captured as an annual average (specification [1] of Table 7, or as an average on the months following the harvest (specifications [2] to [6] in Table 7).

Table 6: Effect of carry-over on post-harvest negative volatility. Continued.

	[8]	[9]	[10]	[11]	[12]	[13]
Lagged volatility	-0,24 ns	0,01 ns	0,02 ns	0,11 ns	-0,14 ns	-0,25 ns
Stock	0,03 ns	0,15 ns	0,03 ns	-0,28 ns	0,01 ns	0,16 ns
Harvest	-0,16 **	-0,12 ns	-0,01 ns	0,02 ns	0,01 ns	0,00 ns
Const	579,20 ***	362,98 **	240,88 ***	142,98 ***	186,92 ***	223,69 ***
Obs	67	127	164	73	130	53
Period for volatility	Dec	Dec-Jan	Dec-Feb	Jan	Jan-Feb	Feb

Table 7: Effect of carry-over on post-harvest positive volatility

	[1]	[2]	[3]	[4]	[5]	[6]
Lagged volatility	0,07 **	-0,04 **	-0,02 **	0,00 ns	0,01 ns	0,01 ns
Stock	0,02 ns	0,10 ns	0,05 ns	0,06 ns	0,04 ns	0,04 ns
Harvest	-0,03 ns	-0,07 ns	-0,06 ns	-0,05 ns	-0,04 ns	-0,03 ns
Const	215,06 ***	449,93 ***	362,15 ***	307,79 ***	267,44 ***	243,67 ***
Obs	212	134	174	193	200	200
Period for volatility	Nov-Oct	Nov-Dec	Nov-Jan	Nov-Feb	Nov-Mars	Nov-Avr

6 Conclusion

Most of the research on the influence of storage decisions on price volatility has focused either on public storage or on speculative storage. The influence of stocks held by farmers on domestic price volatility has not received much attention in the economic literature, despite the strategic importance of grain storage in African countries. In this paper, we build upon an original conceptual model that analyses the effect of farmers' price-anticipation errors on the existence of carry-over and on the occurrence of unexpected price drops. To assess the empirical relevance of such a model, we focus on maize price volatility in a developing country, Burkina Faso, and we analyze the relationship between the levels of stocks held by farmers and price volatility levels observed in 33 local markets in the 2004-2014 period. We differentiate between negative and positive price shocks and provide empirical evidence that carry-overs that result from unanticipated price drops during the lean season increase the occurrence of unexpected price drops at the beginning of a new agricultural season.

If we wish to avoid massive price drops after the harvest, our results constitute an appeal for the implementation of policy measures that ensure that farm stocks will be zero at the end of the agricultural season, or that enable that farmers to retain their grain stock just after the harvest. Based on this work, two objectives may therefore be identified : Enable farmers better access to market information and notably market prices, through more effective market information services and better market infrastructure. Improved access should result in reducing price anticipation errors and thus avoid situations in which farmers possess carry-over at the end of the year.

Promote on-farm storage just after the harvest in order to smooth both price drops after harvests and extreme price increases at the end of the season. This is a challenging endeavor in the context of developing countries because farmers frequently need liquidity during the harvest period, and as a result tend to sell most of their stock at that time. Therefore, encouraging storage through subsidizing storage infrastructures in villages should be accompanied by measures that facilitate farmers access to credit in order to meet their liquidity needs. Warehouse receipt systems are expanding in developing countries precisely because these systems allow farmers both access to liquidity after harvests, as well as increased profitability due to the opportunity to store products to sell later in the year when prices are higher. Systems such as these are of great interest to those involved in these markets, and constitute a valuable avenue for future research.

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7 Appendix

7.1 the initial sales plan

Derivation of the farmer’s program with regard to c_1 gives

$$c_1^{-r} - \sum_{j=1}^k \sum_{i=1}^T \lambda_{ij} = 0$$

Derivation with x_1 gives

$$-\mu_1 + \sum_{j=1}^k \sum_{i=1}^T \lambda_{ij} P_1 = 0$$

Thus

$$c_1 = \left(\frac{P_1}{\mu_1} \right)^{\frac{1}{r}} \tag{22}$$

Derivation with \tilde{c}_2^1 leads to

$$\frac{(\tilde{c}_2^1)^{-r}}{(1+\delta)} - \sum_{i=2}^T \lambda_i - \sum_{j=2}^k \sum_{i=1}^T \lambda_{ij} = 0 \tag{23}$$

Derivation with x_2^1 produces

$$-\mu_1 + \left(\sum_{i=2}^T \lambda_i + \sum_{j=2}^k \sum_{i=1}^T \lambda_{ij} \right) E_1 P_2 = 0$$

$$\tilde{c}_2^1 = \left[\frac{E_1 P_2}{(1 + \delta)\mu_1} \right]^{\frac{1}{r}}$$

Derivation with \tilde{c}_t^1 leads to

$$\frac{(\tilde{c}_t^1)^{-r}}{(1 + \delta)^{t-1}} - \sum_{i=t}^T \lambda_i - \sum_{j=2}^k \sum_{i=1}^T \lambda_{ij} = 0 \quad (24)$$

or

$$\tilde{c}_t^1 = \frac{1}{(1 + \delta)^{\frac{t-1}{r}} \left(\sum_{i=t}^T \lambda_i + \sum_{j=2}^k \sum_{i=1}^T \lambda_{ij} \right)^{\frac{1}{r}}} \quad (25)$$

$$\tilde{c}_t^1 = \left[\frac{E_1 P_t}{(1 + \delta)^{t-1} \mu_1} \right]^{\frac{1}{r}} \quad (26)$$

Using equation (1), and equations (10) to (14), we get obtain after some manipulations the general expression of consumption in month t as the farmer plans it in month 1 :

$$\tilde{c}_t^1 = \frac{y^1 E_1 P_t^{\frac{1}{r}-1}}{P_1^{\frac{1}{r}-1} (1 + \delta)^{\frac{t-1}{r}} + \sum_{i=2}^{T-1} (E_1 P_i)^{\frac{1}{r}-1} (1 + \delta)^{\frac{t-i}{r}} + (E_1 P_T)^{\frac{1}{r}-1} \sum_{i=T}^{12} (1 + \delta)^{\frac{t-i}{r}}}$$

7.2 carry-over

Derivation of the farmer's plan with regard to c_{12} produces

$$c_{12}^{-r} - \lambda_{12} - \sum_{j=2}^k \sum_{i=1}^T \lambda_{ji} = 0 \quad (27)$$

Derivation with x_{12} produces

$$-\mu_1 - \mu_2 - \dots - \mu_k + P_{12}(\lambda_{12} + \sum_{j=2}^k \sum_{i=1}^T \lambda_{ji}) = 0 \quad (28)$$

Thus

$$c_{12} = \left(\frac{P_{12}}{\sum_{j=1}^k \mu_j} \right)^{\frac{1}{r}} \quad (29)$$

Derivation with \tilde{c}_{13}^{12} leads to

$$\frac{(\tilde{c}_{13}^{12})^{-r}}{(1+\delta)} - \sum_{j=2}^k \sum_{i=1}^T \lambda_{ji} = 0 \quad (30)$$

Derivation with \tilde{x}_{13}^{12} leads to

$$-\sum_{j=2}^k \mu_j + \sum_{j=2}^k \sum_{i=1}^T \lambda_{ji} E_{12} P_{13} = 0 \quad (31)$$

$$\tilde{c}_{13}^{12} = \frac{1}{1+\delta} \left(\frac{E_{12} P_{13}}{\sum_{j=2}^k \mu_j} \right)^{\frac{1}{r}} \quad (32)$$

and

$$\tilde{c}_{12+t}^{12} = \frac{1}{(1+\delta)^{\frac{t-1}{r}}} \left(\frac{E_{12} P_{12+t}}{\sum_{j=2}^k \mu_j} \right)^{\frac{1}{r}} \quad (33)$$

The carry-over at the end of year 1, denoted χ^1 and defined by $\chi^1 = y^1 - \sum_{i=1}^{12} x_i$, is given by equation from inequation (10) where $\mu_2 > 0$:

$$\chi^1 = \sum_{i=13}^{12+T} \tilde{x}_i^{12} - \tilde{y}^2 \quad (34)$$

Replacing \tilde{x}_i^{12} , and using (11), we obtain

$$\chi^1 = \sum_{i=1}^T \frac{1}{(1+\delta)^{\frac{i}{r}} E_{12} P_{12+i}} \left(\frac{E_{12} P_{12+i}}{\sum_{j=2}^k \mu_j} \right)^{\frac{1}{r}} + \sum_{i=T+1}^{12} \frac{1}{(1+\delta)^{\frac{i}{r}} E_{12} P_{12+T}} \left(\frac{E_{12} P_{12+T}}{\sum_{j=2}^k \mu_j} \right)^{\frac{1}{r}} - y^2 \quad (35)$$

and noting that $\mu_1 = 0$ when there is carry-over,

$$\chi_1 = c_{12} \left(\sum_{i=1}^T \frac{1}{(1+\delta)^{\frac{i}{r}} E_{12} P_{12+i}} \left(\frac{E_{12} P_{12+i}}{P_{12}} \right)^{\frac{1}{r}} + \sum_{i=T+1}^{12} \frac{1}{(1+\delta)^{\frac{i}{r}} E_{12} P_{12+T}} \left(\frac{E_{12} P_{12+T}}{p_{12}} \right)^{\frac{1}{r}} \right) - y^2 \quad (36)$$

Furthermore, $c_{12} = \frac{y \cdot P_{12}}{\sum_{i=1}^{12} \left(\frac{P_i}{P_{12}} \right)^{\frac{1}{r}-1} (1+\delta)^{\frac{12-i}{r}}}$,

$$\chi^1 = \frac{\sum_{i=1}^T \frac{(E_{12} P_{12+i})^{\frac{1}{r}-1}}{(1+\delta)^{\frac{i}{r}}} + \sum_{i=T+1}^{12} \frac{(E_{12} P_{12+T})^{\frac{1}{r}-1}}{(1+\delta)^{\frac{i}{r}}}}{\sum_{i=1}^{12} \frac{(P_i)^{\frac{1}{r}-1}}{(1+\delta)^{\frac{i-12}{r}}}} y^1 - y^2 \quad (37)$$

7.3 equilibrium prices

We use (??) to obtain

$$\frac{(y^2 + \chi^1 - \sum_{i=13}^{12+t-1} \frac{c_i}{P_i}) P_{12+t}^{\frac{1}{r}-1}}{\sum_{i=12+t}^{12+T} E_i P_t^{\frac{1}{r}-1} (1+\delta)^{\frac{t-i}{r}} + E_t P_T^{\frac{1}{r}-1} \sum_{i=12+T+1}^{24} (1+\delta)^{\frac{t-i}{r}}} = (\gamma P_{12+t})^{-1/\rho}$$

Thus,

$$P_{12+t}^{\frac{1}{r} + \frac{1}{\rho} - 1} = \frac{\sum_{i=12+t}^{12+T} E_i P_t^{\frac{1}{r}-1} (1+\delta)^{\frac{t-i}{r}} + E_t P_T^{\frac{1}{r}-1} \sum_{i=12+T+1}^{24} (1+\delta)^{\frac{t-i}{r}}}{(y^2 + \chi^1 - \sum_{i=13}^{12+t-1} \frac{c_i}{P_i}) \gamma^{1/\rho}}$$

7.4 savings

The introduction of savings in the analysis modifies the constraints in the following way:

$$\lambda_1 \quad S_0 + x_1 P_1 - c_1 \geq 0$$

where S_0 is the cash savings on hand at the time of harvest in the first year.

$$\lambda_2 \quad S_0 + x_1 P_1 - c_1 + \tilde{x}_2^1 E_1 P_2 - \tilde{c}_2^1 \geq 0$$

$$\lambda_t \quad S_0 + x_1 P_1 - c_1 + \sum_{i=2}^t (\tilde{x}_i^1 E_1 P_i - \tilde{c}_i^1) \geq 0$$

$$\lambda_T \quad S_0 + x_1 P_1 - c_1 + \sum_{i=2}^T \tilde{x}_i^1 E_1 P_i - \sum_{i=2}^{12} \tilde{c}_i^1 - \tilde{S}_1 \geq 0 \quad (38)$$

where \tilde{S}_1 is the amount of savings at the end of year 1.

$$\lambda_{13} \quad \tilde{S}_1 + x_{13} P_{13} - c_{13} \geq 0 \quad (39)$$

For year k , month T :

$$\lambda_{kT} \quad \tilde{S}_{k-1} + \sum_{i=1}^T \tilde{x}_{12(k-1)+i}^1 E_1 P_{12(k-1)+i} - \sum_{i=1}^{12} \tilde{c}_{12(j-1)+i}^1 - \tilde{S}_k \geq 0 \quad (40)$$

The intertemporal optimum refers to what a farmer would do if he could simultaneously choose the level of all endogenous variables. The optimal level of savings is given by the stationary state defined by $S_1 = S_0 = S_{k-1} = S$. Derivation with regard to S then leads to

$$\sum_{j=1}^k \sum_{i=1}^{T-1} \lambda_{12(j-1)+i} = 0$$

meaning that

$$\forall j \in [1, k], \quad \forall i \in [1, T-1], \quad \lambda_{12(j-1)+i} = 0 \quad (41)$$

Furthermore, all cash from the sale of grain at T is used for consumption or savings. We thus obtain :

$$\forall j \in [1, k], \quad \lambda_{12(j-1)+T} > 0$$

Equalities (39) mean that for month 1, $S + x_1 p_1 > c_1$, and so on for each month, which implies that $x_1 = 0, x_2 = 0, \dots, x_{T-1} = 0$ because farmers do not sell more grain than is necessary in order to meet their monthly consumption needs during the phase from $t = 1$ to $t = T$ when prices are supposed to increase. As a result, if S is intertemporally optimised, the farmer plans to sell no grain until the expected price peak month T . At T , after which the farmer expects a price drop, he sells all of his remaining surplus and receives cash that is then used throughout the year.

Derivation with \tilde{c}_t^1 leads to

$$\begin{aligned} \forall t < T, \quad \tilde{c}_t^1 &= \frac{1}{(1 + \delta)^{\frac{t-1}{r}} \left(\sum_{i=t}^T \lambda_i \right)^{\frac{1}{r}}} \\ \forall t \geq T, \quad \tilde{c}_t^1 &= \frac{1}{(1 + \delta)^{\frac{t-1}{r}} \lambda_T^{\frac{1}{r}}} \end{aligned} \quad (42)$$

using these equations together with the resource constraint, we obtain the monthly level of consumption:

$$\tilde{c}_t^1 = \frac{y^1 E_1 P_T}{\sum_{i=t}^{T-1} \left(\frac{E_1 P_i}{E_1 P_t} \right)^{\frac{1}{r}} (1 + \delta)^{\frac{t-i}{r}} + \frac{E_1 P_T}{E_1 P_t}^{\frac{1}{r}} \sum_{i=T}^{12} (1 + \delta)^{\frac{t-i}{r}}} \quad (43)$$

This equilibrium yields the highest possible consumption level. Although theoretically appealing, this type of behaviour is only relevant for a limited number of farmers.