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Multidimensional poverty targeting*

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Abstract

The importance of taking into account multidimensionality in poverty measurement has been recently emphasized. The poverty alleviation literature has not, however, yet addressed the important issue of policy design for efficient multidimensional poverty reduction. From a positive perspective, it is regularly observed that different poverty dimensions are often correlated and mutually reinforced, especially over time. From a normative perspective, it can be argued that, in addition to being concerned with impacts on multiple dimensions of poverty, policy should also consider impacts on their *joint* distribution. The paper integrates these two perspectives into a consistent policy evaluation framework. Targeting dominance techniques are also proposed to assess the normative robustness of targeting strategies. The analytical results are applied to data from Vietnam and South Africa and illustrate the role of both normative and positive perspectives in designing efficient multidimensional poverty targeting policies.

Keywords: Targeting; Multidimensional poverty; Efficient policy; Vietnam; South Africa

JEL codes: D63, H21, I38

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1 Introduction

It is commonly argued that an adequate measurement of poverty requires taking into account both the levels of welfare in various dimensions of interest and interactions across those dimensions. This matters both for identifying the multidimensional poor and for measuring the magnitude of their poverty (see for instance Alkire and Foster 2011 and Bourguignon and Chakravarty 2003).

Although the policy importance of taking into account multidimensional linkages has also been stressed, means and objectives have sometimes been confused. The social objective of policy is indeed rarely explicitly set in terms of multidimensional poverty reduction.[1]For instance, although transfer rules regularly rely on multiple means and proxies, reducing unidimensional monetary poverty is often the salient policy objective. A prominent example is the use, in many Latin American countries, of multidimensional eligibility proxies to allocate conditional cash transfers (CCT) to low-income families, with the usual aim of reducing income poverty.

The objectives of the paper are 1) to set poverty reduction formally into a normative multidimensional poverty setting and 2) to design efficient targeting rules in that setting. We are not aware of previous work that does this. The paper thus sets the social objective function in terms of multidimensional poverty reduction and then works towards that normative objective by taking into account the effets of policy on the joint distribution of dimensions of well-being. In doing this, the paper considers three particular (normative and empirical) manners through which targeting may affect multidimensional poverty: through a direct effect on the targeted dimension, through an indirect effect on the level of joint deprivation, and through a spill-over effect on the other dimensions.

The paper thus considers interdependencies of policy effects across multiple deprivations, as advocated in the 2009 Report of the Commission on the Measurement of Economic Performance and Social Progress (see Stiglitz, Sen, and Fitoussi 2009):

> "[T]he consequences for quality of life of having multiple disadvantages far exceed the sum of their individual effects. Developing measures of these cumulative effects requires information on the 'joint distribution' of the most salient features of quality of life across everyone in a country through dedicated surveys. (...) When designing policies in specific fields, impacts on

[1]One exception is the *Chile Solidario* program, which has the explicit objective of reducing multidimensional poverty (Fiszbein and Schady 2009).

> indicators pertaining to different quality-of-life dimensions should be considered jointly, to address the interactions between dimensions and the needs of people who are disadvantaged in several domains." (pp. 15-16)

To assert clearly whether policy is deemed to reduce unidimensional or multidimensional poverty is important.[2] As pointed out by Azevedo and Robles (2010), focussing policy on unidimensional poverty measurement may lead to a sub-efficient fall in multidimensional poverty. Using multidimensional poverty indices to design policies whose objective is rather to reduce unidimensional poverty is also inefficient. This is a point made by Ravallion (2011), who argues that, to reduce income poverty, it is better to target the income poor, and that to reduce deprivation in access to public services, it is analogously better to target independently those that are deprived of such services. Using a multidimensional index of poverty (MIP) that mixes up the two dimensions can lead to a sub-efficient reduction of unidimensional income and public services poverty:

> "The total impact on (*multidimensional*) poverty would be lower if one based the allocation on the MIP [multidimensional index of poverty] rather than the separate poverty measures — one for incomes and one for access to services. It is not the aggregate index that we need for this purpose but its components." (Ravallion 2011, p. 240, our emphasis)

Unlike Ravallion (2011), however, this paper supposes that the policy objective is to reduce multidimensional poverty and not to reduce poverty in separate multiple dimensions (as is meant by the italicized term in the above citation, also known as the "dashboard" approach to multidimensional poverty).

This being said, how to capture 'interactions between dimensions' and measure the importance of 'disadvantages in several domains' (recall the quotation from Stiglitz, Sen, and Fitoussi 2009 above) is an important source of ambiguity and/or arbitrariness in the multidimensional poverty literature. Several multidimensional poverty indices have indeed been proposed, and none of them has emerged as necessarily better than all of the others. To address this difficulty, the paper focuses on 'intersection poverty indices' since these indices

[2]This issue is central in an ongoing United Nations debate on whether the next round of Global Development Goals (initially termed Millennium Development Goals in 2000, set in 2015 to become Sustainable Development Goals, see http://sustainabledevelopment.un.org/) should set multidimensional poverty indices alongside income poverty measures.

can be used to check the ‘dominance’ of targeting policies, thus showing the normative strength of any proposed targeting prescription.

It has also been well known for some time that an appropriate targeting indicator to reduce a poverty index is not necessarily the poverty index itself — see for instance Kanbur (1987) and Besley and Kanbur (1988) in the context of unidimensional poverty reduction. Referring to their MIP, Alkire and Santos (2010) suggest that it “could be used to target the poorest, track the Millennium Development Goals, and design policies that directly address the interlocking deprivations poor people experience.” (p. 1). Although the intention is clear (to reduce a MIP), it is unclear how the MIP itself can be of direct policy use. Rather, it would seem that explicit policy rules need to be derived to reduce efficiently a MIP. As shown in the paper, these rules are generally not straightforward transformations of that MIP.

The paper then proceeds as follows. Section 2.1 presents the multidimensional poverty indices used in the paper. For expositional simplicity, the paper focuses on bidimensional poverty, although the insights and results can be extended to more than two dimensions. Section 2.1 also explains how one can assess where poverty is “robustly” (in a normative sense) greater using these multidimensional indices. This is done by building dominance surfaces based on intersection indices, thus justifying this paper’s subsequent focus on such indices. These links between multidimensional intersection indices and poverty dominance surfaces are used later on to provide targeting policies that are efficient over a wide set of procedures for measuring multidimensional poverty.

Section 2.2 discusses the (theoretical) impact on multidimensional poverty of targeting one dimension, for stylized additive and multiplicative transfers. Section 2.3 derives conditions for determining which population subgroup should be targeted first such as to reduce poverty fastest. Section 2.4 enriches these results by allowing for inter-dimensional spill-over effects. Section 2.5 defines multidimensional targeting dominance surfaces and assesses whether priority rankings for group targeting and other types of targeting schemes are normatively robust over classes of multidimensional poverty indices.

The application of these analytical results is then illustrated in Section 3 with data from Vietnam (1992-1993) and South Africa (1993). Interesting insights emerge. For instance, it is shown that combining direct effects, joint deprivation effects, and spill-over effects can change significantly our understanding of the poverty impact of targeting. It is also observed that efficient rules for the geographical decentralization of targeting funds may differ according to whether it is unidimensional or multidimensional poverty that national authorities wish to reduce. The efficiency of socio-economic allocation rules is also determined by the type of multidimensional poverty indices and the range of poverty frontiers that are the objects of policy as well as by the type of transfers that are envisaged. Section 4 provides a brief conclusion.

2 Framework

2.1 Measurement and robustness

It is one thing to concur that poverty is multidimensional; it is another to agree on a specific procedure to measure it. The literature has been building up a stock of various multidimensional indices over the recent years; see among several others Chakravarty, Mukherjee, and Ranade (1998), Tsui (2002), Bourguignon and Chakravarty (2003), and Alkire and Foster (2011). All such indices have the potential to order the extent of poverty differently across distributions. This also means that they may provide different policy guidelines, especially regarding the design of targeting schemes.

One way to circumvent this problem is to seek unanimity of policy guidance across classes of poverty measurement procedures. To do this, we follow the measurement framework of Duclos, Sahn, and Younger (2006) (DSY, for short), which we now briefly summarize. DSY starts by defining well-being (measured, for expositional simplicity, over two dimensions of well-being, x and y) as a function $\phi(x, y)$ that increases in both x and y. An unknown poverty frontier $\phi(x, y) = 0$ that separates the poor from the rich is supposed to exist, a frontier over which individual well-being is equal to a "poverty level" of well-being, and below which individuals are in poverty. The set of the poor is then given by $\Lambda(\phi) = \{(x, y)\,|(\phi(x, y) \leq 0\}$. Multidimensional additive poverty indices can then be represented by

$$P(\phi) \; = \; \int\int_{\Lambda(\phi)} \pi(x, y; \phi)\, dF(x, y), \qquad (1)$$

where $\pi(x, y; \phi)$ is the contribution to poverty of an individual with well-being indicators x and y and where $F(x, y)$ is the joint distribution of x and y.

Let π^x, π^y and π^{xy} be first-order and cross-derivatives of π with respect to x and y, respectively. DSY then defines a first-order class $\Pi^{1,1}(\phi^*)$ of bidimensional poverty indices as:

$$\Pi^{1,1}(\phi^*) = \left\{ P(\phi) \left| \begin{array}{l} \Lambda(\phi) \subset \Lambda(\phi^*) \\ \pi(x,y;\phi) = 0, \text{ whenever } \phi(x,y) = 0 \\ \pi^x \leq 0 \text{ and } \pi^y \leq 0 \; \forall x, y \\ \pi^{xy} \geq 0, \; \forall x, y. \end{array} \right. \right\} \tag{2}$$

The indices that belong to $\Pi^{1,1}(\phi^*)$ must consider as potentially poor only those individuals that belong to the largest reasonable poverty set, defined by $\Lambda(\phi^*)$. The indices must also be continuous along the poverty frontier, be weakly decreasing in x and in y, and be such that the marginal poverty benefit of an increase in either x or y decreases with the value of the other variable. Atkinson and Bourguignon (1982) refer to this latter property as a property of non-decreasing poverty under a "correlation-increasing switch"; this implies that, *ceteris paribus*, the greater the incidence of multiple deprivation, the higher the level of multidimensional poverty.

Higher-order classes of poverty indices are obtained by imposing further assumptions on the derivatives of $\pi(x,y;\phi)$. For instance, the class $\Pi^{2,2}(\phi^*)$ of second-order indices are convex in x and in y; furthermore, that degree of convexity decreases with the level of the other indicator and at a decreasing rate. Further details can be found in DSY.

To test for whether the poverty ranking of two distributions is robust across all members of a given class of poverty indices, DSY introduces the following bidimensional poverty indices:

$$P(\alpha_x, \alpha_y) = \int_0^{z_x} \int_0^{z_y} \left(\frac{z_x - x}{z_x} \right)^{\alpha_x} \left(\frac{z_y - y}{z_y} \right)^{\alpha_y} dF(x,y), \tag{3}$$

where $\alpha_x \geq 0$, $\alpha_y \geq 0$, z_x and z_y are poverty lines in dimensions x and y respectively, and where $\left(\frac{z_x - x}{z_x}\right)$ and $\left(\frac{z_y - y}{z_y}\right)$ are called normalized "poverty gaps" in the poverty literature, respectively, in x and in y.[3] Tracing (3) over sets of values of z_x and z_y draws a "dominance surface".

DSY then shows that if $P_A(\alpha_x, \alpha_y)$ for some distribution A is greater than $P_B(\alpha_x, \alpha_y)$ for some distribution B over all choices of (z_x, z_y) within $\Lambda(\phi^*)$, then poverty will be unambiguously higher in A than in B for all of the poverty indices that are members of the class $\Pi^{\alpha_x+1, \alpha_y+1}(\phi^*)$ of multidimensional poverty indices of order $(\alpha_x + 1, \alpha_y + 1)$ and for

[3]For expositional simplicity, we use $P(\alpha_x, \alpha_y)$ although making (z_x, z_y) explicit in $P(\alpha_x, \alpha_y; z_x, z_y)$ would be more precise.

all poverty frontiers that lie within $\Lambda(\phi) \subset \Lambda(\phi^*)$. Let $\Delta P = P_A - P_B$; this leads to:

Proposition 1

(Multidimensional poverty dominance)

$$\Delta P(\phi) > 0,\ \forall P(\phi) \in \Pi^{\alpha_x+1,\alpha_y+1}(\phi^*), \tag{4}$$

$$\textit{iff}\ \Delta P(\alpha_x, \alpha_y) > 0,\ \forall (x, y) \in \Lambda(\phi^*). \tag{5}$$

Note that these classes of indices include intersection, union, and intermediate poverty indices, as long as these fit within $\Lambda(\phi^*)$, although the index in (3) is an *intersection* index. The converse is also true: only if $P_A(\alpha_x, \alpha_y)$ is larger than $P_B(\alpha_x, \alpha_y)$ over all values of (z_x, z_y) within $\Lambda(\phi^*)$ can we be certain that poverty is unambiguously larger in A over all members of the class $\Pi^{\alpha_x+1,\alpha_y+1}(\phi^*)$ of multidimensional poverty indices of order $(\alpha_x + 1, \alpha_y + 1)$.

It cannot be argued convincingly that the intersection index in (3) is necessarily better than all other possible multidimensional poverty indices. The superiority of one index over another is generally a matter of value judgment. There are, however, important advantages in focusing on (3), which is what this paper does. First, (3) is a natural generalization of the popular unidimensional FGT indices — see Foster, Greer, and Thorbecke (1984) — defined as

$$P(\alpha_x) = \int_0^{z_x} \left(\frac{z_x - x}{z_x}\right)^{\alpha_x} dF(x) \tag{6}$$

for poverty in x. Second, and through its intersection nature, (3) also focuses on the poorest of the poor, that is, on those that are more likely to suffer from multiple deprivation. Third, and perhaps most importantly, if some policy consistently lowers (3) for a wide range of intersection poverty frontiers, then, by Proposition 1 above, that policy will also reduce poverty for a large class of other poverty indices, possibly with different poverty frontiers. Such a result is unfortunately not available when using other sorts of multidimensional poverty indices.

Much of the paper then rests on how (3) changes when dimensional indicators vary through policies and shocks. We will consider more particularly those cases in which $P(\phi)$ is changed by additive and multiplicative transfers (denoted respectively by γ and λ), sometimes targeted to groups A or B. We will thus denote by $P(\phi, \gamma)$ the value of $P(\phi)$ following an additive transfer and by $P(\phi, \gamma^A)$ the value of $P(\phi)$ when this additive transfer is targeted to group A.

To assess the impact of such transfers, it is useful to extend (3) to cases in which α_x or α_y may equal minus one. Let then

$$P(\alpha_x = -1, \alpha_y) = f\left(z_x\right) \int_0^{z_y} \left(\frac{z_y - y}{z_y}\right)^{\alpha_y} f\left(y \middle| x = z_x\right) dy, \tag{7}$$

where $f\left(z_x\right)$ is the density of x and $f\left(y \middle| x\right)$ is the density of y conditional on x. $P(\alpha_x = -1, \alpha_y)$ is thus the y-dimension FGT poverty of those individuals whose x value borders the x-dimension poverty line, times the density of those individuals in the population. Similarly,

$$P(\alpha_x, \alpha_y = -1) = f\left(z_y\right) \int_0^{z_x} \left(\frac{z_x - x}{z_x}\right)^{\alpha_x} f\left(x \middle| y = z_y\right) dx. \tag{8}$$

It is also useful to rewrite $P(\alpha_x, \alpha_y)$ in a way that shows explicitly the role of the correlation of attributes in the valuation of multidimensional poverty. Letting $f_+ = \max(f, 0)$, we can rewrite (3) as:

$$P(\alpha_x, \alpha_y) = P(\alpha_x)P(\alpha_y) + \mathrm{cov}\left[\left(\frac{z_x - x}{z_x}\right)_+^{\alpha_x}, \left(\frac{z_y - y}{z_y}\right)_+^{\alpha_y}\right]. \tag{9}$$

Thus, bidimensional poverty $P(\alpha_x, \alpha_y)$ equals the product of the two unidimensional poverty indices plus the covariance between the poverty gaps in the two attributes. This latter term captures the importance of the "association" between the two dimensions.

DSY illustrates how this association term can play a crucial role in multidimensional poverty dominance. It can happen, for instance, that urban areas unidimensionally dominate rural areas both in income and in health, but not bidimensionally, because urban areas display greater levels of multiple deprivation. It can also happen that, although unidimensional comparisons may be ambiguous, multidimensional comparisons are not, the ambiguity being resolved by the joint distribution information.

More generally, inspection of (9) shows why a focus on unidimensional poverty ($P(\alpha_x)$, say) may lead to a different policy guidance from that provided by a focus on multidimensional poverty. Not only does $P(\alpha_y)$ multiply $P(\alpha_x)$, but the covariance of multiple deprivation also distinguishes $P(\alpha_x)$ from $P(\alpha_x, \alpha_y)$. The policy consequences of this difference are now considered in Sections 2.2 and 2.3. In Section 2.4, an additional distinction is introduced by considering cases in which transfers in the x dimension have "spill-over" effects on the y dimension.

2.2 The effect of one-dimension targeting

We now consider how changes in either dimension can affect multidimensional poverty. These changes can come from different sources, such as growth and macroeconomic shocks. We focus on the impact of targeting policies, although the results are extendable to other sources of distributional changes.

2.2.1 Additive transfers

Assume that an additive transfer γ is granted to everyone in a population. This is a simplifying framework; it will be enriched later on. We can then re-write (3) as

$$P(\alpha_x, \alpha_y, \gamma) = \int\int \left(\frac{z_x - x - \gamma}{z_x}\right)_+^{\alpha_x} \left(\frac{z_y - y}{z_y}\right)_+^{\alpha_y} dF(x,y) \quad (10)$$

and also express $P(-1, \alpha_y, \gamma)$ and $P(\alpha_x, -1, \gamma)$ in (7) and (8) analogously. For $\alpha_x > 0$, a marginal change in γ will change bidimensional poverty by

$$\left.\frac{\partial P(\alpha_x, \alpha_y, \gamma)}{\partial \gamma}\right|_{\gamma=0} = -\frac{\alpha_x}{z_x} P(\alpha_x - 1, \alpha_y)$$
$$= -\frac{\alpha_x}{z_x} P(\alpha_x - 1) P(\alpha_y) - \frac{\alpha_x}{z_x} \operatorname{cov}\left[\left(\frac{z_x - x}{z_x}\right)_+^{\alpha_x - 1}, \left(\frac{z_y - y}{z_y}\right)_+^{\alpha_y}\right]. \quad (11)$$

$P(\alpha_x - 1)$ in (11) is a multidimensional generalization of the unidimensional poverty impact of targeting derived in Kanbur (1985). It also corresponds to the well-known result that the sensitivity of *unidimensional* (or “dashboard”) FGT poverty to changes in welfare is related to the same FGT index, but with parameter set to $\alpha - 1$. For multidimensional poverty (unlike for *dashboard* poverty), this effect must be multiplied by the level of unidimensional poverty in the other dimension — the term $P(\alpha_y)$ in (11) — although this other dimension is not targeted by the transfer. The multidimensional poverty impact must also incorporate the covariance between the poverty gaps in the dimensions x and y, to the powers $\alpha_x - 1$ and α_y. As we will see later in the illustration, these additional effects can lead to different unidimensional and multidimensional policy prescriptions.

For $\alpha_x = 0$, we have

$$P(0, \alpha_y) = \int_0^{z_x} \int_0^{z_y} \left(\frac{z_y - y}{z_y} \right)^{\alpha_y} dF(x, y), \tag{12}$$

This is the y poverty gap (to the power α_y) of those that are poor both in the x and in the y dimensions. The change in multidimensional poverty following an additive transfer is then given by

$$\left. \frac{\partial P(\alpha_x, \alpha_y, \gamma)}{\partial \gamma} \right|_{\gamma=0} = -P(\alpha_x = -1, \alpha_y). \tag{13}$$

The targeting impact is thus proportional to the density of individuals *around* z_x times the unidimensional FGT index in dimension y, for those at $x = z_x$. The targeting impact is therefore quite different from the value of the index itself. It can also differ significantly from the x headcount index in the x dimension. The *per capita* cost of a universal additive transfer is $R(\gamma) = \gamma$, with $\partial R(\gamma)/\partial \gamma = 1$. The change in aggregate poverty per additional dollar spent *per capita* is thus also given by (11) and (13).

2.2.2 Multiplicative transfers

An alternative and commonly-modeled form of targeting increases a pre-transfer indicator x by some proportion λ. (The poverty impact of inequality-neutral growth in x can be similarly modeled.) Algebraically, post-transfer poverty can be written as

$$P(\alpha_x, \alpha_y, \lambda) = \int \int \left(\frac{z_x - x(1+\lambda)}{z_x} \right)_+^{\alpha_x} \left(\frac{z_y - y}{z_y} \right)_+^{\alpha_y} dF(x, y). \tag{14}$$

When $\alpha_x > 0$, the derivative of (14) with respect to λ is given by

$$\left. \frac{\partial P(\alpha_x, \alpha_y, \lambda)}{\partial \lambda} \right|_{\gamma=0} = -\alpha_x [P(\alpha_x - 1, \alpha_y) - P(\alpha_x, \alpha_y)]. \tag{15}$$

The *per capita* cost of such a multiplicative transfer is

$$R(\lambda) = \lambda \overline{x}, \tag{16}$$

where $\overline{x}$ is the average of x. The change in aggregate poverty per dollar spent *per capita* is then:

$$\frac{\partial P(\alpha_x, \alpha_y, \lambda)}{\partial \lambda} \bigg/ \frac{\partial R(\lambda)}{\partial \lambda}\bigg|_{\lambda=0} = -\frac{\alpha_x}{\overline{x}}[P(\alpha_x - 1, \alpha_y) - P(\alpha_x, \alpha_y)]. \tag{17}$$

The expression above is always negative since $P(\alpha_x - 1, \alpha_y) > P(\alpha_x, \alpha_y)$ for $\alpha_x > 0$. (17) compares the value of two bidimensional indices. Poverty reduction following a multiplicative transfer is faster the greater the difference between $P(\alpha_x - 1, \alpha_y)$ and $P(\alpha_x, \alpha_y)$. Intuitively, this occurs when multiplicative transfers decrease the poverty gaps of the "most important poor" fast — who are these normatively "most important poor" depends on the value of the poverty aversion parameter α_x. This requires the x values of the poor to be not too close to 0 and the incomes x not to be too large either, again depending on α_x.

If $\alpha_x = 0$, the change in the bidimensional headcount per dollar spent is

$$\frac{\partial P(\alpha_x, \alpha_y, \lambda)}{\partial \lambda} \bigg/ \frac{\partial R(\lambda)}{\partial \lambda}\bigg|_{\lambda=0} = -\frac{z_x}{\overline{x}} P(\alpha_x = -1, \alpha_y). \tag{18}$$

Comparing (11) to (17), and (13) to (18), it is not possible to say *a priori* whether, for every *per capita* dollar spent, an additive transfer reduces poverty faster than a multiplicative transfer. For relatively poor societies — *viz*, where $\overline{x}$ is below the poverty line z_x — a multiplicative transfer will reduce poverty faster if $\alpha_x = 0$. For $\alpha > 0$, the comparative effects will also depend on the values of $P(\alpha_x - 1, \alpha_y)$ and $P(\alpha_x - 1, \alpha_y) - P(\alpha_x, \alpha_y)$.

2.3 Socio-economic targeting

In addition to taking various forms (such as additive and multiplicative ones), targeting is rarely uniform across population groups. Socio-demographic characteristics are in particular often used to design targeting schemes, leading to "socio-economic targeting". We thus turn to how we may rank the poverty alleviation efficiency of such socio-economic targeting schemes.

2.3.1 Additive transfers

Developing the framework above, we can provide insights as to which population subgroup should be first targeted in order to reduce population poverty faster per dollar spent.

For simplicity, assume that the total population is divided into two exclusive groups, A and B (such as urban and rural areas, or regions/provinces in the empirical illustrations below). Population poverty is then given by

$$P(\alpha_x, \alpha_y, \gamma^A, \gamma^B) = \omega^A P^A(\alpha_x, \alpha_y, \gamma^A) + \omega^B P^B(\alpha_x, \alpha_y, \gamma^B), \tag{19}$$

where ω^A and ω^B are the population shares of groups A and B, γ^A and γ^B are additive transfers targeted specifically to members of groups A and B, and P^A and P^B are poverty levels for groups A and B, respectively.

To assess whether, for efficient population-level poverty reduction, an additive transfer is better targeted towards group A or group B, we need to check whether

$$\frac{\partial P(\alpha_x, \alpha_y, \gamma^A)}{\partial \gamma^A} \Big/ \frac{\partial R(\gamma^A, \gamma^B)}{\partial \gamma^A} \lesseqgtr \frac{\partial P(\alpha_x, \alpha_y, \gamma^B)}{\partial \gamma^B} \Big/ \frac{\partial R(\gamma^A, \gamma^B)}{\partial \gamma^B}, \tag{20}$$

where the *per capita* cost of an additive transfer is given by

$$R = \omega^A \gamma^A + \omega^B \gamma^B. \tag{21}$$

We start with the case of $\alpha_x > 0$. We then have

$$\frac{\partial P(\alpha_x, \alpha_y, \gamma^A)}{\partial \gamma^A} \Big/ \frac{\partial R}{\partial \gamma^A}\bigg|_{\gamma^A=\gamma^B=0} = -\frac{\alpha_x}{z_x} P^A(\alpha_x - 1, \alpha_y) \tag{22}$$

and, similarly,

$$\frac{\partial P(\alpha_x, \alpha_y, \gamma^B)}{\partial \gamma^B} \Big/ \frac{\partial R}{\partial \gamma^B}\bigg|_{\gamma^A=\gamma^B=0} = -\frac{\alpha_x}{z_x} P^B(\alpha_x - 1, \alpha_y). \tag{23}$$

The largest aggregate poverty reduction per dollar spent *per capita* (namely, per population head) is then obtained by targeting that group that has the highest $P(\alpha_x - 1, \alpha_y)$ index. Looking back to (11), note that this will be the case for the group that displays the highest $P(\alpha_x - 1)$ index, the largest $P(\alpha_y)$ index, and/or the highest covariance between $\alpha_x - 1$ and α_y unidimensional gaps. It is clear that choosing the group to target on the basis simply of the $P(\alpha_x)$ indices will generally not lead to efficient multidimensional poverty reduction strategies.

For $\alpha_x = 0$, $\frac{\alpha_x}{z_x} P^A(\alpha_x - 1, \alpha_y)$ and $\frac{\alpha_x}{z_x} P^B(\alpha_x - 1, \alpha_y)$ in (22) and (23) above are replaced respectively by $P^A(-1,\alpha_y)$ and $P^B(-1,\alpha_y)$. Again, the multidimensional poverty index itself is not the right guide to selecting the better group to target. Instead, the efficient targeting rule uses the y-dimension FGT index of those that are around the x poverty line, multiplied by the density of the group's individuals at the x-dimension poverty line.

2.3.2 Multiplicative transfers

Let us now identify efficient group selection rules under multiplicative targeting schemes. The *per capita* cost of such a scheme is given by

$$R = \omega^A \overline{x}^A + \omega^B \overline{x}^B \tag{24}$$

and, when $\alpha_x > 0$, changes in poverty due to a multiplicative transfer λ in groups A and B respectively are given by

$$\frac{\partial P(\alpha_x,\alpha_y,\lambda^A)}{\partial \lambda^A} \Bigg/ \frac{\partial R}{\partial \lambda^A}\Bigg|_{\lambda^A=\lambda^B=0} = -\frac{\alpha_x}{\overline{x}^A}[P^A(\alpha_x-1,\alpha_y) - P^A(\alpha_x,\alpha_y)] \tag{25}$$

and

$$\frac{\partial P(\alpha_x,\alpha_y,\lambda^B)}{\partial \lambda^B} \Bigg/ \frac{\partial R}{\partial \lambda^B}\Bigg|_{\lambda^A=\lambda^B=0} = -\frac{\alpha_x}{\overline{x}^B}[P^B(\alpha_x-1,\alpha_y) - P^B(\alpha_x,\alpha_y)]. \tag{26}$$

For $\alpha_x = 0$, these expressions become

$$\frac{\partial P(\alpha_x,\alpha_y,\lambda^A)}{\partial \lambda^A} \Bigg/ \frac{\partial R}{\partial \lambda^A}\Bigg|_{\lambda^A=\lambda^B=0} = -\frac{z_x}{\overline{x}^A} P^A(\alpha_x=-1,\alpha_y) \tag{27}$$

and

$$\frac{\partial P(\alpha_x,\alpha_y,\lambda^B)}{\partial \lambda^B} \Bigg/ \frac{\partial R}{\partial \lambda^B}\Bigg|_{\lambda^A=\lambda^B=0} = -\frac{z_x}{\overline{x}^B} P^B(\alpha_x=-1,\alpha_y). \tag{28}$$

Again, the case in which the transfer is a proportion of dimension x is less straightforward to interpret than the case of an additive transfer. Looking back to (25) and (26), the reduction in multidimensional poverty per dollar spent is the largest for those groups with the lowest average income and the greatest distance between $P(\alpha_x-1,\alpha_y)$ and $P(\alpha_x,\alpha_y)$. Those groups living in more deprived conditions in dimension x will have a lower $\overline{x}$; the difference in poverty of orders α_x-1 and α_x is also likely to be larger for those groups, but not necessarily so. In addition, those groups are also likely to show higher poverty in other dimensions, but again not necessarily so; the assessment must further take into account the correlation across dimensions (recall (9)).

For $\alpha_x = 0$, multidimensional population poverty falls fastest per dollar spent when targeting favors those groups whose $P(-1, \alpha_y)$ is largest and/or whose average income is lowest, the explicit trade-off being shown in (27). A large $P(-1, \alpha_y)$ value is observed when the density around the x poverty line is large, and/or when those around that poverty line have a large y poverty gap of order α_y.

2.4 Targeting with dimensional spill-overs

Now suppose that dimension y is also indirectly affected by transfers γ made to dimension x. We suppose that this spill-over effect on y is captured by a function $\sigma(y, \gamma)$, which is equal to y in the absence of spill-over effects and thus with $\sigma(y, 0) = y$. We may re-write (10) as

$$P(\alpha_x, \alpha_y, \gamma) = \int\int \left(\frac{z_x - x - \gamma}{z_x}\right)_+^{\alpha_x} \left(\frac{z_y - \sigma(y,\gamma)}{z_y}\right)_+^{\alpha_y} dF(x, y). \tag{29}$$

For expositional purposes, let us think of x and y as income and health, respectively, two dimensions in which welfare analysts are often jointly interested. (29) shows that a policy that targets income explicitly (for instance, through a cash transfer) affects multidimensional poverty directly through its impact on the poverty gap in dimension x, through its multiplying effect on the gap in the other dimension y, and through its spill-over effect on that other dimension, captured in (29) by $\sigma(y, \gamma)$.

For $\alpha_y > 0$, the marginal *spill-over effect* on bidimensional poverty of a change in γ is then given by

$$\left.\frac{\partial P(\alpha_x,\alpha_y,\gamma)}{\partial\gamma}\right|_{\text{spill-over effect},\gamma=0} = -\frac{\alpha_y}{z_y} P(\alpha_x) \int_0^{z_y} \left.\frac{\partial\sigma(y,\gamma)}{\partial\gamma}\right|_{\gamma=0} \left(\frac{z_y - y}{z_y}\right)^{\alpha_y - 1} dF(y) \tag{30}$$
$$-\frac{\alpha_y}{z_y} \operatorname{cov}\left[\left(\frac{z_x - x}{z_x}\right)_+^{\alpha_x}, \left.\frac{\partial\sigma(y,\gamma)}{\partial\gamma}\right|_{\gamma=0} \left(\frac{z_y - y}{z_y}\right)_+^{\alpha_y - 1}\right],$$

and, for $\alpha_y = 0$, by

$$\left.\frac{\partial P(\alpha_x,\alpha_y,\gamma)}{\partial\gamma}\right|_{\text{spill-over effect},\gamma=0} = \tag{31}$$
$$-\left.\frac{\partial\sigma(y,\gamma)}{\partial\gamma}\right|_{y=z_y,\gamma=0} f(\sigma(y,\gamma) = z_y) \int_0^{z_x} \left(\frac{z_x - x}{z_x}\right)^{\alpha_x} dF\left(x \middle| y = z_y\right).$$

This spill-over effect adds to the other effects described above, either through the impact of an additive or of a multiplicative transfer on dimension x. For instance, the net multidimensional poverty effect of an additive transfer to dimension x would be the sum of (11) (or (13) for $\alpha_x = 0$) and either (30) or (31). For a multiplicative transfer, expression (11) is replaced by (15), and analogously for $\alpha_x = 0$.

The formulation of $\sigma(y, \gamma)$ is sufficiently general to allow for several types of spill-over effects on the second dimension. Special cases include additive spill-over effects, when $\sigma(y, \gamma) = y + \gamma$, or multiplicative ones, when $\sigma(y, \gamma) = (1 + \gamma)y$. In all cases, the spill-over effect is given by the mean of the product of the y poverty gaps to the power $\alpha - 1$ and the marginal change in $\sigma(y, \gamma)$, weighted by the x poverty gaps to the power α_x.

Importantly, whether this indirect effect favors targeting the more severely poor depends on whether the severely poor's welfare indicator y is more sensitive to γ. That may or may not be the case. It also depends on whether the more severely poor in the x dimension are also poor in the y dimension, which again may or may not be the case.

These spill-over effects can then be normalized by the *per capita* cost of targeting dimension x. This is done in the same way as in Section 2.3. Doing so makes it possible to assess which population subgroup should be targeted first in order to reduce multidimensional poverty as quickly as possible, subject to resource constraints. If a *per capita* targeting cost can also be assessed for each of the two dimensions, x and y, then such a normalization further allows establishing which *dimension* (in addition to which *group*) should preferably be targeted by public expenditures.

2.5 Targeting dominance

As in Section 2.1 for comparing poverty across two distributions, we might also want to ensure that our targeting conclusions and policy recommendations are robust to the choice of multidimensional poverty indices and to the choice of multidimensional poverty frontiers. As in Section 2.1, we can do this for classes of indices denoted by $\Pi^{\alpha_x+1,\alpha_y+1}(\phi^*)$. To test for whether a targeting preference for a group is robust to the choice of a multidimensional poverty index within one such class of poverty indices, we can use "targeting dominance surfaces". These surfaces are given by expressions such as (13), (17), (22), (25) and (30) (for spill-over effects) over areas of intersection poverty frontiers (z_x, z_y).

For instance, to rank robustly the impact of additive and proportional transfer policies over the class $\Pi^{\alpha_x+1,\alpha_y+1}(\phi^*)$ of multidimensional poverty indices (with $\alpha_x > 0$), the targeting dominance surfaces given by (11) and (17) are compared over an area of intersection poverty frontiers (z_x, z_y) lying within $\Lambda(\phi^*)$. Formally, assuming no spill-over effect:

Proposition 2

(Dominance of additive over multiplicative targeting)

For all $P(\phi) \in \Pi^{\alpha_x+1,\alpha_y+1}(\phi^*)$ *(with* $\alpha_x > 0$*) and for* $\gamma = \lambda\overline{x}$*,* $P(\phi,\gamma) \leq P(\phi,\lambda)$ *for marginal* γ *and* λ *if and only if*

$$-\frac{\alpha_x}{z_x}P(\alpha_x-1,\alpha_y) \leq -\frac{\alpha_x}{\overline{x}}[P(\alpha_x-1,\alpha_y)-P(\alpha_x,\alpha_y)]\ \forall(z_x,z_y) \in \Lambda(\phi^*). \tag{32}$$

This says that additive targeting will decrease poverty faster, *per capita* dollar spent, than multiplicative targeting for all indices of poverty in $\Pi^{\alpha_x+1,\alpha_y+1}(\phi^*)$ if and only if expression (11) is always found to be lower than (17) regardless of the choice of intersection poverty frontiers, as long as these frontiers lie within the maximum domain of poverty frontiers within which a multidimensional poverty assessment can reasonably be made. As above, the dominance tests compare additive and multiplicative impacts on multidimensional *intersection* indices, although robustness is obtained over indices that include intersection, union, and intermediate poverty indices.

Extensions of Proposition 2 can be made straightforwardly by allowing for spill-over effects, by considering classes of order $\alpha_x + 1 = 1$ in dimension x, or by assessing whether robust socio-economic targeting conclusions can be obtained over classes of indices. An example of dominance of additively targeting socio-economic group A over group B is given by Proposition 3 (assuming no spill-over effect):

Proposition 3

(Dominance of additively targeting group A *instead of group* B*)*

For all $P(\phi) \in \Pi^{\alpha_x+1,\alpha_y+1}(\phi^*)$ *and for* $\omega^A\gamma^A = \omega^B\gamma^B$ *,* $P(\phi,\gamma^A) \leq P(\phi,\gamma^B)$ *for marginal* γ^A *and* γ^B *if and only if*

$$-\frac{\alpha_x}{z_x}P^A(\alpha_x-1,\alpha_y) \leq -\frac{\alpha_x}{z_x}P^B(\alpha_x-1,\alpha_y)\ \forall(z_x,z_y) \in \Lambda(\phi^*). \tag{33}$$

Both Propositions 2 and 3 have the potential to generate robust targeting prescriptions. It may however be that the targeting dominance surfaces happen to be not statistically different from each other over the entire area $\Lambda(\phi^*)$, or that they may even cross over that area. In such cases, the normative and statistical validity of the targeting prescription will depend on which subset of poverty indices within $\Pi^{\alpha_x+1,\alpha_y+1}(\phi^*)$ will be preferred. In such cases, inspection of the subareas of targeting dominance surfaces over which dominance can be inferred can also serve to indicate the strength of the robustness of targeting prescriptions.

3 Illustrations

3.1 Multiple deprivation and dimensional spill-over effects

As discussed above, the correlation — and more generally, the joint distribution — of dimensions is important both for measurement and for policy purposes. From an empirical perspective, much of this correlation usually reflects a “natural” distribution of dimensions. An example is the correlation between child nutrition and schooling performance (and adult labor outcomes): child malnutrition (especially if experienced during the first two years of life) is usually associated with smaller school achievements and reduced lifetime income (see, for example, Glewwe and King 2001, Heckman 2008 and Alderman, Hoddinott, and Kinsey 2006 for discussion and evidence). Negative correlations can also occur (as when an increase in child school attendance decreases household income, at least in the short term). The interactions between welfare dimensions can be especially strong in cases of severe deprivation over a long period, as in the case of “continuing multi-dimensional poverty traps” (see for instance Thorbecke 2005).

Some of that joint distribution between dimensions of well-being can also be driven (at least partly) by policy in a number of different ways. Subsidized provision of education, health and housing may be one way to alleviate poverty in each of its multiple dimensions as well as jointly. Public investments in perinatal care (for instance, through pre-natal health visits and nutritional programs for pregnant women) can improve the health status of newborn children and their later life prospects in several dimensions. Policy can thus serve to reduce both dimensional deprivation statuses and their correlation, and can thus also reduce the prevalence of multiple deprivations.

The popularized conditional cash transfer (CCT) programs intend for instance to break down the multidimensional (and inter-generational) poverty traps both by alleviating monetary poverty and by increasing levels of human capital (health and education). A key mechanism that is employed is the multidimensional conditionality of the transfers. The cross-dimension effects of this have been most extensively demonstrated in the context of Latin American countries. For example, Fiszbein and Schady (2009) show plenty of cross-country evidence of CCT's positive impacts on various health indicators and access to health services, school enrolment and attendance, and — most prominently because of the nature of the programs — on income poverty.

Note that the effect on health poverty of a cash transfer conditioned on family investments in child health is likely to be higher than one without conditionality; the short-term effect on monetary poverty may, however, be reduced by conditionality, if, for instance, some of the transfers cannot then be used for short-term income production purposes. Hence, conditionality may not be efficient for monetary poverty reduction, but may be efficient for reducing multidimensional poverty, especially if substantial spill-over effects exist.

The correlation across well-being attributes and the ability of policy to modify it also depend on the quality of markets. Where markets are inexistant or highly imperfect, social programs may not be effective at producing positive spill-over effects on dimensions other than the targeted one. For example, in remote areas where appropriate schooling infrastructure is missing or is of poor quality, social cash transfers for children may have meagre effects on school outcomes (see for instance Kakwani, Soares, and Son 2006 and Cockburn, Fofana, and Tiberti 2010).

All of this points to the usefulness of a consistent multidimensional framework for assessing the context-dependent impact of policy. It is not possible, of course, to take empirically into account all of the possible effects of policy on multidimensional poverty. It is nevertheless feasible and, we believe, useful to apply the analytical framework developed above to illustrate how these effects can feed into policy design and evaluation. We do this in three different ways. We first assess the poverty impact and the efficiency of simple targeting rules set on the basis of socioeconomic characteristics, following the strong targeting tradition found in the unidimensional poverty literature. We then enrich those simple rules with a more realistic assessment of the impact of policies, policies that can have spill-over effects beyond the dimensions that are targeted. We finally test the robustness of targeting prescriptions using dominance results of the types shown in Propositions 2 and 3.

3.2 Data and estimation procedures

We apply the analytical approach presented above to two separate datasets from Vietnam and South Africa. These are the Vietnam Living Standard Survey (VLSS) 1992-1993 and the South Africa Integrated Household Survey (SAIHS) 1993. These two data sets include information on household consumption and anthropometric measures, which is a major reason for their use here. This information enables the construction of *per capita* household consumption (deflated by appropriate spatial and temporal price deflators) and height-for-age *z* scores (*HAZ*), standardized by the growth standards found in WHO (2006). These indicators of monetary welfare and of health are used for income poverty and health poverty respectively. The analysis focuses on children under five years old. It is supposed that policy can target consumption (dimension x in the above analytical framework), but that the multidimensional poverty effectiveness of that policy depends on its impact on the joint distribution of consumption and *HAZ* (dimension y).[4]

The spill-over effect on health of targeting consumption is obtained through the following regression model:

$$y_i = \alpha + \beta_x x_i + \sum_k \beta_k z_{k,i} + \epsilon_i, \tag{34}$$

where y_i is the z-score for child i, x_i is log *per capita* household consumption, β_x is the coefficient associated to *per capita* consumption, z_k is determinant k, β_k is the associated coefficient, and ϵ_i is an error term. The model is borrowed from Wagstaff, van Doorslaer, and Watanabe (2003), with OLS estimation and community-level fixed effects at the level of the child's commune. Note that the model is intended to provide a simple, reduced-form, representation of potentially complex mechanisms linking consumption to children's health. These mechanisms will generally depend on household composition and intra-household allocation rules, rules that are rarely observable for the analyst. An example is the distribution of cash transfers for the benefit of children. These can be directly distributed to adults, with a potentially diluted effect on the targeted children. The transfers can alternatively take the form of nutritional transfers, which could in principle be potentially better targeted to children; with these transfers, there also exist, however, strategies that parents can use in order to substitute away from children some of the additional resources intended for them.

Table 1 shows descriptive statistics on *HAZ* and on the explanatory variables appearing in the *HAZ* regressions. The estimated coefficients of the *HAZ* regression are shown in Table 2. Most of the coefficients take the expected sign in all two surveys. *Per capita* consumption is positively associated with child health; child health is negatively (and con-

[4]It is assumed that child consumption is increased by the value of the cash transfer. We thus abstract from important intra-household allocation issues — also see below.

vexly) linked to child age; in South Africa, being male is associated with worse health, while having access to improved sanitation facilities improves health statistically only in Vietnam 1992-93. Somewhat surprisingly, the estimated parameters on access to safe water sources and maternal schooling are not statistically significant.

The spill-over parameters of child consumption on child HAZ are produced by the estimates of Table 2 are 0.0171 percent for VLSS 1992-1993 and 0.1766 percent for SAIHS 1993. These parameters are obtained as ratios between $ln(pc\_consumption)$'s coefficients in Table 2 and the exponential of the mean of $ln(pc\_consumption)$. They are then calculated as $0.2470/\exp(7.2705)$ for VLSS 1992-1993 and $0.2842/\exp(5.0808)$ for SAIHS 1993. These spill-over effects are used below in valuing the impact of a variation in child consumption onto *HAZ* values for children.

3.3 Efficient multidimensional poverty targeting

We proceed by separating the total population into separate sub-population geographical groups — see their definition in Table 3. This makes it possible to interpret many of the results below as guidance for geographical targeting and possibly for decentralization of targeting funds. As suggested in WHO (2006), out-of-range values (<-5 and >3) for the z-scores are dropped. For ease of exposition, a value of 10 is added to the *HAZ* variable and to the poverty lines in the health dimension; such a transformation does not affect any of the substantive results since we are interested in absolute multidimensional poverty, not relative multidimensional poverty or inequality.

For benchmarking purposes, a reference annual monetary poverty line of 1790 thousands Dong (in 1998 prices) is used for the Vietnamese survey, while a monthly monetary poverty line of 164 Rand is used for South Africa. These values correspond to around 385 and 75 dollars (in 2005 'international' dollars) respectively. For health, a poverty threshold of -2 standard deviations is used for each of the two countries — this threshold is often used to identify moderate-to-severe stunting (following the transformation of the *HAZ* variable, the reference health poverty threshold is set to 8). These poverty lines are used for reference purposes. For dominance, ranges of poverty lines are needed and these will be discussed in section 3.4.

We focus on impacts on bidimensional poverty with $\alpha_x = \alpha_y = 0$ and $\alpha_x = \alpha_y = 1$, normalized by the *per capita* cost of the policy. The geographical units are ordered according to the importance of the marginal poverty reduction that follows a marginal increase in a consumption cash transfer.

3.3.1 Vietnam 1992-1993

We start with Vietnam 1992, using $\alpha_x = \alpha_y = 0$ and the reference poverty lines mentioned above. We first consider additive transfers. The results are shown in the upper Panel A of Table 4, a panel that is split into four different sets of columns. The first column of Table 4 shows the priority ranking that must be assigned to the groups shown in the other columns. (All of the rankings shown are statistically significant at the conventional 5% level; the analytical procedures and the Stata routines for checking this are available on request). The set of the next three columns then shows the unidimensional results, namely, those results based only on the monetary impact of the transfer. The second set of three columns multiplies this unidimensional impact by poverty in the second dimension. The third set of three columns incorporates the impact of the monetary transfer on the covariance of deprivation. The last set of three columns shows the total multidimensional poverty impact of the transfer, adding to the earlier effects the spill-over effect on the non-targeted dimension.

Focusing first on unidimensional poverty, a statistically significant larger reduction in total poverty per dollar spent is obtained by targeting group 1 as opposed to groups 6, 5, 3 and 7. The second-best group to be targeted is group 2, whose unidimensional poverty impact per dollar spent is significantly larger than 3 and 7. A statistical ranking cannot be established with respect to any other geographical groups.

Let us now add the health poverty component. The effect of this is shown in the second set of columns in Panel A of Table 4. A significant re-ranking across the geographical groups is obtained. Groups 1 and 2 continue to be most efficiently prioritized but comparisons with other groups have changed: group 1 is now also preferred to groups 8 and 9 but not anymore to groups 3 and 6; as seen in Table 6, the reason is that groups 3 and 6 show the largest health headcount. Taking health poverty into account then moves groups 3 and 6 upward in terms of priority, but not enough to outrank groups 1 and 2. Targeting group 2 is now statistically preferable to targeting groups 9, 8 and 5. The next groups to be prioritized are groups 3, 10 and 6; targeting these groups provide a statistically larger poverty reduction than targeting group 5.

For multidimensional poverty reduction, considering poverty in separate dimensions is not enough; we must also take into account joint deprivation. This is done by adding the covariance term to obtain the third set of columns in Table 4. A few changes in the ranking of priority groups are immediately observable. Groups 1 and 2 are still the first priority groups; these groups are now statistically preferred to groups 5, 3, 8, 6, 4 and 7. Targeting group 1 thus becomes statistically better than targeting groups 3, 6 or 4, but not better anymore than targeting group 9. Similarly, group 2 is now also statistically preferred to groups 3, 6, 4 and 7, but not anymore to group 9. Group 10 follows in the ranking and is statistically preferred to group 7. Finally, group 9 is preferred to groups 6 and 7 since health poverty for those around the consumption poverty line is larger for that group — see equations (7) and (13).

The last set of columns shows the impact of adding spill-over effects, as indicated in equation (31). Groups 1 and 2 then lose their statistical priority over group 8. Group 10 is now also preferred to group 6, while targeting group 5 allows for a statistically larger reduction in multidimensional population poverty than targeting group 7.

Panel B of Table 4 shows priority rankings with $\alpha_x = \alpha_y = 1$; they also vary again when moving away from unidimensional towards multidimensional poverty alleviation. For instance, the rankings of groups 4, 6 and 7 depend on whether it is unidimensional or multidimensional poverty that is alleviated. The same is true for many other priority rankings for targeting.

As is well-known from the poverty literature, the use of different poverty indices can affect quantitatively and qualitatively the nature of poverty comparisons. As is less well known, that can also affect the comparative evaluation of targeting schemes. This can be observed by comparing Panels A and B in Table 4. In particular, looking at the last set of columns (*Total impact with spill-over*), targeting groups 3 and 7 is a statistically significant priority with $\alpha_x = \alpha_y = 1$ (the multidimensional poverty gap) but clearly not with $\alpha_x = \alpha_y = 0$ (the multidimensional headcount). Conversely, there is no reason to prefer group 9 with the multidimensional poverty gap, while a priority for group 9 over groups 6 and 7 can be statistically inferred with the multidimensional headcount.

More generally speaking, the use of multidimensional poverty gaps yields more precise targeting guidance than the use of multidimensional headcounts. Greater statistical precision emerges because greater sample information is used with the poverty gap than with the headcount: when it comes to estimating standard errors, all observations below the poverty lines are important, not only those close to those lines. Greater normative strength is also obtained with the multidimensional poverty gap: the priority ranking with the multidimensional poverty gap is established by looking at the average welfare impact across all of the poor, and not only by considering whether that impact is large enough to lift some of the poor out of multidimensional poverty.

The results following a proportional transfer are shown in Table 5. The priority rankings differ significantly relative to those of additive transfers in Table 4. With $\alpha_x = \alpha_y = 0$, for instance, proportional transfers to group 1 are preferred to proportional transfers to groups 2, 8 and 9, which is not the case for additive transfers. The transfer schemes are thus important in establishing social-economic targeting priorities. These Vietnamese results are driven by the large average consumption of groups 2, 8 and 9 (see Table 6). As seen in equation (18), a $\overline{x}$ larger than z_x (as in the case of groups 2, 8 and 9) makes proportional targeting less efficient.

3.3.2 South Africa

Let us now turn to regional targeting in South Africa. The main results for additive targeting are shown in Table 7. Let us focus on $\alpha_x = \alpha_y = 1$ (Panel B) and on some of the more interesting findings. Take group 5, for instance; it has a relatively large health headcount (see Table 8) as well as a large average health poverty gap, but its level of consumption poverty is relatively low. Hence, with unidimensional poverty, a statistically significant preference for targeting groups 3, 9, 13, and 6 over group 5 can be established; with multidimensional poverty, this is not the case anymore. Conversely, targeting group 9 (which has high consumption poverty) is better than targeting any of groups 11, 2, 5, 16, 8 or 1 for unidimensional poverty reduction but this is nevertheless not the case anymore for multidimensional poverty.

Moving from $\alpha_x = \alpha_y = 0$ to $\alpha_x = \alpha_y = 1$ again changes policy guidance dramatically. This is easily seen by comparing panels A and B of Table 7. As an example, group 3 is dominated by most other geographical groups when $\alpha_x = \alpha_y = 0$, while, with $\alpha_x = \alpha_y = 1$, it dominates 16 out of 17 possible groups (group 5 is the only group not statistically outranked by group 3). While group 3 shows an extraordinarily large consumption headcount and health poverty gap (which explains its high priority ranking under $\alpha = 1$: Figure 1, Panel A), nearly nobody lies around the consumption poverty line (which explains the small bidimensional impact when $\alpha = 0$, see Panel B of Figure 1). This important distinction between the incidence and the intensity of multidimensional poverty, and between levels of multidimensional poverty and efficient strategies for multidimensional poverty alleviation, explains the important reversals of priority rankings when moving from $\alpha_x = \alpha_y = 0$ to $\alpha_x = \alpha_y = 1$.

3.4 Targeting dominance

The results above show how a switch from unidimensional to multidimensional poverty can change the nature of efficient poverty reduction strategies. They also show that priority rankings can sometimes depend on how multidimensional poverty is measured. We now use the methods of Section 2.5 to construct multidimensional targeting dominance surfaces and thus assess whether priority rankings for group targeting are robust over classes of multidimensional poverty indices.

Recall that distribution *A* dominates distribution *B* if that A's dominance surface is lower than that of *B* over a sufficiently large area of poverty frontiers. In terms of targeting dominance, the same applies but through comparing the targeting dominance surfaces of groups A and B, as in Proposition 3. Prioritizing a group with a more negative targeting surface will lead to a faster reduction in multidimensional poverty per dollar spent.

For practical purposes, ten equally-spaced different poverty lines (equal to or lower than the reference poverty lines) are used for each of the two dimensions, yielding 100 possible combinations of poverty line.[5] We specify 10 different poverty lines for each of the two dimensions, giving an area of poverty frontiers set over 100 possible combinations of consumption and health poverty lines. The 10 poverty lines in each dimension are set at the minimum values of the indicators plus the deciles of the distance between the official poverty lines and those minimum values. The upper limit of those lines (the upper right corners in the forthcoming figures) corresponds to the official poverty lines, while the lower poverty lines are at the lower left corners. The dominance results are shown in Figures 2, 3, 4 and 5; they show the p-values of differences in poverty impact across alternative targeting strategies.

For Vietnam 1992-1993 and for the $\Pi^{1,1}$ class of indices, Figure 2a shows that targeting group 2 should be prioritized relative to group 5 as this would allow a larger reduction in multidimensional poverty over most of the bidimensional poverty domain. Move now to 1993 South Africa. Figure 2b says that group 15 should be preferred to group 16: the reduction in total multidimensional poverty that follows from targeting group 15 is statistically always greater (at a 5% level) over the entire area of poverty frontiers shown in that Figure. Given the results of Propositions 2 and 3, this says that a priority for group 15 over group 16 in South Africa can be established on the basis of the entire class $\Pi^{1,1}(\phi^*)$ of multidimensional poverty indices, for all the poverty areas that fit within the $\Lambda(\phi^*)$ shown

[5] The findings are robust to choosing a larger number of lines.

in the Figure.

Let us now move to the class of bidimensional poverty indices $\Pi^{2,2}(\phi^*)$. Figure 3 shows that targeting group 3 is preferable to targeting group 8 over the whole area of poverty frontiers shown in that figure. This says that the reduction in the multidimensional poverty gap is faster when group 3 is targeted. It also says that all of the multidimensional poverty indices that are members of the $\Pi^{2,2}(\phi^*)$ class will fall faster if group 3 is targeted instead of group 8. Relative to group 2, targeting group 3 is statistically dominant only over upper health and consumption poverty lines. Group 3 dominates group 4 for intermediate areas of poverty lines.

Consider now Figure 4 for South Africa. Panel B of Table 7 showed that it was better to target group 3 instead of groups 13 and 14 for efficient multidimensional poverty gap ($\alpha_x = \alpha_y = 1$) reduction at the reference poverty lines. Figure 4 shows that this is not necessarily true for all poverty frontiers and for all indices in the $\Pi^{2,2}(\phi^*)$ class. Targeting group 3 dominates targeting group 13 only over the area of consumption poverty lines above around 70 Rand and health poverty lines above around 7.5. A more detailed examination of the results shows that while the product of consumption and health poverty (the first term on the right-hand side of (11)) does allow a robust ranking even for lower poverty lines, this is not anymore the case when the joint deprivation effect (the second term on the right-hand side of (11)) is added in. Targeting group 3 is, however, preferable to targeting 14 over the the entire range of poverty lines shown in Figure 4, thus indicating targeting dominance of group 3 over group 14.

Figure 5 for 1993 South Africa shows a case in which taking into account multidimensional deprivation helps sharpen targeting prescriptions. Figure 5a shows the usual p-values of the differences in the targeting dominance surfaces of two groups, in that case groups 13 and 9, for additive transfers and over the class $\Pi^{2,2}(\phi^*)$ of indices. Figure 5b shows p-values of the differences in the consumption and in the health unidimensional targeting dominance curves. Although, for most poverty lines, neither univariate targeting dominance is statistically observed (with the exception of health poverty lines lower than about 7.3, which are quite low), for a large area of multidimensional combinations of these poverty lines the poverty reduction through targeting group 13 dominates statistically that from targeting group 9. The fundamental reason for this is lower health poverty in group 9 than in group 13 (0.024 versus 0.039 — see Table 8 — as estimated at the reference poverty line), lower deprivation in group 9 than 13 (0.002 versus 0.004) as well as a smaller spill-over effect in group 9.

4 Conclusion

The paper derives targeting rules, both theoretically and empirically, that can reduce poverty as quickly as possible per overall *per capita* dollar spent. Simple transfer schemes are considered, such as additive and multiplicative transfers, but generalizations of these as well as transfers that have spill-over effects on other dimensions can also be analyzed. Those targeting rules can help identify which socioeconomic groups (such as provinces or regions, smaller or larger families, wage workers or farmers) should be prioritized for efficient poverty reduction. It is also shown how targeting dominance techniques can help check the normative robustness of targeting rules. Applications of this framework to the alleviation of child poverty in Vietnam and South Africa show how these tools can help monitor and maximize the reduction in multidimensional consumption and health poverty.

An important and intuitively reasonable message that runs across the paper is that the nature of efficient targeting rules may depend on whether it is unidimensional or multidimensional poverty that policy is intended to reduce. In contrast to unidimensional poverty — where it is only the impact on a single dimension that matters —, the paper emphasizes three possible effects of targeting on multidimensional poverty, denoted as a direct effect on the targeted dimension, an indirect effect on joint deprivation and a possible spill-over effect on the other dimensions. Because of this, some targeting schemes may end up being more efficient at reducing univariate poverty but less so at alleviating multidimensional poverty, and *vice versa*. The value of targeting prescriptions also depends on the structure of the transfers; whether a group should be prioritized may depend, for instance, on the nature of the transfers that are being contemplated (such as whether the transfers will be additive or multiplicative).

The paper further points out that the appropriate indicators to use to design efficient targeting schemes are not the poverty indices themselves. For multiplicative transfers, for instance, it is the level of average welfare plus the distance between two multidimensional indices that should be used to identify which group it is most efficient to target. This makes it necessary *inter alia* to consider non-obvious but important trade-offs between the effect of targeting on the poorest of the poor and the effect of targeting on the speed of income increase among the not-so-poor.

The social value of targeting schemes also depends on the choice of poverty measures that policy is intended to reduce. The arbitrariness involved in choosing one specific poverty index and one specific poverty frontier and the possible sensitivity of targeting prescriptions to that choice make it desirable to use targeting dominance tools. These tools are developed and applied in the paper; apart from being linked to simple intersection poverty indices, they also help assess the normative strength of targeting prescriptions.

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Appendix A Tables

Table 1: Means and standard deviations of variables included in the *HAZ* regressions

	VLSS92-93	SAIHS93
HAZ	-2.20	-1.22
	(1.35)	(1.47)
ln(pc_consumption)	7.27	5.08
	(0.52)	(0.92)
age_months	32.02	31.32
	(17.43)	(16.71)
age_months2	1328.86	1260.27
	(1118.27)	(1063.74)
gender	0.50	0.50
	(0.50)	(0.50)
safe_water	0.79	0.83
	(0.41)	(0.37)
safe_sanitation	0.14	0.35
	(0.35)	(0.48)
schooling_mother	6.51	5.56
	(3.44)	(3.61)
# of observations	2754	3858

Note: standard deviations are reported in parentheses. Means and standard deviations are estimated on the sample of children 0-5 years old retained for the regression analysis.
Source: authors' analysis based on VLSS 1992-1993 and SAIHS 1993.

Table 2: *HAZ* regressions' coefficients

explanatory variables	**VLSS92-93**	**SAIHS93**
ln(pc_consumption)	0.2470	0.2842
	(3.61)	(6.47)
age_months	-0.0764	-0.0567
	(-12.55)	(-9.8)
age_months2	0.0010	0.0008
	(10.88)	(8.53)
gender	0.0262	-0.1232
	(0.54)	(-2.71)
safe_water	0.0543	-0.1752
	(0.5)	(-1.72)
safe_sanitation	0.2405	0.1404
	(2.68)	(0.95)
schooling_mother	0.0167	0.0135
	(1.61)	(1.74)
constant	-3.0117	-1.7532
	(-6.27)	(-7.14)
Adj. R^2	0.1551	0.1696
# of observations	2754	3858

Note: *t*-stats are reported in parentheses. Explanatory variables are not necessarily comparable across surveys since their definition may differ.

Source: authors' analysis based on VLSS 1992-1993 and SAIHS 1993.

Table 3: Numbering of the geographical groups, VLSS92-93 and SAIHS93

(a) *VLSS92-93*		**area**	
		urban	rural
region	RedRiverDelta	8	6
	Northeast	3	3
	Northwest	4	4
	NorthCentralCoast	7	7
	SouthCentralCoast	5	10
	CentralHighlands	1	1
	Southeast	5	2
	MekongRiverDelta	9	2

(c) *SAIHS93*			**area**	
		metro	urban	rural
province	Western Cape	1	1	1
	Northern Cape	2	2	
	Eastern Cape	3	4	5
	KwaZulu-Natal	6	7	8
	Free State	9	10	
	Mpumalanga	11	12	
	Limpopo	13	14	
	North West	15	16	
	Gauteng	17	17	18

Note: The geographical groups appearing in the tables were obtained as a combination of regions/provinces and areas. *E.g.*, group "1" in *VLSS92-93* corresponds to the combination of urban and rural areas in the Central Highlands region.

Table 4: Impact of additively targeting consumption on bidimensional poverty: Vietnam 1992-1993 (x 10^{-4})

Ranking	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated
***Panel A*: $\alpha_x = \alpha_y = 0$**												
	Unidimensional			Product of two dimensions			Multidimensional			Total impact with spill-over		
	$-P(\alpha_x = -1)$			$-P(\alpha_x = -1)P(\alpha_y)$			$-[P(\alpha_x = -1)P(\alpha_y) + \text{cov}(.)]$					
1	1	-5.98	*6:5:3:7*	1	-3.59	*7:9:8:5*	1	-3.58	*5:3:8:6:4:7*	1	-3.76	*5:3:6:7:4*
2	2	-4.52	*3:7*	2	-2.49	*9:8:5*	2	-2.52	*5:3:8:6:4:7*	2	-2.70	*5:3:6:7:4*
3	10	-3.89		3	-2.27	*5*	10	-2.04	*7*	9	-2.28	*6:7*
4	9	-3.54		10	-2.22	*5*	9	-1.95	*6:7*	10	-2.22	*6:7*
5	4	-3.48		6	-2.21	*5*	5	-1.53		8	-2.00	
6	6	-3.27		4	-2.16		3	-1.32		5	-1.84	*7*
7	5	-3.24		7	-1.94		8	-1.26		3	-1.36	
8	3	-3.20		9	-1.59		6	-1.05		6	-1.16	
9	8	-3.14		8	-1.35		4	-1.03		7	-1.08	
10	7	-2.83		5	-1.28		7	-1.00		4	-1.06	
***Panel B*: $\alpha_x = \alpha_y = 1$**												
	$-(\alpha_x/z_x)[P(\alpha_x - 1)]$			$-(\alpha_x/z_x)[P(\alpha_x - 1)P(\alpha_y)]$			$-(\alpha_x/z_x)[P(\alpha_x - 1)P(\alpha_y) + \text{cov}(.)]$			Total impact with spill-over		
1	4	-5.20	*7:6:10:1:2:9:5:8*	3	-0.50	*6:4:10:2:9:5:8*	3	-0.52	*6:4:10:2:5:9:8*	3	-0.57	*6:4:10:2:5:9:8*
2	3	-4.99	*6:10:1:2:9:5:8*	7	-0.48	*6:4:10:2:9:5:8*	7	-0.50	*6:4:10:2:5:9:8*	7	-0.55	*6:4:10:2:5:9:8*
3	7	-4.81	*10:1:2:9:5:8*	6	-0.39	*10:2:9:5:8*	1	-0.43	*2:5:9:8*	1	-0.47	*5:9:8*
4	6	-4.63	*10:2:9:5:8*	1	-0.37	*9:5:8*	6	-0.40	*2:5:9:8*	6	-0.44	*2:5:9:8*
5	10	-3.94	*9:5:8*	4	-0.37	*2:9:5:8*	4	-0.37	*5:9:8*	4	-0.41	*5:9:8*
6	1	-3.88	*9:5:8*	10	-0.29	*9:5:8*	10	-0.33	*5:9:8*	10	-0.36	*5:9:8*
7	2	-3.74	*9:5:8*	2	-0.27	*9:5:8*	2	-0.29	*5:9:8*	2	-0.32	*5:9:8*
8	9	-2.26		9	-0.09	*8*	5	-0.11	*8*	5	-0.12	*8*
9	5	-1.70		5	-0.07		9	-0.07		9	-0.08	
10	8	-1.16		8	-0.04		8	-0.04		8	-0.04	

Source: authors' analysis based on data from the VLSS 1992-1993.

Note: The "groups dominated" (in *italics*) are those groups for which the difference in poverty impact is significant at 5 percent.

Table 5: Impact of proportionately targeting consumption on bidimensional poverty: Vietnam 1992-1993 (x 10^{-6})

Ranking	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated
***Panel A**: $\alpha_x = \alpha_y = 0$*												
	Unidimensional			Product of two dimensions			Multidimensional			Total impact with spill-over		
	$-(z_x/\overline{x})P(\alpha_x = -1)$			$-(z_x/\overline{x})P(\alpha_x = -1)P(\alpha_y)$			$-(z_x/\overline{x})[P(\alpha_x = -1)P(\alpha_y) + \text{cov}(.)]$					
1	1	-6.8	*6:7:9:5:8*	1	-4.1	*2:9:5:8*	1	-4.1	*2:3:4:6:7:9:5:8*	1	-4.1	*2:3:4:6:7:9:5:8*
2	4	-5.2	*5:8*	3	-3.2	*9:5:8*	2	-2.5	*6:7:9:5:8*	2	-2.5	*6:7:9:5:8*
3	3	-4.5	*9:5:8*	4	-3.2	*9:5:8*	10	-2.4	*5:8*	10	-2.4	*5:8*
4	10	-4.5	*9:5:8*	6	-2.9	*9:5:8*	3	-1.9	*8*	3	-1.9	*8*
5	2	-4.4	*9:5:8*	7	-2.6	*9:5:8*	4	-1.5		4	-1.5	
6	6	-4.2	*9:5:8*	10	-2.6	*9:5:8*	6	-1.4		6	-1.4	
7	7	-3.8	*5:8*	2	-2.4	*9:5:8*	7	-1.3		7	-1.3	
8	9	-2.3		9	-1.1		9	-1.3		9	-1.3	
9	5	-2.0		5	-0.8		5	-0.9		5	-0.9	
10	8	-1.7		8	-0.7		8	-0.7		8	-0.7	
***Panel B**: $\alpha_x = \alpha_y = 1$*												
	$-(\alpha_x/\overline{x})[P(\alpha_x - 1) - P(\alpha_x)]$			$-(\alpha_x/\overline{x})[(P(\alpha_x - 1) - P(\alpha_x))P(\alpha_y)]$			$-(\alpha_x/\overline{x})[(P(\alpha_x - 1) - P(\alpha_x))P(\alpha_y) + \text{cov}(.)]$			Total impact with spill-over		
1	4	-4.9	*7:6:10:1:2:9:5:8*	3	-0.5	*4:6:1:10:2:9:5:8*	3	-0.5	*4:6:1:10:2:5:9:8*	3	-0.5	*4:6:1:10:2:9:8*
2	3	-4.5	*6:10:1:2:9:5:8*	7	-0.4	*6:1:10:2:9:5:8*	7	-0.4	*6:1:10:2:5:9:8*	7	-0.5	*6:1:10:2:9:8*
3	7	-4.2	*10:1:2:9:5:8*	4	-0.4	*10:2:9:5:8*	4	-0.4	*10:2:5:9:8*	4	-0.4	*10:2:9:8*
4	6	-4.0	*10:1:2:9:5:8*	6	-0.3	*10:2:9:5:8*	6	-0.3	*10:2:5:9:8*	6	-0.4	*10:2:9:8*
5	10	-2.9	*9:5:8*	1	-0.3	*9:5:8*	1	-0.3	*5:9:8*	1	-0.3	*9:8*
6	1	-2.8	*9:5:8*	10	-0.2	*9:5:8*	10	-0.2	*5:9:8*	10	-0.3	*9:8*
7	2	-2.5	*9:5:8*	2	-0.2	*9:5:8*	2	-0.2	*5:9:8*	2	-0.2	*9:8*
8	9	-1.1		9	-0.0		5	-0.1		5	-0.1	
9	5	-0.8		5	-0.0		9	-0.0		9	-0.0	
10	8	-0.5		8	-0.0		8	-0.0		8	-0.0	

Source: authors' analysis based on data from the VLSS 1992-1993.

Note: The "groups dominated" (in *italics*) are those groups for which the difference in poverty impact is significant at 5 percent.

Table 6: Population shares and poverty gaps in consumption and health dimensions: Vietnam 1992-1993

Groups	**Population shares**	**Consumption**			**Health**		
		P0	P1	mean	P0	P1	mean
1	0.034	0.695	0.249	1565.785	0.600	0.096	7.617
2	0.260	0.669	0.211	1834.894	0.551	0.073	7.958
3	0.160	0.893	0.321	1262.547	0.710	0.100	7.459
4	0.036	0.930	0.336	1209.299	0.620	0.071	7.711
5	0.070	0.304	0.077	2936.803	0.397	0.041	8.414
6	0.172	0.829	0.277	1387.726	0.678	0.083	7.698
7	0.145	0.862	0.307	1329.38	0.686	0.101	7.521
8	0.021	0.207	0.031	3348.073	0.431	0.031	8.239
9	0.032	0.404	0.119	2699.172	0.449	0.038	8.431
10	0.068	0.706	0.261	1552.488	0.572	0.074	7.884
Population	1	0.729	0.247	1679.308	0.607	0.080	7.798

Source: authors' analysis based on data from the VLSS 1992-1993.

Table 7: Impact of additively targeting consumption on bidimensional poverty: South Africa 1993 (x 10^{-3})

Ranking	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated
			Panel A: $\alpha_x = \alpha_y = 0$									
	Unidimensional			Product of two dimensions			Multidimensional			Total impact with spill-over		
	$-P(\alpha_x = -1)$			$-P(\alpha_x = -1)P(\alpha_y)$			$-[P(\alpha_x = -1)P(\alpha_y) + \text{cov}(.)]$					
1	4	-4.58	*18:3*	17	-1.33	*2:12:16:1:3:14:18:7*	4	-3.07	*3*	4	-3.15	*3*
2	6	-3.85	*12:1:7:2:16:18:3*	5	-1.25	*18:7*	15	-2.95	*13:1:17:18:16:2:3*	15	-3.08	*13:1:17:18:16:2:3*
3	8	-3.83	*1:2:18:3*	6	-1.25	*2:12:16:1:3:14:18:7*	6	-2.53	*1:17:18:16:2:3*	6	-2.68	*1:17:18:16:2:3*
4	15	-3.65	*1:7:2:16:18:3*	4	-1.19		9	-2.21		9	-2.33	
5	5	-3.48	*1:2:18:3*	8	-1.09	*7*	8	-2.21	*18:3*	8	-2.31	*3*
6	17	-3.23	*18:3*	15	-1.05	*12:1:3:14:18:7*	10	-2.08	*3*	12	-2.26	*3*
7	10	-3.19	*18:3*	13	-1.00	*2:12:1:3:14:18:7*	11	-2.07	*18:3*	10	-2.25	*3*
8	11	-3.04	*18:3*	10	-0.98	*12:1:3:14:18:7*	5	-2.01	*3*	11	-2.24	*3*
9	13	-2.88	*18:3*	11	-0.85	*1:3:18:7*	12	-2.00	*3*	5	-2.15	*3*
10	9	-2.59		9	-0.73		13	-1.93	*3*	14	-2.10	*3*
11	12	-2.58	*3*	2	-0.67	*18:7*	14	-1.93		13	-2.03	*3*
12	14	-2.35		12	-0.50		7	-1.84	*3*	7	-2.03	*3*
13	1	-2.25	*3*	16	-0.50		1	-1.54	*3*	1	-1.79	*3*
14	7	-2.15		1	-0.47		17	-1.43	*3*	17	-1.78	*3*
15	2	-1.89		3	-0.45		18	-1.35	*3*	18	-1.59	*3*
			Panel B: $\alpha_x = \alpha_y = 1$									
	$-(\alpha_x/z_x)[P(\alpha_x - 1)]$			$-(\alpha_x/z_x)[P(\alpha_x - 1)P(\alpha_y)]$			$-(\alpha_x/z_x)[P(\alpha_x - 1)P(\alpha_y) + \text{cov}(.)]$			Total impact with spill-over		
1	3	-5.44	*13:6:15:11:4:10:2:5: 12:16:17:14:8:7:1:18*	3	-0.26	*13:6:4:2:15:10: 9:11:8:17:16:12: 1:18:7:14*	3	-0.27	*13:6:10:15:4:8: 2:9:11:16:12:17: 18:14:1:7*	3	-0.31	*13:6:10:15:9:4: 8:2:11:16:12:17: 14:18:1:7*
2	9	-4.78	*11:2:5:12:16:17:14:8: 7:1:18*	13	-0.18	*9:11:8:17:16:12: 1:18:7:14*	13	-0.20	*9:11:12:17:18:14: 1:7*	13	-0.24	*9:2:11:12:17:14: 7*
3	13	-4.60	*6:15:11:2:5:12:16: 17:14:8:7:1:18*	6	-0.16	*11:8:17:16:12:1: 18:7:14*	5	-0.20		5	-0.22	
4	6	-3.96	*5:12:16:17:14:8: 7:1:18*	5	-0.13		6	-0.17	*12:17:18:14:1:7*	6	-0.18	*12:17:14:18:1:7*
5	15	-3.84	*12:16:17:14:8:7:1:18*	4	-0.13	*12:1:18:7:14*	10	-0.16	*12:17:18:14:1:7*	10	-0.18	*12:17:14:18:1:7*
6	11	-3.71	*12:17:14:8:7:1:18*	2	-0.12	*12:1:18:7:14*	15	-0.15	*12:17:18:14:1:7*	15	-0.18	*12:17:14:18:1:7*
7	4	-3.61	*12:17:14:8:7:1:18*	15	-0.12	*12:1:18:7:14*	4	-0.14	*17:18:14:1:7*	9	-0.16	*12:17:14:18:1:7*
8	10	-3.60	*12:17:14:8:7:1:18*	10	-0.12	*12:1:18:7:14*	8	-0.14	*7*	4	-0.15	*14:18:1:7*
9	2	-3.14	*17:8:7:1:18*	9	-0.11	*12:1:18:7:14*	2	-0.13	*18:14:1:7*	8	-0.15	*18:1:14*
10	5	-2.33		11	-0.10	*12:1:18:7:14*	9	-0.13	*17:18:14:1:7*	2	-0.15	*18:1:14*
11	12	-2.03	*18*	8	-0.08		11	-0.12	*17:18:14:1:7*	11	-0.14	*17:14:18:1:7*
12	16	-1.94		17	-0.07	*18:7:14*	16	-0.10		16	-0.11	
13	17	-1.79		16	-0.05		12	-0.07	*7*	12	-0.08	*7*
14	14	-1.74		12	-0.04		17	-0.06	*7*	17	-0.08	*7*
15	8	-1.67		1	-0.03		18	-0.05	*7*	14	-0.06	
16	7	-1.65		18	-0.02		14	-0.05		18	-0.05	*7*

Source: authors' analysis based on data from the SAIHS 1993.

Notes: The "groups dominated" (in *italics*) are those groups for which the difference in poverty impact is significant at 5 percent; Lower ranked groups that do not dominate any group are not reported in the table for lack of space.

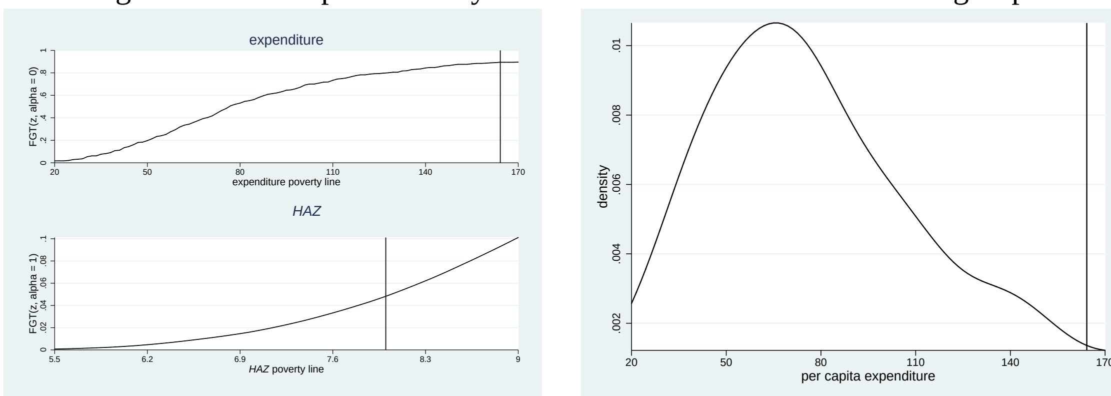
Table 8: Population shares and poverty gaps in the consumption and health dimensions: South Africa 1993

Groups	**Population shares**	**Consumption**			**Health**		
		P0	P1	mean	P0	P1	mean
1	0.068	0.251	0.078	473.337	0.210	0.021	9.109
2	0.013	0.515	0.207	194.412	0.356	0.040	8.356
3	0.146	0.894	0.473	107.529	0.384	0.048	8.474
4	0.016	0.593	0.209	256.718	0.261	0.036	8.993
5	0.015	0.383	0.125	226.900	0.361	0.057	8.404
6	0.140	0.649	0.232	158.029	0.324	0.041	8.646
7	0.038	0.271	0.085	365.650	0.115	0.014	9.211
8	0.022	0.274	0.097	294.640	0.286	0.046	8.903
9	0.024	0.785	0.399	130.403	0.284	0.024	8.901
10	0.034	0.591	0.250	238.046	0.306	0.033	8.740
11	0.063	0.610	0.252	189.990	0.279	0.028	8.772
12	0.025	0.333	0.096	402.422	0.196	0.018	8.781
13	0.151	0.755	0.355	138.297	0.346	0.039	8.517
14	0.011	0.285	0.118	376.269	0.171	0.012	9.221
15	0.057	0.631	0.274	199.732	0.287	0.031	8.727
16	0.012	0.318	0.125	354.744	0.273	0.026	9.359
17	0.029	0.294	0.123	252.045	0.412	0.038	8.375
18	0.137	0.196	0.063	600.252	0.193	0.021	9.301
National	1	0.554	0.241	263.750	0.292	0.034	8.781

Source: authors' analysis based on data from the SAIHS 1993.

Appendix B Figures

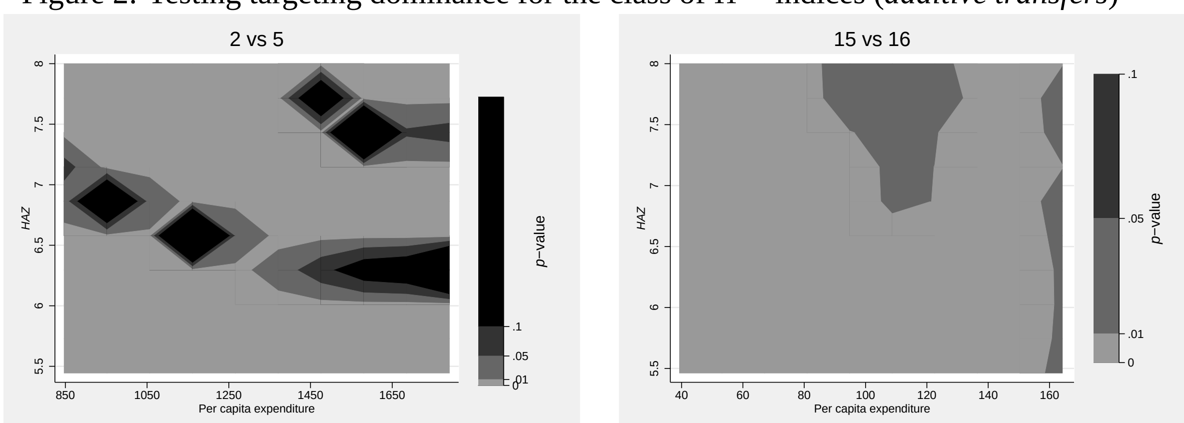
Figure 1: Consumption density and FGT indices for South Africa's group 3



(a) FGT indices (b) Consumption density

Source: authors' analysis based on data from SAIHS 1993.

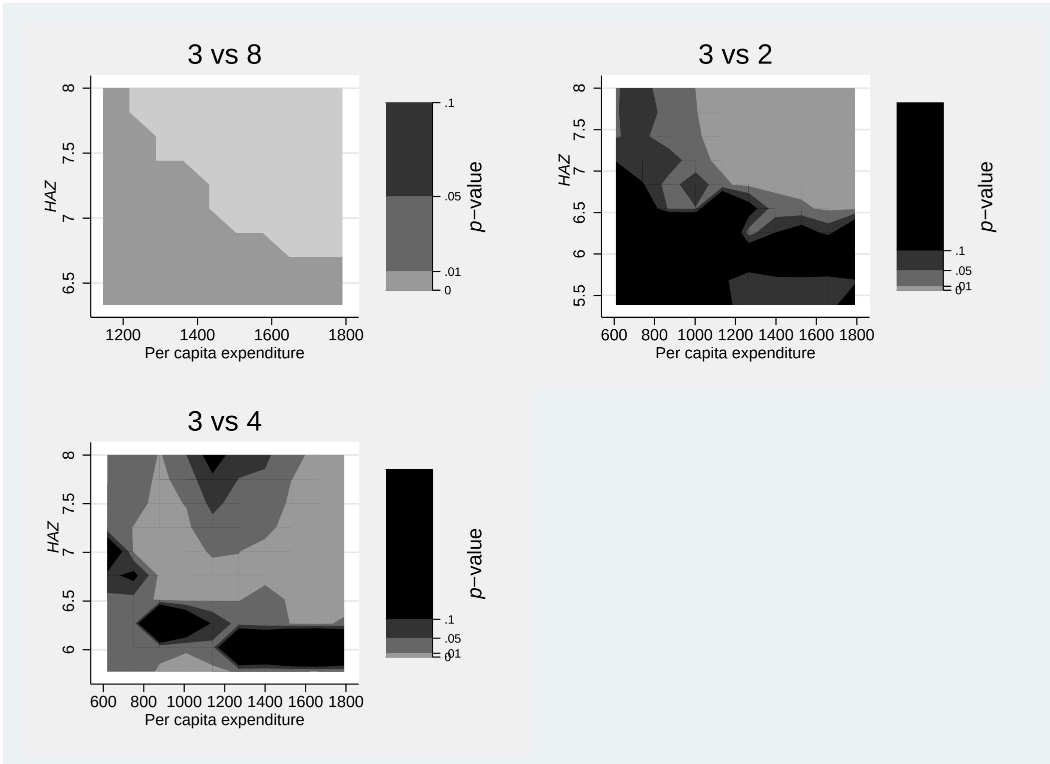
Figure 2: Testing targeting dominance for the class of $\Pi^{1,1}$ indices (*additive transfers*)



(a) Vietnam 1992-1993 (b) South Africa 1993

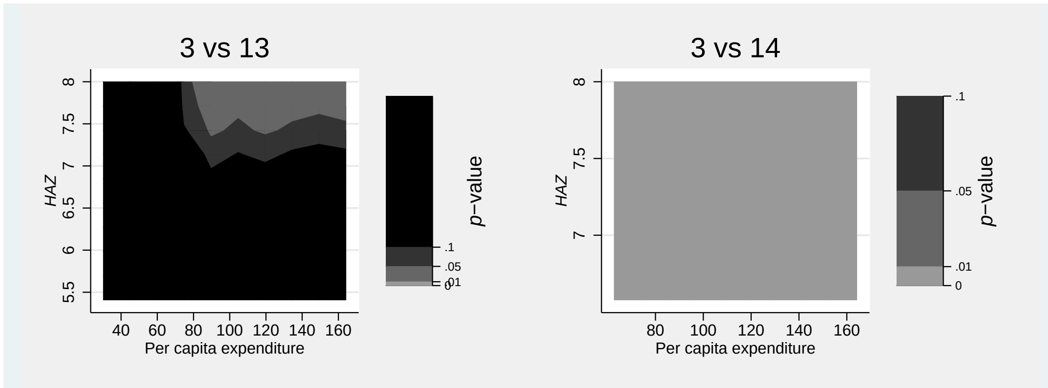
Note: the first graph shows the p-values of the differences in poverty impact between targeting group 2 and targeting group 5 in Vietnam; the second graph shows the p-values of the differences in poverty impact between targeting group 15 and targeting group 16 in South Africa. Lighter areas indicate where it is statistically more likely that targeting the first group (in each of the two graphs) will reduce poverty faster.
Source: authors' analysis based on data from the VLSS 1992-1993 and SAIHS 1993.

Figure 3: Testing targeting dominance of group 3 over other groups for the class of $\Pi^{2,2}$ indices, Vietnam 1992-1993 (*additive transfers*)



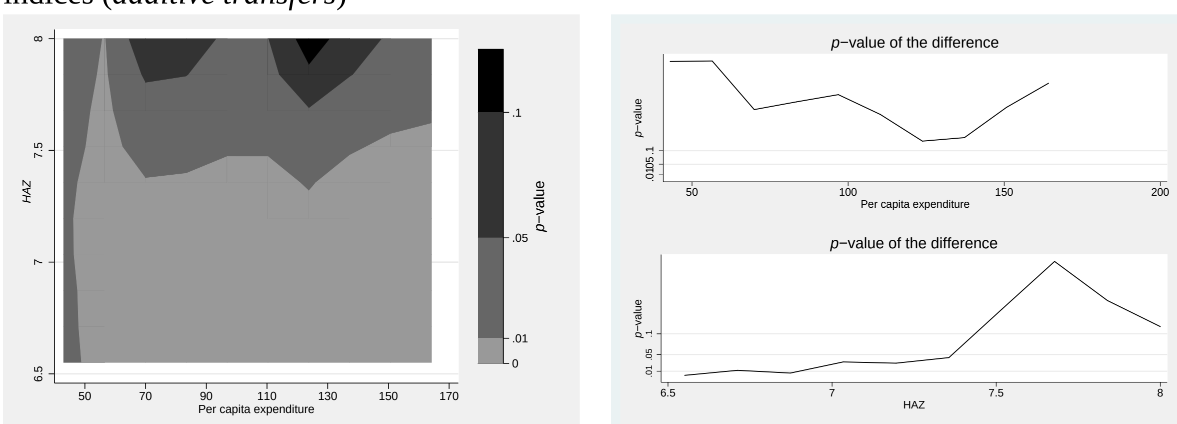
Note: the graphs show the p-values of the differences in poverty impact between targeting group 3 and targeting other groups; the lighter areas indicate where it is statistically more likely that targeting group 3 will reduce poverty faster.
Source: authors' analysis based on data from the VLSS 1992-1993.

Figure 4: Testing targeting dominance of group 3 over other groups for the class of $\Pi^{2,2}$ indices (*additive transfers*), South Africa 1993



Note: the graphs show the p-values of the differences in poverty impact between targeting group 3 and targeting other groups; the lighter areas indicate where it is statistically more likely that targeting group 3 will reduce poverty faster.
Source: authors' analysis based on data from the SAIHS 1993.

Figure 5: Testing the dominance of targeting group 13 over group 9 for the class of $\Pi^{2,2}$ indices (*additive transfers*)



(a) multidimensional (b) unidimensional

Note: the first graph shows the p-values of the differences in poverty impact between targeting group 13 and targeting group 9; the lighter areas indicate where it is statistically more likely that targeting group 13 will reduce poverty faster. The second graph shows the p-values of the difference in the unidimensional poverty impact between targeting group 13 and targeting group 9, in the dimension of consumption and health poverty, respectively.
Source: authors' analysis based on data from the SAIHS 1993.

"Sur quoi la fondera-t-il l'économie du monde qu'il veut gouverner? Sera-ce sur le caprice de chaque particulier? Quelle confusion! Sera-ce sur la justice? Il l'ignore."

Pascal

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