Group insurance against common shocks *

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Abstract

We study insurance against common shocks in cooperatives and other productive groups of individuals. In those groups, and due to strategic interactions among group members, insurance decisions may be preferably taken at the group level rather than the individual level. We highlight two kinds of potential problems with individual insurance : the first one is a coordination problem that occurs because it may be unprofitable for an individual to take insurance if the others in the group do not ; the second one is an underprovision problem because due to strategic interactions, insurance decisions exert a positive externality on other group members. Both types of problems can be resolved if insurance is offered at the group level.

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1 Introduction

Risk reduction remains a major challenge in increasing productivity and enhancing livelihoods among smallholders in developing countries. In the agricultural sector in particular, risk is often seen as a major ingredient of poverty traps. After the occurence of a negative income shock, agricultural households can reduce their children's school frequentation (Jacoby and Skoufias, 1997) or can be obliged to sell productive capital at low prices (Rosenzweig and Wolpin, 1993). In anticipation of the shocks, they can diversify their income by investing time in low productivity activities (Dercon and Krishnan, 1996), they can also fail to adopt high-productivity but high-risk varieties (Eswaran and Kotwal, 1990).

The revenue of agricultural households in a community is subject to different kinds of shocks. A typology of shocks is possible according to the statistical properties of their distribution. The two polar types are idiosyncratic shocks and common shocks. Idiosyncratic shocks are independently distributed in the community. In a first approximation, health shocks are an example of idiosyncratic shocks. Common shocks affect everyone in the community. Price fluctuations and weather shocks are, again in a first approximation, examples of common shocks. Insurance against idiosyncratic shocks is possible via mutualization. In a celebrated empirical study, Townsend (1994) finds that village communities in three ICRISAT villages manage to achieve an outcome close to full insurance against idiosyncratic shocks via mutualization. Insurance against common shocks is much more difficult to achieve. It usually necessitates the intervention of third-parties like insurance companies and/or financial markets. For instance, insurance against price fluctuations is possible on derivative financial markets (Moschini and Lapan, 1995). Insurance against weather shocks requires the availability of suitable weather derivatives or the existence of re-insurance companies willing to diversify their portfolio. Of course the idiosyncratic or common nature of the shock depends on the perimeter of the community considered.

In this paper we are interested in the analysis of the demand for insurance against common shocks with a particular emphasis on insurance against weather shocks.

In the recent years there has been a growing interest in weather insurance for agricultural households in developing countries (see Barnett and Mahul, 2007, and World Bank, 2009). In particular, academic researchers and practitionners have argued that index-based policies can be used to insure poor rural households against weather shocks. Those policies make use of a meteorological index like the temperature or the cumulative rainfall in a given area, strongly correlated with the losses, and condition the payments to the insuree on the realized value of the index. Because the index measures a purely exogenous variable, offering these policies is not subject to moral hazard issues that usually plague insurance markets. Moreover, because information on past values of the index can be made public, there is no reason to suspect that adverse selection is a concern. Finally, the recent availability of relatively cheap and reliable automatic meteorological stations has decreased the fixed cost associated to such policies. Index-based policies seem to be the right tool for insurance to reach the poor (Skees, Hartell and Yao, 2006). Since 2003, there has been a number of experimentations of index-based policies in Malawi (World Bank CRMG, 2009), Morocco (Stoppa and Hess, 2003), Peru and Vietnam (Skees, Hartell and Goes, 2007), India (Manuamorn, 2007) and many other developing countries. Yet, individual uptake for those policies has been disappointingly low (see for instance Giné and Yang, 2009, or Carter et al., 2010).

In this paper we shall argue that the demand for weather insurance, and other types of insurance against common shocks, could rise if policies are sold at the community level rather than the individual level.

Several explanations have been proposed to shed light on the low individual demand for weather insurance. It has been argued that insurance policies are complex products and that poorly educated agricultural households face difficulties to understand their interest. It has also been argued that weather risk typically involves small probabilities that people are likely to misevaluate. It also involves very long term cost-benefit analysis and present-biased households may underevaluate their potential losses. Others argued that agricultural households already adopted a wide array of risk coping strategies such as credit and savings and that weather insurance does not bring much to them (see Gollier, 1995). Finally, some authors have argued that interlinked transactions must be taken into account in order to understand the demand for weather insurance. In particular, it is plausible that, in village communities, formal weather insurance interacts with informal risk sharing. Informal risk sharing makes use of repeated interactions among group members. The fear of being excluded from the group and unable to benefit from the mutualization of risks in the future is used as a disciplining device. When people have the opportunity to obtain insurance from outside, they rise their reservation utility and informal risk sharing can be jeopardized. This argument originates in Attanasio and Rios-Rull (2000) and has been used by Clarke and Dercon (2009) to explain low individual demand for weather insurance.

This last explanation suggests that risk coping decisions of individuals in village communities are likely to exert externalities on other community members. And that offering insurance against common shocks at the community level can internalize those externalities. In this paper we propose a simple and tractable model to scrutinize this point. We highlight two distinct characteristics of insurance against common shocks in communities. The first one is that the value of this kind of insurance can be positive or negative for an individual, depending on the insurance decisions of the other community members. The game played by community members when they choose whether or not to take insurance is, in some circumstances, a coordination game. And community members may fail to coordinate on the Pareto dominant outcome in which they all choose to take insurance. The second characteristic is that even if community members manage to coordinate and all take insurance, the demand for insurance may still be plagued by a free-rider problem. It is plausible that the sum of the individual willingnesses to pay for insurance is less than the group willingness to pay for insurance.

Our model is built on the following specification for individual preferences : the utility of individual i in a group of N members depends on his own wealth and on the aggregate wealth of the group. Therefore, those individuals have social preferences. This is a rather natural hypothesis for village communities where interlinked transactions lead people to care about the wealth of others in the community. We propose the following rationale for this specification. If the group of individuals that we consider produces some local public good, then equilibrium utilities of individuals depend on those two variables. This remains true for several widely used individual preferences and under several decision rules for the provision of the local public good. We believe that this specification is particularly suitable to study the demand for weather insurance in agricultural cooperatives or other productive groups of individuals.

When the two variables (i.e. own wealth and aggregate wealth) that enter the utility function of individuals are complements, a risk averse individual may prefer to stay uninsured if other group members do not take insurance. This occurs because individuals prefer to be rich when the group as a whole is rich and poor when the group as a whole is poor rather than poor when the group is rich and rich when the group is poor.

Even if group members coordinate and decide to insure themselves, individual insurance decisions may exert a positive externality on others and create a free-riding problem. This may occur because the decision by one individual to take insurance involves a reduction in the risk associated to the aggregate wealth in the sense of second-order stochastic dominance. This will be valued by other group members provided the premium paid to get insured is not too high. As a consequence, the sum of the individual inverse demands for insurance, i.e. the sum of the individual risk premia, may be lower than what the group as a whole would be ready to pay, i.e. the group risk premium. Offering the insurance policy at the group level may rise demand.

The paper is organized as follows. In section 2 we present the model and some justifications. Examples of public good games are detailed to provide a rationale for the specification of individual preferences according to which both his own wealth and the aggregate wealth are taken into account by an individual. We also justify the symmetry assumption that we make in sections 3 and 4 by studying the mutual insurance possibilities in the group. The core of the paper is sections 3 and 4. In section 3 we show that insurance against common shocks can have a negative value and we identify a sufficient condition on the utility functions for such a result to hold. In section 4 we compare individual and group risk premia and show that the sum of individual premia can be lower than the group premium. Section 5 presents concluding remarks.

2 Social preferences and mutual insurance

2.1 Indirect utility

The community we consider is a group of N individuals. Each individual $i \in \{1, ..., N\}$ in the group is endowed with a wealth w_i. We denote by $W = \sum_{i=1}^{N} w_i$ the aggregate wealth in the group, and $W_{-i} = \sum_{j \neq i} w_j$ the sum of the wealth of individuals different from i. To take into account the fact that group members interact, we assume that each individual i has preferences given by the (indirect) VonNeumann-Morgenstern utility function

$$u_i(w_i, W) \tag{1}$$

With this specification, each individual cares about his own wealth but also about the aggregate wealth of others in the group. This kind of (indirect) social preferences is very plausible when the group considered is a cooperative or any other productive group. It is the result of having community members involved in several interlinked transactions with each others. We first provide different examples of settings where individuals have such preferences.

Example 1 : Voluntary contribution to a public good

Consider a public good provision game among the N individuals. There are two goods in the economy : a private good c and a public good G. Each individual i can use his wealth w_i to buy a quantity c_i of the private good and contribute G_i to the public good. We normalize the price of both goods to be 1 so that the player's budget constraint is $w_i = c_i + G_i$. The utility of player i is given by

$$U_i(c_i, G) = \alpha_i \log(c_i) + \beta_i \log(G) \tag{2}$$

where

$$G = \sum_{i=1}^{N} G_i$$

is the total amount of public good that is produced by the players. The parameters α_i and β_i are strictly positive. Suppose the players contribute non-cooperatively to the public good. We can find the unique equilibrium of the voluntary contribution game following Bergstrom, Blume and Varian (1986) and Cornes and Hartley (2007). With the specification given in equation (2) for the utility of players, the marginal rate of substitution of player i is given by

$$MRS_i(c_i, G) = \frac{\alpha_i}{\beta_i}\frac{G}{c_i}.$$

For an aggregate level of public good provision equal to G, player i maximizes its payoff if and only if his contribution G_i satisfies

$$G_i = r_i(G) = \max\{0, w_i - \frac{\alpha_i}{\beta_i}G\},$$

because either his contribution equals zero and the marginal rate of substitution is less than 1 (corner solution) or the marginal rate of substitution is equal to 1 (interior solution). By summing over i we obtain :

$$\sum_{i=1}^{N} r_i(G) = \sum_{i=1}^{N} \max\{0, w_i - \frac{\alpha_i}{\beta_i}G\}.$$

Let us denote

$$I(G) = \{i \in N : w_i - \frac{\alpha_i}{\beta_i}G > 0\}.$$

Equilibrium conditions are therefore

$$G^* = \frac{\sum_{i\in I(G^*)} w_i}{1 + \sum_{i\in I(G^*)} \frac{\alpha_i}{\beta_i}},$$

$$G_i^* = \max\{0; w_i - \frac{\alpha_i}{\beta_i}G^*\},$$

$$i \in I(G^*) \quad \text{if and only if} \quad w_i - \frac{\alpha_i}{\beta_i}G^* > 0.$$

When the players are not too asymmetric, we will have $I(G^*) = N$ i.e. all the players contribute a strictly positive amount to the public good. In that particular case, equilibrium conditions give:

$$G^* = \frac{\sum_{i=1}^{N} w_i}{1 + \sum_{i=1}^{N} \frac{\alpha_i}{\beta_i}},$$

$$c_i^* = w_i - G_i^* = \frac{\alpha_i}{\beta_i}\frac{\sum_{i=1}^{N} w_i}{1 + \sum_{i=1}^{N} \frac{\alpha_i}{\beta_i}}.$$

So that the equilibrium utility of a particular player i is :

$$\alpha_i \log\left(\frac{\alpha_i}{\beta_i}\right) + (\alpha_i + \beta_i)\log\left(\frac{1}{1 + \sum_{i=1}^{N} \frac{\alpha_i}{\beta_i}}\right) + (\alpha_i + \beta_i)\log\left(\sum_{i=1}^{N} \omega_i\right)$$

and the indirect utilities are, up to a linear and increasing transformation, of the form

$$u_i(w_i, W) = \log(W) \tag{3}$$

Here, the indirect utilities depend only on the aggregate wealth of the group. □

Example 2 : Mandatory contribution to a public good

Consider the same setting as in Example 1, but assume that contributions to the public good are mandatory, such as studied for instance in Epple and Romano (2003). They result from a collective choice rule adopted by the group. For simplicity let us assume that public good provision is financed through a proportional tax enforced at the group level.

Let us denote T the tax rate. With such a tax, the utility of agent i is

$$u_i = \alpha_i \log((1-T)\omega_i) + \beta_i \log\left(T\sum_{j=1}^{N}\omega_j\right),$$

Those functions are concave in T and the most preferred tax rate of agent i is

$$\bar{T}_i = \frac{\beta_i}{\alpha_i + \beta_i}.$$

We will assume that the group's choice is driven by the median voter's preferences. Agents are ranked according to their $\frac{\beta_i}{\alpha_i+\beta_i}$, with

$$\frac{\beta_1}{\alpha_1+\beta_1} \leq \frac{\beta_2}{\alpha_2+\beta_2} \leq \ldots \leq \frac{\beta_N}{\alpha_N+\beta_N}.$$

The median voter si determined by the parameters α and β exclusively and does not change with the distribution of wealth in the population.[1] Let us denote m the median voter. The equilibrium condition is :

$$G^{eq} = \frac{\beta_m}{\alpha_m+\beta_m}\left(\sum_{i=1}^{N}\omega_i\right).$$

The equilibrium utility of agent i is therefore :

$$\alpha_i \log\left(\frac{\alpha_m}{\alpha_m+\beta_m}\right) + \beta_i \log\left(\frac{\beta_m}{\alpha_m+\beta_m}\right) + \alpha_i \log(\omega_i) + \beta_i \log\left(\sum_{j=1}^{N}\omega_j\right)$$

And, up to an increasing and linear transformation, the indirect utilities take the form

$$u_i(w_i, W) = \log w_i + a_i \log W \text{ with } a_i > 0, \tag{4}$$

With mandatory contributions, the equilibrium (or indirect) utility depends both on the agent's wealth and the aggregate wealth. □

[1] This has the important implication that the vote on the tax rate can take place before or after the occurence of the shocks on wealth : this would not change the result of the vote.

Example 3 : Other specifications

Other specifications for the preferences of the individuals lead to indirect utility functions with w_i and W as arguments. Suppose the utility of individuals, instead of being given by equation (2), is given by

$$U_i(c_i, G) = c_i^{\alpha_i} G^{\beta_i} \tag{5}$$

where $\alpha_i \in]0, 1[$, $\beta_i \in]0, 1[$ and $\alpha_i + \beta_i < 1$. These utility functions are increasing and concave transformations of the utilities given in equation (2). Therefore they represent the same preferences in riskless environments. The equilibria of the voluntary contribution game and the mandatory contribution vote are unchanged. It is straightforward to show that the corresponding indirect utilities are now, up to an affine transformation, as follows. For the voluntary contribution game

$$u_i(w_i, W) = W^{\alpha_i + \beta_i} \tag{6}$$

For the mandatory contribution game

$$u_i(w_i, W) = w_i^{\alpha_i} W^{\beta_i} \tag{7}$$

Suppose now that the utility of individuals is given by a constant elasticity of substitution function of the form

$$U_i(c_i, G) = (\lambda_i c_i^{\alpha_i} + (1 - \lambda_i) G^{\alpha_i})^{\frac{1}{\beta_i}} \tag{8}$$

where $0 < \alpha_i \leq \beta_i < 1$. In that case, the marginal rate of substitution of agent i is given by

$$MRS_i(c_i, G) = \frac{\lambda_i}{1 - \lambda_i} \left(\frac{G}{c_i} \right)^{1-\alpha_i}.$$

In an interior equilibrium of the voluntary contribution game, there is a linear relation between c_i and G with

$$G = \left(\frac{1 - \lambda_i}{\lambda_i} \right)^{\frac{1}{1-\alpha_i}} c_i.$$

The same argument as developped for Example 1 establishes that, in an interior equilibrium, the public good quantity G is proportional to the aggregate wealth W with

$$G = W \left(1 + \sum_{i=1}^{N} \left(\frac{\lambda_i}{1 - \lambda_i} \right)^{\frac{1}{1-\alpha_i}} \right)^{-1}.$$

Therefore, in an interior equilibrium, the indirect utility function of individuals is of the form

$$u_i(W) = W^{\frac{\alpha_i}{\beta_i}} \tag{9}$$

If the contributions are mandatory and fixed by a tax rate T, the indirect utility function of individuals is of the form

$$u_i(w_i, W) = (w_i^{\alpha_i} + b_i W^{\alpha_i})^{\frac{1}{\beta_i}}\ , \quad b_i > 0 \tag{10}$$

The specification given in equation (1) is sufficiently general to encompass several instances of public good contribution games. □

In agricultural cooperatives and other village communities, infrastructures and capital shared by the members are local public goods. They benefit everyone in the community and generate free-riding problems. Whether the contribution to those public goods is voluntary or mandatory depends on the degree of institionalization of the collective decision processes in the community. Mandatory contributions are more likely in communities that rely on more formal transactions.

2.2 Mutual insurance

In this subsection we elaborate on the indirect utilities described in equation (1) to study the demand for insurance in the group. As a first step, we study what kind of insurance the group itself can provide to its members. This will lead us to isolate the common component of the shock that impacts individual wealths and will provide a rationale for the symmetric setting we study in the next section.

In order to introduce risk in the environment, we assume that the initial wealth profile $w = (w_1, ..., w_N)$ is a stochastic variable that takes values in $[\underline{w}, \bar{w}]^N$ and is distributed according to the joint density g. We denote by E_g the associated expectation operator. The wealth of individuals is therefore subject to shocks that can be idiosyncratic and/or common.

Risk-sharing inside the group, i.e. mutual insurance, can be used to provide insurance against idiosyncratic shocks. Mutual insurance consists in redistributing wealth among individuals without changing the aggregate wealth of the group. It will be valued by all individuals in the group provided their indirect utility exhibits risk aversion with respect to own wealth and individuals are not too different concerning their risk exposure, i.e. the shocks they face are not too asymmetric. We formalize this with the two following assumptions:

Assumption 1 : For each i, the indirect utility function $u_i(w_i, W)$ is increasing and strictly concave in the first argument.

Assumption 2 : For each i, the conditional expected value of w_i satisfies

$$E_g(w_i | \sum_{j=1}^{N} w_j = W) = \frac{W}{N}$$

.

Under Assumption 2 , given that the aggregate wealth is W, the expected wealth of agent i is W/N : a symmetry requirement.

Proposition 1 *Under* Assumptions 1 *and* 2*, each individual in the group values from an ex ante perspective a mutual insurance agreement that results in an equal sharing of the aggregate wealth, i.e. each individual receiving W/N in all states of nature.*

Proof: In order to make use of Assumption 2, it is useful to decompose the expected utility as

$$E_g u_i(w_i, W) = E_g[E_g(u_i(w_i, W)|W)]$$

where $E_g(\cdot|W)$ denotes the conditional expectation operator. Under Assumption 1 we know that

$$E_g(u_i(w_i, W)|W) \leq u_i(E_g(w_i|W), W)$$

which, under Asssumption 2, induces

$$E_g u_i(w_i, W) \leq E_g u_i \left(\frac{W}{N}, W \right).$$

As a consequence, from an ex ante perspective, each individual values a mutual insurance agreement. □

Under Assumptions 1 and 2, group members are likely to agree on an ex ante mutual risk sharing agreement. So that the only remaining source of wealth variation will be common risk, i.e. variation in aggregate wealth. The group is unable to provide insurance to its members against this common risk. Such an insurance can only be provided by an external agent such as an insurance company.

Notice that when indirect utility is given by equations (3), (6) or (9), mutual insurance is unnecessary. In particular, when indirect utility depends only on aggregate wealth because it comes from a public good voluntary contribution game, individuals are already insured against idiosyncratic shocks by the public good contribution game. This is a direct consequence of the well-known fact that private provision of public good is independent of the distribution of income (Warr, 1983). When public good contribution is mandatory, however, the individual's wealth w_i directly influences his utility and mutual insurance is valuable.

3 Coordination and the value of insurance against common shocks

In order to study the demand for insurance against common shocks, we assume for simplicity a symmetric environment. The wealth of each agent is given by the same stochastic variable w. We denote by g the distribution of w, E_g the expectation operator with respect to that distribution and $\hat{w}$ the mean value or expectation of w. Such a symmetric environment is natural if individuals agreed on a mutual risk sharing agreement. It is also natural if the main source of income variation in a group of identical individuals is a common shock such as a weather shock. In this setting, what we call insurance is the possibility for an individual to replace his own stochastic wealth w by its mean value $\hat{w}$.

We will highlight conditions under which insurance can have a negative value for individuals, even if they are risk averse. Notice first that this is unlikely to happen if the shocks are independently distributed or if individuals care only about their own wealth. This can only happen when individuals have social preferences and shocks are (at least partly) common. The intuition for this point is the following. Consider the stochastic variable that describes the wealth profile of individuals $(w_1, ..., w_N)$. When shocks on wealth are independent, i.e. the random variables w_j, $j = \{1, ..., N\}$ are independently distributed, replacing the stochastic wealth of any individual by its expected value $\hat{w}$ induces a reduction in risk. In other terms when the stochastic variables w_j are independent, the distribution of the stochastic variable $(w_1, ..., w_i, ..., w_N)$ is a mean-preserving spread of the distribution of the stochastic variable $(w_1, ..., \hat{w}, ..., w_N)$. This no longer holds if shocks are correlated, i.e. if the stochastic variables w_j are correlated. In particular, when the shock is common and the individual wealths are given by the same stochastic variable w, it is not true that the distribution of the stochastic wealth profile $(w, ..., w, ..., w)$ is a mean-preserving spread of the distribution of the stochastic wealth profile $(w, ..., \hat{w}, ..., w)$. Indeed, when one combines the lottery that gives $(w, ..., \hat{w}, ..., w)$ with a family of zero-mean lotteries, the obtained lottery cannot put positive probability on $(w, ..., w, ..., w)$ with (let say) $w < \hat{w}$ without putting positive probability on an outcome $(w, ..., \tilde{w}, ..., w)$ with $\tilde{w} > \hat{w}$. It is impossible to obtain a distribution that puts positive weights only on the diagonal elements $(w, ..., w, ..., w)$.

Therefore, the fact that one individual insures himself against his own wealth variations does not imply a reduction in risk concerning the distribution of the whole wealth profile. As a consequence, when the utility of an individual depends not only on his own wealth but on the whole wealth profile, and even if this utility function is concave because the individual is risk-averse, insurance may be unvaluable.

To go a little further, notice that the distribution of $(\hat{w}, ..., \hat{w}, w, \hat{w}, ..., \hat{w})$ is always a mean-preserving spread of the (degenerate) distribution of $(\hat{w}, ..., \hat{w}, \hat{w}, \hat{w}, ..., \hat{w})$. Therefore, taking insurance has a positive value for a risk-averse individual if all the others take insurance. The problem is one of coordination because the value of being insured may depend on the decision of others.

Notice also that if individuals only care about W the aggregate wealth, insurance has a positive value whatever the number of group members that take insurance. Indeed, if we denote by k the number of group members that take insurance, the aggregate wealth is a stochastic variable

$$W_k = k\hat{w} + (N-k)w.$$

It is easily proved that W_k is a mean-preserving spread of W_{k+1} for $0 \leq k < N$.[2] If indirect utilities are given by equation (3), (6) or (9), then insurance cannot have a negative value.[3]

Let us detail below a relevant example where insurance has a negative value.

It illustrates the fact that the demand for insurance against common shocks can be plagued with multiple equilibria with either all agents or none being insured. Indirect utilities are given by equation (7) and are identical across individuals, i.e. for all i :

$$u_i(w_i, W) = w_i^{\alpha} W^{\beta}, \quad \alpha > 0, \ \ \beta > 0, \ \ \alpha + \beta < 1 \tag{11}$$

Suppose that agents can obtain full insurance for free, i.e. they can choose to exchange their stochastic wealth w for a certain wealth equal to the expected value $\hat{w}$. We are interested in the strategic form game in which individuals simultaneously choose to take insurance or not to take insurance. The payoffs in this strategic form game are as follows. If k other individuals choose to take insurance, individual i gets

$$E_g u_i(\hat{w}, (k+1)\hat{w} + (N-k-1)w) = \hat{w}^{\alpha} E_g \left((k+1)\hat{w} + (N-k-1)w\right)^{\beta}$$

if he takes insurance and

$$E_g u_i(w, k\hat{w} + (N-k)w) = E_g w^{\alpha} \left(k\hat{w} + (N-k)w\right)^{\beta}$$

if he does not. We start the analysis with a useful lemma.

Lemma 1 *For all $a > 0$ and all α, β such that $\alpha + \beta < 1$, the function $h_a(x) = x^{\alpha}(x+a)^{\beta}$ is concave.*

[2] To see this point, notice that the cumulative density F_{W_k} of W_k is decreasing with k (resp. increasing with k) below $N\hat{w}$ (resp. above $N\hat{w}$). Therefore $\int_{-\infty}^{x} F_{W_k}(y) - F_{W_k+1}(y)dy$ is positive for all $x \in \mathbb{R}$, and W_{k+1} second-order stochastically dominates W_k.

[3] This last point also shows that in those cases, insurance decision by one individual exerts an externality on the other individuals. We come back to this in Section 4.

Proof: the second order derivative of the function h_a is

$$h_a''(x) = x^{\alpha-2}(x+a)^{\beta-2}(\alpha(\alpha-1)(x+a)^2 + 2\alpha\beta x(x+a) + \beta(\beta-1)x^2),$$

which can be rearranged as

$$h_a''(x) = x^{\alpha-2}(x+a)^{\beta-2}[\alpha(\alpha+\beta-1)x(x+2a) + \beta(\alpha+\beta-1)x^2 + \alpha(\alpha-1)a^2].$$

All the terms in the brackets are negative and $h_a''(x) < 0$. □

If all agents except agent i take the insurance, it is in the interest of agent i to take the insurance as well because in this case his utility is given by $h_{(N-1)\hat{w}}(w)$ which according to Lemma 1 is a concave function. Therefore the insurance game in which the agents simultaneously choose to take or not the full insurance always has an equilibrium in which all agents take the insurance. But it may not be the only equilibrium of that game.

To see this suppose that no other individual takes the insurance. If individual i does not take the insurance his payoff is

$$E_g w^\alpha (Nw)^\beta,$$

while if he takes the insurance, it is

$$\hat{w}^\alpha E_g(\hat{w} + (N-1)w)^\beta.$$

Proposition 2 *Suppose indirect utilities are given by equation (11) and w is distributed on $\{0, \bar{w}\}$ with probabilities $\{p, 1-p\}$, $p > 0$. For N large enough, there is an equilibrium of the insurance game in which nobody takes the insurance.*

Proof : We have to compare the payoff of individual i without insurance

$$(1-p)N^\beta \bar{w}^{\alpha+\beta},$$

to the payoff with insurance

$$(1-p)^\alpha \bar{w}^\alpha (p((1-p)\bar{w})^\beta + (1-p)((1-p)\bar{w} + (N-1)\bar{w})^\beta.$$

After simple manipulations, we obtain that individual i prefers no insurance whenever

$$[(1-p)N^\beta - (1-p)^{1+\alpha}(N-p)^\beta - p(1-p)^{\alpha+\beta}] > 0,$$

which is verified for N sufficiently large. □

In that case, the insurance game possesses two equilibria : one with full insurance, i.e. insurance taken by all agents, the other with no insurance, i.e. insurance taken by no agent. Of course, the full insurance equilibrium Pareto dominates the no insurance equilibrium, nevertheless there is a priori no guarantee that agents will manage to coordinate on the full insurance equilibrium.[4] Group insurance can solve the coordination problem because it would let the group choose between the two equilibrium outcomes : full insurance or no insurance. There would be a unanimous agreement on the full insurance outcome.

Complementarities between the individual's wealth and the aggregate wealth of the other members of the group are key to explain the negative value of insurance at the individual level. Because of those complementarities, it is preferable for individual i that his own wealth is subject to the same shocks as the wealth of other group members, rather than being insured against these shocks. Beyond the example provided above, we now present sufficient conditions on the indirect utility functions that guarantee that insurance against common shocks can have a negative value.

Assumption 3 : For each i, the indirect utility function $u_i(w_i, W)$ is increasing in the second argument, differentiable and such that for all w_i,

$$\lim_{W \to +\infty} \frac{\partial u_i}{\partial w_i}(w_i, W) = +\infty.$$

The last part of Assumption 3 is not equivalent to the hypothesis of constant sign of the cross-partial derivative $\frac{\partial^2 u_i}{\partial w_i \partial W}$, it neither implies or is implied by single-crossing. But it is linked to it. It is another way to capture some elements of complementarity between the two variables. Assumption 3 is satisfied by the preferences given in equation (11).

Proposition 3 *Suppose the indirect utility functions of individuals satisfy* Assumptions 1 *and* 3*, then insurance against a common shock can have a negative value for all individuals.*

Proof : We assume that the shock on individual wealths is common and given by the random variable w which takes value in $\{0, \bar{w}\}$, with $\bar{w} > 0$. The distribution of w is such that $w = 0$ with probability $1/2$ and $w = \bar{w}$ with probability $1/2$. Consider individual i and suppose the others do not take insurance. His expected payoff if he does not take insurance is

$$\frac{1}{2} u_i(\bar{w}, N\bar{w}) + \frac{1}{2} u_i(0, 0),$$

while if he takes insurance he gets

$$\frac{1}{2} u_i \left(\frac{\bar{w}}{2}, \frac{\bar{w}}{2} + (N-1)\bar{w} \right) + \frac{1}{2} u_i \left(\frac{\bar{w}}{2}, \frac{\bar{w}}{2} \right).$$

[4] The game theory literature has repeatedly pushed forward the fact that in coordination games there is a priori no reason to focus exclusively on the Pareto dominant equilibrium, see for instance Harsanyi and Selten (1987), Carlsson and van Damme (1993).

The agent strictly prefers not to take insurance whenever

$$u_i(\bar{w}, N\bar{w}) - u_i\left(\frac{\bar{w}}{2}, (N - \frac{1}{2})\bar{w}\right) > u_i\left(\frac{\bar{w}}{2}, \frac{\bar{w}}{2}\right) + u_i(0, 0).$$

Because the function u_i is increasing in its second argument and differentiable, we know that

$$u_i(\bar{w}, N\bar{w}) - u_i\left(\frac{\bar{w}}{2}, (N - \frac{1}{2})\bar{w}\right) \geq u_i(\bar{w}, N\bar{w}) - u_i\left(\frac{\bar{w}}{2}, N\bar{w}\right) = \int_{\frac{\bar{w}}{2}}^{\bar{w}} \frac{\partial u_i}{\partial w_i}(x, N\bar{w})dx.$$

Because u_i is concave in w_i we obtain

$$u_i(\bar{w}, N\bar{w}) - u_i\left(\frac{\bar{w}}{2}, (N - \frac{1}{2})\bar{w}\right) \geq \frac{\bar{w}}{2}\frac{\partial u_i}{\partial w_i}(\bar{w}, N\bar{w}).$$

Under Assumption 3, the right-hand side of the last equation goes to $+\infty$ as N goes to $+\infty$. Suppose now that starting from an initial group of size N_0 with possible heterogenous individuals, we replicate this economy by creating k avatars of each initial individual-type. As k goes to $+\infty$, the size of the replicated group, $N = kN_0$, goes to infinity while keeping the number of individual-type fixed. Therefore it is possible to find k sufficiently high such that for all individual i in the group

$$u_i(\bar{w}, N\bar{w}) - u_i\left(\frac{\bar{w}}{2}, (N - \frac{1}{2})\bar{w}\right) > u_i\left(\frac{\bar{w}}{2}, \frac{\bar{w}}{2}\right) + u_i(0, 0)$$

□

When indirect utilities are given by equation (10), Assumption 3 is satisfied. Therefore when individuals have indirect utilities that come from a constant elasticity of substitution function, insurance against common shocks can have a negative value.

4 Individual and group risk premia

In the previous section, we highlighted a coordination problem that may arise when insurance against common shocks is offered at the individual level. We showed that, when individuals care about the wealth of other group members, the value of insurance for one individual may depend on the insurance decision of other members of the group. In particular, the value of insurance can be negative if nobody else takes insurance, while positive if every other group member takes it. This result already suggests that there are some externalities exerted by the insurance decisions of individuals. Those externalities may lead to inefficient insurance decisions when taken at the individual level.

In this section we assume that the coordination problem about insurance decisions can be solved at no cost. Still we want to study the following question. Is there a potential interest for an insurance company to sell insurance directly to the group rather than to the individuals ?

Specifically, we would like to know when the group is ready to pay more for insurance than the sum of what each individual is ready to pay. To study this question, we consider a group of N identical individuals whose indirect utility function is given as before by $u(w_i, W)$. We still assume that the individual wealths w_i are subject to the same common shock, i.e. for all i, $w_i = w$ with w a random variable distributed according to g. The notation E_g (resp. $\hat{w}$) is used for the expectation operator (resp. the expected value of w). An insurance company proposes to fully insure the common risk, i.e. to replace the individual wealth w_i by its expected value $\hat{w}$, for a positive premium.

Assumption 4 : The indirect utility function $u(w_i, W)$ is increasing in both arguments and strictly concave.

Suppose that insurance is offered to the group and that its price is shared equally among group members. Let us denote c the per member price of insurance. Individuals in the group will buy insurance whenever

$$E_g u(w, Nw) \geq u(\hat{w} - c, N(\hat{w} - c)).$$

We now define c^g the group risk premium, i.e. the highest price group members are ready to pay when insurance is offered at the group level. This risk premium is given by

$$E_g u(w, Nw) = u(\hat{w} - c^g, N(\hat{w} - c^g)) \tag{12}$$

It exists and is positive and unique under Assumption 4.

Suppose now that insurance is offered to the individuals. Each individual i is offered insurance at a price c_i. Each individual i will buy insurance whenever

$$E_g u\left(w, w + (N-1)\hat{w} - \sum_{j \neq i} c_j\right) \geq u\left(\hat{w} - c_i, N\hat{w} - \sum_{j \neq i} c_j - c_i\right).$$

Lemma 2 *Under* Assumption 4*, the maximal amount individuals are ready to pay for insurance is the same for all individuals in the group. It is given by* c^i *which solves*

$$E_g u\left(w, w + (N-1)(\hat{w} - c^i)\right) = u(\hat{w} - c^i, N(\hat{w} - c^i)) \tag{13}$$

Proof : We prove by contradiction that individuals cannot differ in the maximal amount they are ready to pay for insurance, given that all the other pay their maximal amount and get insurance. Let us denote c^i_j the maximal amount individual j is ready to pay and assume that there exists k and l such that $c^i_k > c^i_l$. we know that

$$E_g u\left(w, w + (N-1)\hat{w} - \sum_{j \neq k} c^i_j\right) = u\left(\hat{w} - c^i_k, N\hat{w} - \sum_{j} c^i_j\right).$$

The fact that u is increasing in w_i ensures that

$$E_g u\left(w, w+(N-1)\hat{w}-\sum_{j\neq k} c_j^i\right) < u\left(\hat{w}-c_l^i, N\hat{w}-\sum_j c_j^i\right),$$

or

$$E_g u\left(w, w+(N-1)\hat{w}-\sum_{j\neq k} c_j^i\right) < E_g u\left(w, w+(N-1)\hat{w}-\sum_{j\neq l} c_j^i\right).$$

When u is increasing in its second argument, this last equation is in contradiction with $c_k^i > c_l^i$. Therefore, the individual risk premium is necessarily the same for all agents and solves equation (13). □

Lemma 3 *Under* ASSUMPTION 4*, the individual risk premium that solves equation (13) is such that*

$$E_g u(w, w+(N-1)(\hat{w}-c^i)) > E_g u(w, Nw) \tag{14}$$

if and only if the per member group premium c^g is such that $c^g > c^i$.

Proof : According to the equations (12) and (13) that define the group and individual premia, equation (14) is equivalent to

$$u(\hat{w}-c^g, N(\hat{w}-c^g)) < u(\hat{w}-c^i, N(\hat{w}-c^i)),$$

which is verified only if $c^i < c^g$ because u is increasing in both arguments. □

Notice that the condition (14) is slightly more restrictive than the hypothesis of a positive externality of free insurance which we could write as

$$E_g u(w, w+(N-1)\hat{w}) > E_g u(w, Nw).$$

This occurs because when an individual takes insurance, he reduces the variability of the aggregate wealth (provided all the others take insurance also), but he pays a premium which reduces the expected aggregate wealth.

We now provide an example in which $c^g > c^i$. We focus on the case where the individual indirect utility functions are given by equation (4) and are identical for all individuals in the group so that they are given by:

$$u_i(w_i, W) = \log(w_i) + a\log(W). \tag{15}$$

We first determine the risk premium associated to the lottery w, i.e. the maximal amount c that an agent is ready to pay to replace the lottery w by a constant wealth $\hat{w}$.

Suppose first, that insurance is offered at the group level and that the premium Nc is shared equally among agents. In that case, agent i is ready to pay an amount up to c^g with

$$E_g \log(w) = \log(\hat{w} - c^g) \tag{16}$$

Suppose now that insurance is offered at the individual level. We focus on the risk premium that agent i is ready to pay, when the other agents take full insurance and pay a premium that results in a constant aggregate level of wealth $W_{-i} > 0$. Now, if agent i does not insure he gets

$$E_g \log(w) + aE_g \log(w + W_{-i}),$$

while if he takes insurance for a premium c he gets

$$\log(\hat{w} - c) + a \log(\hat{w} - c + W_{-i}).$$

Let us denote c_i^i the risk premium for agent i.

Proposition 4 *Suppose that individual utility functions are given by equation (15), for all individual i, $c_i^i < c^g$.*

Proof : The proof is by contradiction. Suppose that $c_i^i \geq c^g$, then we can use equation (16) together with the fact that a is positive, to establish that

$$E_g \log(w + W_{-i}) \leq \log(\hat{w} - c_i^i + W_{-i}).$$

In words, c_i^i is less than the risk premium an individual with utility given by $\log(\cdot + W_{-i})$ would be ready to pay. Let us denote $c_i^* > c_i^i$ such an hypothetic risk premium. Now we use the fact that the log function exhibits decreasing abolute risk aversion, so that an individual with preferences given by $\log(\cdot + W_{-i})$ is (strictly) less risk averse than an individual with utility given by $\log(\cdot)$. A standard result in risk theory (see Pratt (1964), or Gollier (2001) for a synthetic presentation) tells us that for any lottery, the risk premium of the latter individual is larger than the risk premium of the former, i.e. $c_i^* < c^g$. We therefore obtain $c_i^* > c_i^i \geq c_g > c_i^*$, a contradiction. And we must have $c_i^i < c_g$. □

Here the public good provision game generates some externalities among agents. In turn, those externalities induce that when an agent decides to buy insurance coverage, he exerts a positive externality on the welfare of others. When insurance is offered at the individual level, nobody internalizes those externalities and the equilibrium (potentially) results in underprovision, i.e. in our setting, $c_i^* < c^g$.

So far in this papragraph we focused on indirect utility functions that depend both on the individual wealth w_i and the aggregate wealth W. However, in some cases and in particular when indirect utilities come from a voluntary contribution game, it is natural to assume that the functions u_i depend only on W. A result similar to Proposition 4 is available for those cases.

Proposition 5 *When the indirect utilities of agents are given by equations (3), (6) or (9), we have for all i* $c^g > c_i^*$.

Proof : The proof is similar to that of Proposition 4. It is sufficient to notice that the functions x^y with $0 < y < 1$ and $\log x$ both exhibit decreasing absolute risk aversion. □

EXAMPLE 4 : To evaluate the difference between c^i and c^g, let us consider the following numerical application. The utility of individuals is $u(W) = \log W$, the individual wealth w takes value in $\{1, 2\}$ with equal probability 1/2. The group risk premium c^g solves

$$\frac{1}{2}\log 2 = \log\left(\frac{3}{2} - c^g\right),$$

which gives

$$c^g = \frac{3}{2} - \sqrt{2} \approx 0,0858.$$

The individual risk premium c^i solves

$$\frac{1}{2}\left(\log\left(1 + (N-1)(\frac{3}{2} - x)\right) + \log\left(2 + (N-1)(\frac{3}{2} - x)\right)\right) = \log\left(N(\frac{3}{2} - x)\right).$$

For $N = 3$, straightforward computations give

$$c^i \approx 0,0283,$$

which implies that, in a group of three individuals, the sum of the individual premia is three times less than the group premium. □

A direct consequence of Propositions 4 and 5 is that the premium that the group is willing to pay for full insurance exceed the sum of the premia the individuals are ready to pay. This remains true for a wide array of decision mechanisms at the group level because in our model, c^g is the same for all individuals.

5 Final remarks

Practitionners in the weather insurance sector are aware of the potential interest of dealing directly with cooperatives. As E. Meherette, Nyala Insurance S.C.'s deputy CEO explains:

> "Nyala has found that farmers' unions serve as effective delivery channels for the weather insurance products. By working with cooperative unions, Nyala insures all farmers who belong to the cooperative under the same contract. The cooperative is responsible for both paying the premium and distributing potential payouts to each insured farmer, reducing transaction costs for Nyala" (Meherette, 2009).

From the point of view of insurance companies, group policies certainly decrease the transaction costs. They also contribute to the scaling up necessary to recover fixed costs. Beyond these offer side advantages, we showed in this paper that group policies may also increase the demand for insurance.

In the microfinance sector, group contracts have already been pushed forward for credit. Group loans were suspected to explain the success of micro-credit institutions. Theoretical arguments have been proposed to explain their superiority over individual loans in terms of overcoming adverse selection and moral hazard (liability) problems (see Armendariz de Aghion and Morduch, 2005, for a synthesis of the different arguments).[5] The case for group insurance, as developed in this paper, relies on largely distinct arguments. In particular, group insurance must be targeted at groups that already exist and share a common interest. What occurs after the insurance contract is signed is not changed by the fact that it is a group contract. What is changed is mainly what occurs in interlinked transactions.

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