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Demand for Insurance

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Abstract

Farmers in low income countries presently use a variety of mechanisms to manage risk, including risk related to variation in crop income. There's been recent interest in introducing new financial assets to improve farmers' ability to manage such risks, such as contracts that have payoffs related to rainfall.

One of the most important lessons from finance is that the value of a new asset depends on how its returns are related to the returns on an entire *portfolio* of assets. We formulate the portfolio problem facing the farmer, and describe methods adapted from the finance literature which may help to value these new assets.

It's been observed elsewhere that demand for these new weather-contracts is low. But just because demand for one "insurance" instrument is low doesn't mean that insurance is unimportant. We further describe a framework which can be used to measure the *total* insurance provided by the portfolio of assets held by the farmer.

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1. Introduction

Farmers in low income countries operate in an environment with risk: the resources they have available at any date for consumption or investment may depend on shocks which are outside their control. If those farmers are risk averse, then (other things equal) they will prefer to "smooth" their consumption, so that consumption doesn't depend on any shock.

Since most people who live in the rural areas of low-income countries have livelihoods related to agriculture, it seems reasonable to suppose that weather-related shocks are apt to be important in determining consumption, and this observation has led to experiments with weather-based "index" insurance.

But just because weather may affect agricultural income doesn't necessarily mean that there's much unsatisfied demand for weather insurance. Precisely *because* weather is important, farmers are likely to have found other ways to at least partially address the risks they face with respect to variation in crop income. For example, if the farmers have access to credit institutions outside the village (whether formal or informal), then credit may be useful in eliminating much of the risk induced by variation in the weather. Both weather-index insurance and credit instruments are examples of financial assets which may be useful for households managing risk, and the demand for any assets will generally depend on the returns associated with other assets available to farmers.

Adapting some simple ideas from the finance literature, I pose the farmer's optimal portfolio problem, and then provide a method of characterizing the effect of introducing a new asset such weather-index insurance on the risk of the farmer's overall portfolio. This characterization of risk may also be useful for designing new sorts of contracts or assets.

To date, farmers' demand evidenced for weather-index insurance contracts has tended to fall short of expectations. But this does not necessarily mean that insurance is unimportant. I describe some further methods which may be useful in measuring the *total* insurance provided by the farmer's overall portfolio of assets. I apply these methods to ask the counterfactual question of whether farmers in the Indian ICRISAT villages would have benefited from having a simple rainfall contract available during the period 1976-82, and find evidence that they would have so benefited, though the magnitude of the benefits would have varied considerably across villages.

2. Review of optimal portfolio problem

To understand demand for crop insurance, we need to think about the more general portfolio problem a farmer solves.

Assume that the farmer has access to M distinct assets. These may be financial assets such as debt (negative holdings of bonds), equities, or futures contracts, or they may take the form of crops, real estate, or human capital.

Assume that there are a finite set of possible states of nature S. In the farmer's assessment (which in general may differ from others' assessments) the probability of a particular state s being realized is equal to π_s. In state s the returns to asset m are R_{sm}, and the vector of returns to all assets is an M-vector R_s. Thus, the collection of returns for different assets in every different state of nature form an S x M matrix $\mathbf{R}$. If the farmer holds an asset portfolio x (an M-vector), then the returns he realizes in every state are given by $\mathbf{R}x$.

The farmer's expected returns are equal to $\pi^{\top}\mathbf{R}x$ (where π is a vector of the farmer's beliefs about probabilities of different states). However, we assume that the farmer is risk-averse, and derives utility from his returns in state s equal to $U(R_s x)$.
The farmer is assumed to begin the period with wealth $\bar{x}$. He is assumed to be a subjective expected utility maximizer. His problem is to solve

$$V(\bar{x}) = \max_x \sum_{s=1}^{S} \pi_s U(R_s x)$$

such that $\sum_{m=1}^{M} x_m = \bar{x}$. The first order conditions associated with the farmer's problem are simple:

$$\sum \pi_s U'(R_s x) R_{ms} = \mu \text{ for all } m = 1, \ldots, M,$$

where μ is the Lagrange multiplier associated with his wealth constraint.

2.1. Solving the optimal portfolio problem.
To obtain a solution to the portfolio problem, it's useful to express these first order conditions in matrix form:

$$R^{\top} \Pi [\mu_s] = \bar{\mu}$$

where $\mu_s = U'(R_s x), \bar{\mu} = \mu \ell_M$,
and $\Pi = diag(\pi)$, so that $[\mu_s]$ is an S-vector of marginal utilities in dierent states.

We then solve for the optimal portfolio x^* in two steps: (i) Solve the first order conditions to obtain $\{\mu_s\}$ (marginal utility in each state); and (ii) Invert U' to obtain x. We discuss each of these in turn.

2.1.1. *How to solve for* μ_s.
It's convenient to define $\Phi_s = \mu_s / \pi_s$. There are three cases to consider, depending on the column rank of the matrix $\mathbf{R}$. In the simplest case, the column rank is simply equal to the number of states S. In this case the set of assets "spans" the possible states, and it's possible for farmers to *fully insure* themselves in such a way that marginal utilities will be constant across states. Since in this case $\mathbf{R}$ has full row rank, "the right inverse" of $\mathbf{R}$ exists: $(\mathbf{R}\mathbf{R}^{\top})^{-1}\mathbf{R}$, and we have

$$\Phi = (\mathbf{R}\mathbf{R}^{\top})^{-1}\mathbf{R}\ell;$$

and the solution to the optimal portfolio problem is simply

$$x = (\mathbf{R}\mathbf{R}^{\top})^{-1}\mathbf{R}(1/\mu).$$

In the second case rank $(\mathbf{R}) = M < S$. In this case assets don't span states, and so there's necessarily less than full insurance. For our purposes this is the interesting case, in which $\mathbf{R}$ has full *column* rank, so that the "left inverse" $((\mathbf{R}^{\top}\mathbf{R})^{-1}\mathbf{R}^{\top})$ of $\mathbf{R}$ exists, and we have

$$\Phi = (\mathbf{R}^{\top}\mathbf{R})^{-1}\mathbf{R}^{\top}\ell;$$

and so

$$x = (\mathbf{R}^{\top}\mathbf{R})^{-1}\mathbf{R}^{\top}(1/\mu):$$

Notice here the similarity to a least squares regression of the reciprocal of marginal utility on returns.

In the third case rank $(\mathbf{R}) < M < S$. Here, there's a "redundant" asset (returns are linearly dependent). Proceed by identifying and eliminating it, from the matrix of returns, and repeat until the matrix of returns has full column rank.

2.2. Interpreting the solution to the optimal portfolio problem.
By following the procedures above, we obtain a "solution" x to the optimal portfolio problem, and obtain marginal utilities μ which a farmer holding that portfolio would realize in every state. But caution is called for in interpreting these. First, for a given $\mathbf{R}$ it's entirely possible that some of the computed marginal utilities will be *negative*. This seeming impossibility is telling us that the matrix of returns allows the farmer to earn risk-free infinite returns. Such a matrix can't be an equilibrium object! We say that a matrix of returns $\mathbf{R}$ is *admissible* if all elements of $(\mathbf{R}\mathbf{R}^{\top})^{-1}\mathbf{R}\mathbf{1}$ are positive.

It's also quite possible that some elements of x will be negative. This implies that construction of the optimal portfolio involves shorting the assets corresponding to the negative elements in x. There's nothing wrong with this, though one should note that if there's shorting the farmer's portfolio can't be the same as everyone *else's* portfolio (since not everyone can hold negative quantities of any asset).
Is it realistic to think of farmers taking short asset positions? That's a behavior we tend to associate with sophisticated investors in financial

markets. But taking a short credit position is simply the act of borrowing; taking a short land position can be accomplished by simply leasing out one's land; and both of these are financial behaviors very commonly observed among farmers in many low-income environments.

2.3. Measuring the risk of a portfolio.

Define the *consumption risk* for the farmer of a portfolio x given a matrix of returns $\mathbf{R}$ by

$$U(\pi^{\mathsf{T}}\mathbf{R}x) - \pi^{\mathsf{T}}U(\mathbf{R}x)\,;$$

If U concave, then by Jensen's inequality this is non-negative; consistent with a Rothschild-Stiglitz ordering of risks.

2.3.1. *Additional assets & Generalized Beta.*

Suppose that the farmer constructs an optimal portfolio x^* given returns $\mathbf{R}$, resulting in marginal utilities μ^*. But then another asset becomes available with returns Z, taking realized values $(z_1, z_2, \ldots, z_S)$.

The farmer's demand for this new asset will depend on the relationship between its returns and the returns on the existing portfolio. The value of the new asset to the farmer will depend on the covariance of its returns with marginal utility μ^*. One way to measure this relationship is via the "generalized beta" of the new asset, defined as

$$\beta(x^*,Z) = \frac{\mathrm{cov}(\mu^*,Z)}{\mathrm{cov}(\mu^*,Rx^*)} \tag{1}$$

(Ingersoll, Jr., 1987, Chapter 5). This is also sometimes called the "systematic risk" of the asset Z.

When utility is quadratic, μ^* can be regarded as a linear function of consumption $c = Rx^*$; accordingly we obtain

$$\beta(x^*,Z) = \frac{\mathrm{cov}(Rx^*,Z)}{\mathrm{var}(Rx^*)}$$

Notice that this is just equal to the coefficient β that one would obtain in an ordinary least squares regression of

$$Z = \alpha + \beta(Rx^*) + \varepsilon\,;$$

whence the term "beta" in the traditional mean-variance analysis of finance.

With log utility, we have μ^* as a hyperbolic function of consumption. Accordingly, in this case we obtain

$$\beta(x^*,Z) = \frac{\mathrm{cov}(1/Rx^*,Z)}{\mathrm{cov}(1/(Rx^*),\, Rx^*)} = \frac{\mathrm{cov}(1/Rx^*,Z)}{1 - \mathrm{E}(Rx^*)\mathrm{E}(1/Rx^*)}$$

By Jensen's inequality the denominator of this last expression is non-positive; thus, the sign of the generalized depends only on the covariance between Z and $1/(Rx^*)$.

3. Using simple regressions to design insurance products

Suppose that we believe that rainfall may be an important source of risk for farmers, and wish to evaluate the value of a particular index-insurance contract for reducing this risk.

We'd begin by describing the returns associated with the contract in each state, giving us some vector Z. We're looking for a way to apply (1) to the case. We cannot typically expect to observe even the portfolio x, much less the matrix of returns $\mathbf{R}$ to all assets in every state. But more reasonably we may observe consumption, the inner product of these two objects. Let C denote this random variable. We have also to observe the farmers' marginal utilities of consumption; let us assume that $\mu = 1/C$, as is the case if utility of consumption is logarithmic. Then (1) can be written

$$\beta(x^*,Z) = \frac{\mathrm{cov}(1/C,Z)}{1 - \mathrm{E}(C)\mathrm{E}(1/C)} \tag{2}$$

For a given set of returns Z β can be calculated directly; recall that this gives a value proportional to the expected excess returns for the asset.

But alternatively, suppose that the Z are the returns associated with a contract the payoffs of which depend on some other random variable W. Our job is to try to *design* some $Z = f(W)$ so as to create the most valuable asset possible.

The denominator of (2) is just a function of the existing portfolio, and so doesn't depend on Z. The numerator is simply the covariance of $1/C$ with $Z = f(W)$. Let

$$f(W;\delta) = \sum_{k=0}^{K} \delta_k f_k(W),$$

for some set of known "basis" functions $\{f_k(W)\}$, where $f_0(W)$ is a constant. Then we seek the vector $\delta = (\delta_1, \ldots, \delta_K)^T$ which will maximize the covariance of $f(W;\delta)$ with the marginal utility $1/C$.

The solution to this problem is extremely straightforward: we simple calculate δ using ordinary least squares in the estimating equation

$$1/C = \delta^T [f_k(W)] + \varepsilon.$$

By the properties of least squares, conditional on our choice of basis functions the coecients $\hat{\delta}$ will maximize the covariance in the numerator of (2).

4. Evaluating rainfall insurance in the ICRISAT villages

We have historical data on rainfall for the ICRISAT villages (1975-82); also data on aggregate consumption. How valuable would rainfall insurance have been *given* existing arrangements?

Suppose that a simple rainfall index contract had been available during the period 1975-82 during which data was being collected from farmers in the ICRISAT villages, where the simple contract simply paid a rupee for every millimeter of rain during the year. How well would taking a short position have served farmers as insurance against risk in the rest of their portfolio?

Table 1. Measures of portfolio risk in the ICRISAT villages, along with calculated (generalized) betas associated with a simple rainfall contract.

Village	$E1/cEc$	Std(c)	$\beta(rain)$
Aurepalle	-0.048	2161	0.099
Shirapur	-0.086	2658	0.154
Kanzara	-0.055	2691	0.072
Pooled	-0.074	2763	0.061

Table 1 provides some simple results, both by village and pooled across the three villages. Rainfall was measured at three different rainfall stations, one per village. The first two columns provide two different measures of existing portfolio risk for households in these villages. Interestingly, these different measures provide different orderings across villages. The hyperbolic measure that we prefer on theoretical grounds indicates that the greatest levels of risk are to be found in Shirapur, while a simple calculation of the standard deviation of consumption indicates greater levels of risk in Kanzara. One way of thinking about the reason for these differences is using the standard deviation of consumption as a way of ranking risky utilities is tantamount to assuming quadratic utility, and these preferences feature increasing absolute risk aversion. Kanzara is considerably wealthier than the other two villages, and so the variation farmers face there receives greater weight when one uses the standard deviation metric.

The issue of who faces greater risk aside, which village would have benefited most from having rainfall insurance available? The answer here (computed assuming logarithmic utility) is quite clear: systematic risk in Shirapur was related to rainfall, to such an extent that had actuarially fair rainfall insurance been available, farmers would have been willing to invest 40% of their entire portfolio in the asset.

5. Conclusions

In this brief note, we've used some standard asset pricing techniques from finance to describe some simple methods for measuring the value of new assets such as index insurance. Though these tools were developed with financial markets in high-income countries in mind, these methods can nevertheless be adapted to the problems facing farmers in low income settings with little difficulty.

One of the central insights from the consumption capital asset pricing model (consumption CAPM) is that the demand for an asset should depend not on its variance, but rather on the relation of the asset's returns with marginal utilities. We use this insight to introduce a sort of "generalized beta," which gives us a measure of the way in which returns are related to marginal utilities; larger betas (in absolute value) are more valuable.

Finally, we use data on household consumption and village-level rainfall from the Indian ICRISAT village to illustrate our methods. We imagine a very simple contract on aggregate rainfall, and then calculate the generalized beta this asset would have had if it had been available to these farmers at the time. There would have been demand for this asset at its actuarially fair price in all three villages, but particularly in Shirapur, where our estimates indicate that it would optimally have comprised approximately 40% of the total village portfolio.

6. References

- **Ingersoll, Jr., J.** (1987). *Theory of Financial Decision Making*. New York: Rowman and Littleeld Publishers.

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