

# More on multidimensional, intertemporal and chronic poverty orderings

**Florent Bresson<sup>†</sup>**      **Jean-Yves Duclos<sup>‡</sup>**

<sup>†</sup>: CERDI, CNRS – Université d'Auvergne

<sup>‡</sup>: CIRPÉE, Université Laval & FERDI

# Disclaimer

 **Very first draft!**

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Which measure is appropriate?

# The problem

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Poverty is lower with distribution *B* when compared with distribution *A* if:

$$P_B(\lambda) - P_A(\lambda) \leq 0$$

with:

*P*: the poverty measure,

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Contingency of the result with respect to  $\lambda$  and  $P$ .

Robustness implies using criteria that make it possible to obtain rankings that do not depend on  $\lambda$  and  $P$ .

# State of the art

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However, they all assume poverty indices are continuous while many poverty indices are not.

# Contribution

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- ▶ Relax the continuity assumption and propose dominance conditions for bidimensional poverty indices that comply with restricted continuity,
  - ▶ Highlight the changes induced by relaxing the continuity assumption in comparison with the unidimensional case.
  - ▶ Show how the framework can easily be adapted for the ordering of the chronic component of intertemporal poverty.

# Outline

1. Framework
2. Main results
3. Chronic poverty
4. Concluding remarks

## Notations

- ▶  $x := (x_1, x_2)$  is an individual profile.
  - ▶  $\lambda : \mathfrak{R}_+^2 \rightarrow \mathfrak{R}$  is a non-decreasing welfare function.
  - ▶  $\mathcal{L}$  is the set of non-decreasing functions on  $\mathfrak{R}_+^2$ .
  - ▶  $\Gamma \in \mathfrak{R}_+^2$  is the poverty domain.
  - ▶  $\Lambda$  is the poverty frontier.
  - ▶  $\Gamma_1(\lambda) \subset \Gamma(\lambda) | x_1 < x_2$ .
  - ▶  $F(x_1, x_2)$  is the joint cumulative distribution function.
  - ▶  $H$  is a headcount index.

# The DSY approach of poverty identification

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How shall we separate the poor from the non-poor in a bidimensional context?

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How shall we separate the poor from the non-poor in a bidimensional context? Duclos, Sahn & Younger (2006) defines the poor as those such that:

$$\lambda(x_1, x_2) \leq 0.$$

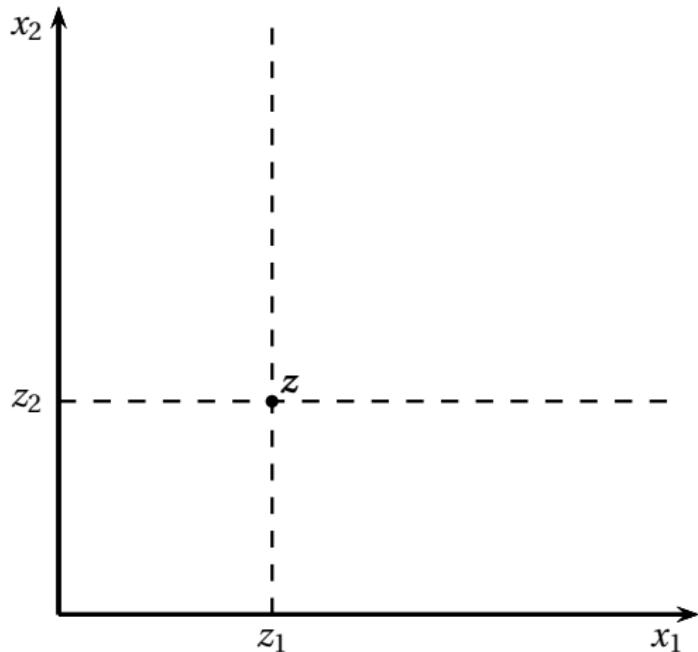


Figure 1: The definition of the poverty domain.

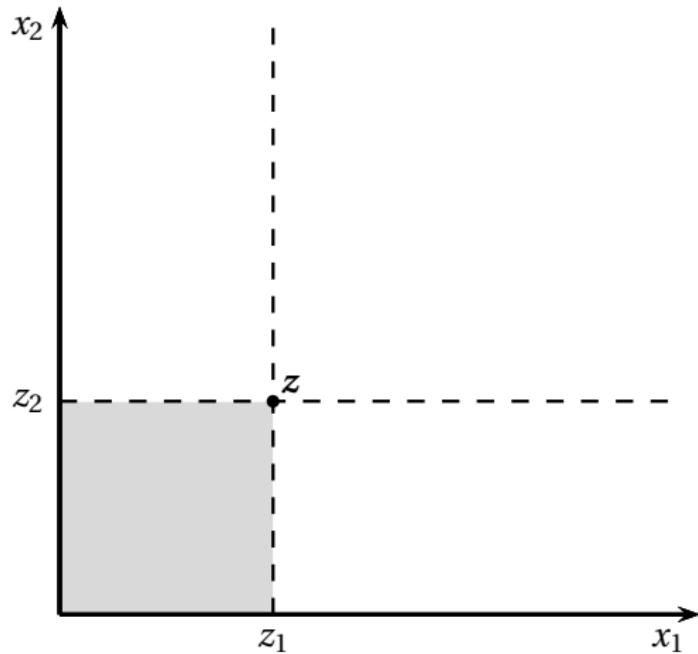


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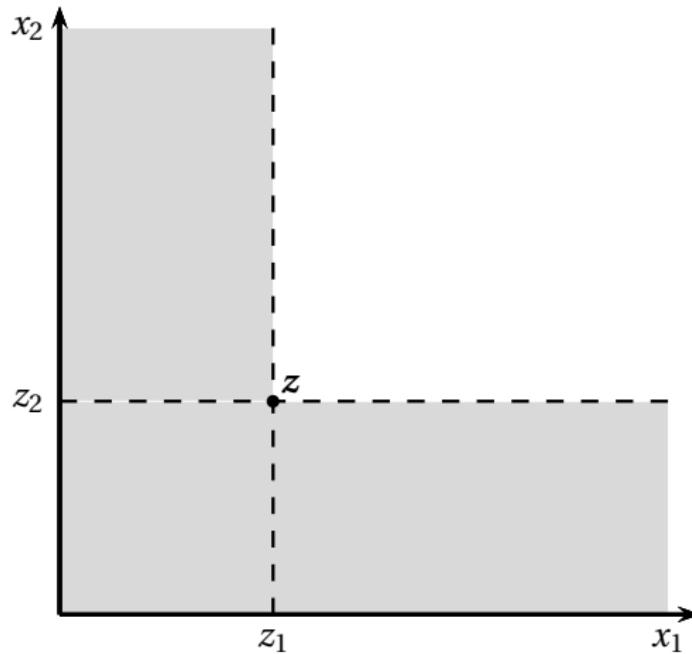


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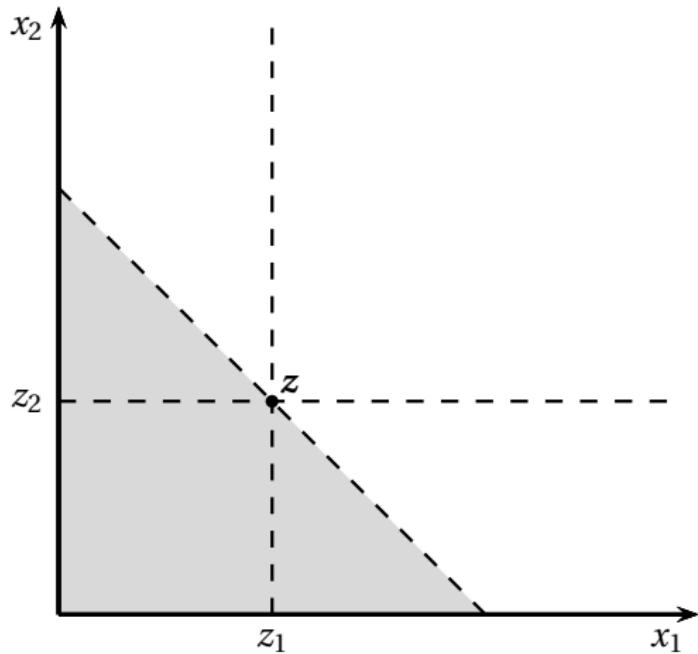


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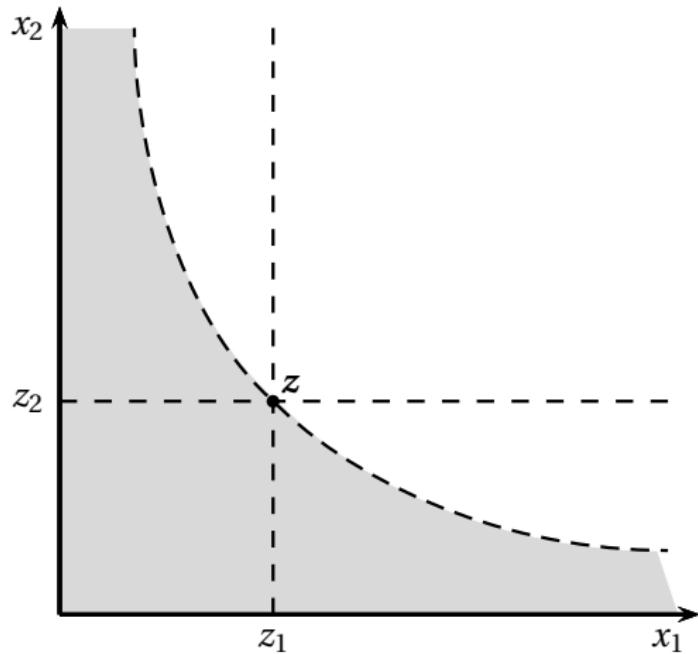


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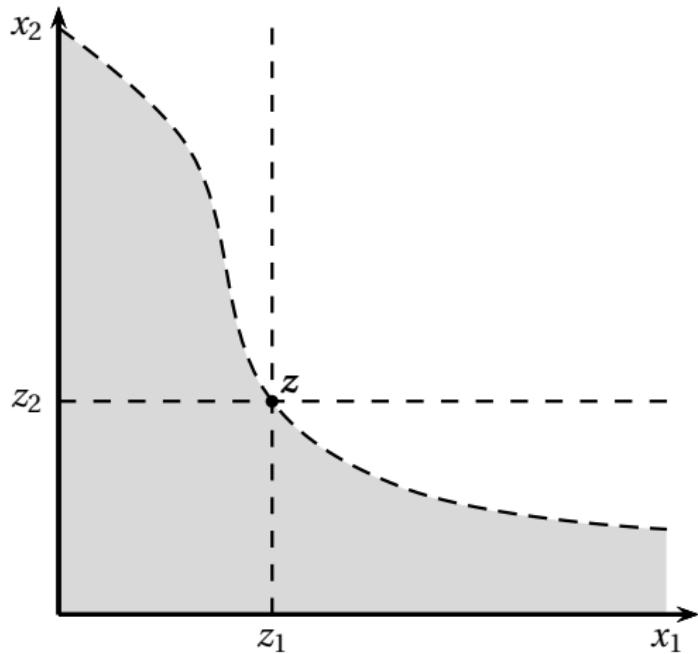


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# The DSY framework

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Let consider bidimensional additive poverty measures of the form:

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DSY define the class  $\tilde{\Pi}(\lambda^+)$  of bidimensional poverty indices  $P(\lambda)$  as:

$$\ddot{\Pi}(\lambda^+) = \left\{ P(\lambda) \begin{array}{l} \Gamma(\lambda) \subseteq \Gamma(\lambda^+) \\ \pi(x_1, x_2; \lambda) = 0, \text{ whenever } \lambda(x_1, x_2) = 0 \\ \pi^{(1)}(x_1, x_2; \lambda) \leq 0 \text{ and } \pi^{(2)}(x_1, x_2; \lambda) \leq 0 \quad \forall x_1, x_2 \\ \pi^{(1,2)}(x_1, x_2; \lambda) \geq 0, \forall x_1, x_2 \end{array} \right\}$$

## The DSY solution

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## Theorem DSY (Duclos, Sahn & Younger, 2006)

$$P_A(\lambda) > P_B(\lambda), \forall P(\lambda) \in \ddot{\Pi}(\lambda^+),$$

$$\text{iff } F_A(x_1, x_2) > F_B(x_1, x_2), \forall (x_1, x_2) \in \Gamma(\lambda^+).$$

## The issue

Many existing indices do not belong to  $\ddot{\Pi}(\lambda^+)$ !

Restricted continuity

# Relaxing continuity

Restricted continuity means continuity within the poverty domain.

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Restricted continuity means continuity within the poverty domain.

⇒ Allows the poverty index to show a discontinuity at the poverty line. [example](#)

Restricted continuity

# A class of poverty indices with restricted continuity

## A class of poverty indices with restricted continuity

Let  $z^*$  be the value of permanent income on the poverty frontier, i.e.  $\lambda(z^*, z^*) = 0$ .  $z_1(x_2)$  is the continuous non-negative function so that  $\lambda(z_1(x_2), x_2) = 0 \quad \forall x_2$ .  $z_2(x_1)$  is the inverse of  $z_1(x_2)$ .

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We consider the class  $\Pi(\lambda^+)$  of bidimensional poverty indices  $P(\lambda)$  as:

$$\Pi(\lambda^+) = \left\{ P(\lambda) \left| \begin{array}{l} \Gamma(\lambda) \subset \Gamma(\lambda^+), \\ \pi(x_1, x_2; \lambda) \geq 0, \text{ whenever } \lambda(x_1, x_2) = 0, \\ \pi^{(x_1)}(x_1, z_2(x_1)) \leq 0, \forall x_1 \in [0, z^*], \\ \pi^{(x_2)}(z_1(x_2), x_2) \leq 0, \forall x_2 \in [0, z^*], \\ \pi^{(1)}(x_1, x_2; \lambda) \leq 0 \text{ and } \pi^{(2)}(x_1, x_2; \lambda) \leq 0 \forall x_1, x_2, \\ \pi^{(1,2)}(x_1, x_2; \lambda) \geq 0, \forall x_1, x_2 \end{array} \right. \right\}$$

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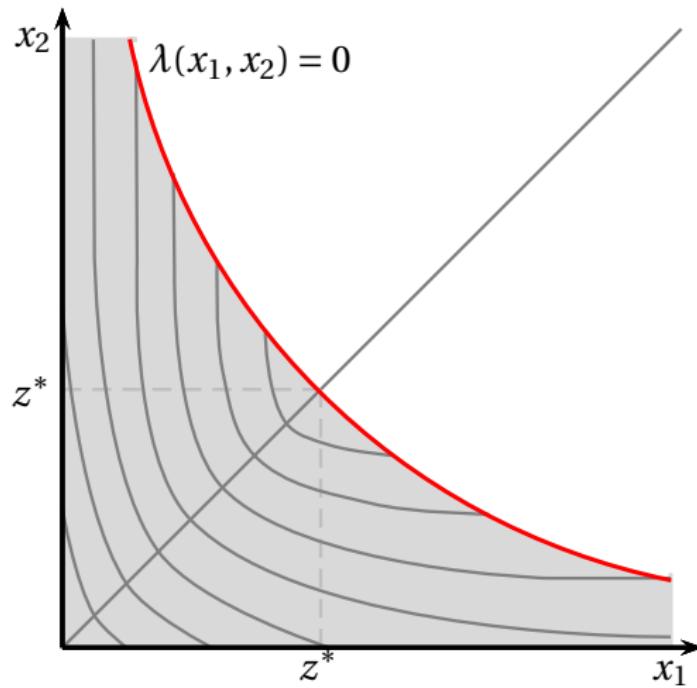


Figure 2: A member of  $\Pi(\lambda^+)$

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The general case

# Main result

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Let  $H(\lambda)$  be the bidimensional headcount, i.e.

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## Theorem 1

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⇒ More demanding than Theorem DSY!

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# Increasing the ordering power

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- ▶ Consider higher-orders of dominance (but Zheng, 1999).
- ▶ Add more structure (like symmetry).
- ▶ Consider specific families for  $\lambda$ .
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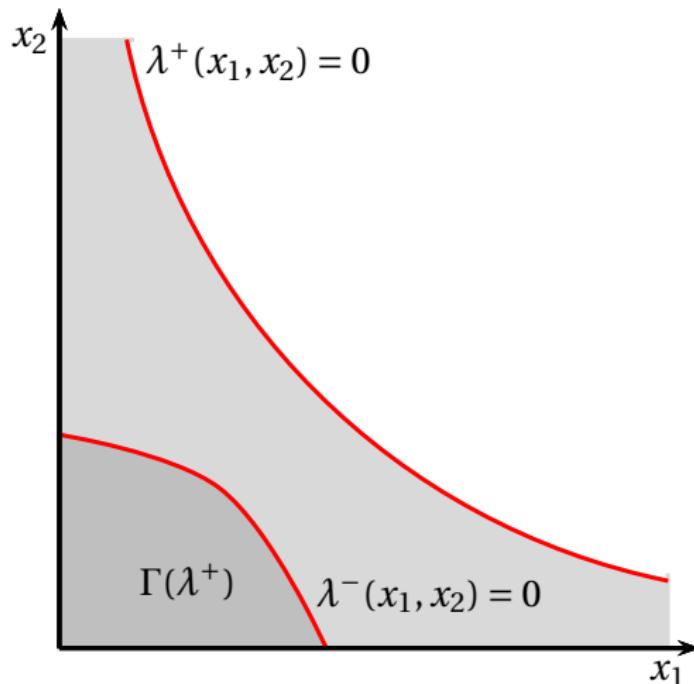


Figure 3: A minimal poverty domain.

$\bar{\Gamma}(\lambda^+, \lambda^-) := \Gamma(\lambda^+) \cap \Gamma(\lambda^-)$  be the domain where the poverty frontier is assumed to be located.

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# Partial headcount

## Partial headcount

$F_{\lambda,1}$  and  $F_{\lambda,2}$  are *partial headcount* indices defined as:

$$F_{\lambda,1}(x_1) := \int_0^{x_1} F(z_2(y_1)|y_1) f_1(y_1) dy_1,$$
$$F_{\lambda,2}(x_2) := \int_0^{x_2} F(z_1(y_2)|y_2) f_2(y_2) dy_2.$$

The general case

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### Theorem 2

$$P_A(\lambda) \geq P_B(\lambda), \quad \forall P(\lambda) \in \Pi(\lambda^+), \quad \lambda \in \mathcal{L} \text{ s. t. } \Lambda(\lambda) \subseteq \bar{\Gamma}(\lambda^+, \lambda^-),$$

- iff  $F_A(x_1, x_2) \geq F_B(x_1, x_2), \quad \forall (x_1, x_2) \in \Gamma(\lambda^+),$   
and  $H_A(\lambda) \geq H_B(\lambda) \quad \forall \lambda \in \mathcal{L} \text{ s. t. } \Lambda(\lambda) \subseteq \bar{\Gamma}(\lambda^+, \lambda^-),$   
and  $F_{\lambda, t}^A(x) \geq F_{\lambda, t}^B(x) \quad \forall t \in \{1, 2\}, \quad x \in [0, z^*], \quad \lambda \in \mathcal{L} \text{ s. t. } \Lambda(\lambda) \subseteq \bar{\Gamma}(\lambda^+, \lambda^-).$

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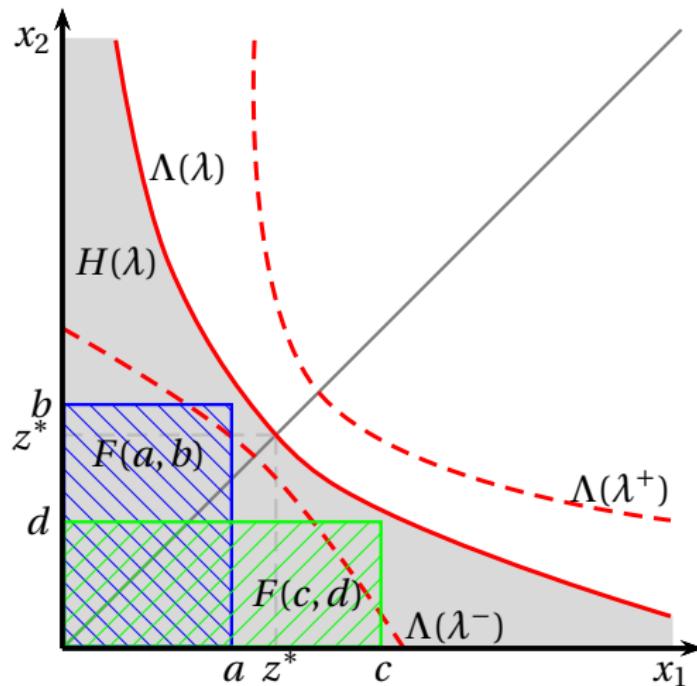
$$P_A(\lambda) \geq P_B(\lambda), \quad \forall P(\lambda) \in \Pi(\lambda^+), \quad \lambda \in \mathcal{L} \text{ s. t. } \Lambda(\lambda) \subseteq \bar{\Gamma}(\lambda^+, \lambda^-),$$

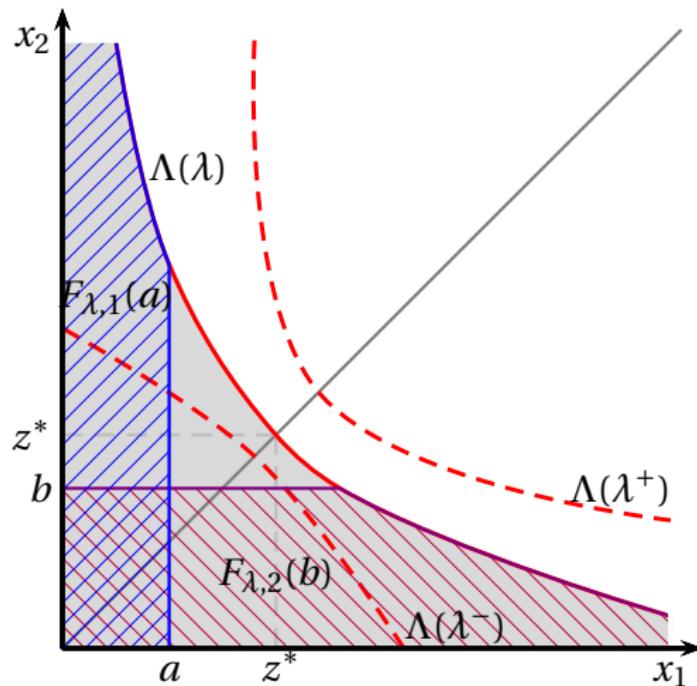
iff  $F_A(x_1, x_2) \geq F_B(x_1, x_2), \quad \forall (x_1, x_2) \in \Gamma(\lambda^+),$

and  $H_A(\lambda) \geq H_B(\lambda) \quad \forall \lambda \in \mathcal{L} \text{ s. t. } \Lambda(\lambda) \subseteq \bar{\Gamma}(\lambda^+, \lambda^-),$

and  $F_{\lambda, t}^A(x) \geq F_{\lambda, t}^B(x) \quad \forall t \in \{1, 2\}, \quad x \in [0, z^*], \quad \lambda \in \mathcal{L} \text{ s. t. } \Lambda(\lambda) \subseteq \bar{\Gamma}(\lambda^+, \lambda^-).$

Third condition vanishes if we assume  $\pi(z_1(x_2), x_2) = c \geq 0 \quad \forall x_2.$





## Remarks

- ▶ Contrary to the unidimensional case, imposing a minimal set for the poverty domain raises the ordering power.
- ▶ In the specific case of intersection poverty domains, conditions in Theorems 1 and 2 boil down to Theorem DSY's condition.

Symmetry and asymmetry

## Additional axioms

Symmetry and asymmetry investigated in Bresson & Duclos (2012) for continuous indices.

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- ▶ Asymmetry means (for instance)  $\pi(x_1, x_2) \geq \pi(x_2, x_1)$   
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Both properties permits the ordering of a larger set of distributions because they make it possible to benefit from compensations effects within the poverty domain.

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$$\Pi_S(\lambda_S^+) = \left\{ P(\lambda) \in \Pi(\lambda_S^+) \mid \begin{array}{l} \lambda \in \mathcal{L}_S, \\ \pi(x_1, x_2; \lambda_S) = \pi(x_2, x_1; \lambda_S), \forall (x_1, x_2) \in \Gamma(\lambda_S) \end{array} \right\}.$$

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## Theorem 3

$$P_A(\lambda) \geq P_B(\lambda), \quad \forall P(\lambda) \in \Pi_S(\lambda_S^+),$$

iff  $F_A(x_1, x_2) + F_A(x_2, x_1) \geq F_B(x_1, x_2) + F_B(x_2, x_1), \quad \forall (x_1, x_2) \in \Gamma_1(\lambda_S^+)$   
 and  $H_A(\lambda) \geq H_B(\lambda) \quad \forall \lambda \in \mathcal{L}_S \text{ s. t. } \Gamma(\lambda) \subseteq \Gamma(\lambda_S^+)$

and  $\sum_{t=1}^2 F_{\lambda,t}^A(x) \geq \sum_{t=1}^2 F_{\lambda,t}^B(x) \quad \forall x \in [0, z^*], \quad \lambda \in \mathcal{L}_S \text{ s. t. } \Gamma(\lambda) \subseteq \Gamma(\lambda_S^+).$

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# Asymmetry

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We still assume that the poverty frontier is symmetric.

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$$\Pi_{AS}(\lambda_S^+) = \left\{ P(\lambda) \in \Pi(\lambda_S^+) \middle| \begin{array}{l} \lambda \in \mathcal{L}_S \\ \pi^{(x_1)}(x_1, z_2(x_1)) \leq \pi^{(x_1)}(z_1(x_1), x_1) \leq 0, \forall x_1 \in [0, z^*] \\ \pi^{(1)}(x_1, x_2; \lambda) \leq \pi^{(2)}(x_2, x_1; \lambda) \quad \text{if } x_1 \leq x_2 \\ \pi^{(1,2)}(x_1, x_2; \lambda) \geq \pi^{(1,2)}(x_2, x_1; \lambda) \quad \text{if } x_1 \leq x_2 \end{array} \right\}$$

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Symmetry and asymmetry

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## Theorem 5

$$P_A(\lambda) > P_B(\lambda), \forall P(\lambda) \in \Pi_{AS}(\lambda_S^+),$$

iff  $F_A(x_1, x_2) > F_B(x_1, x_2), \forall (x_1, x_2) \in \Gamma_1(\lambda_S^+)$

and  $F_A(x_1, x_2) + F_A(x_2, x_1) > F_B(x_1, x_2) + F_B(x_2, x_1), \forall (x_1, x_2) \in \Gamma_1(\lambda_S^+)$

and  $H_A(\lambda) \geq H_B(\lambda) \quad \forall \lambda \in \mathcal{L}_S \text{ s. t. } \Gamma(\lambda) \subseteq \Gamma(\lambda_S^+)$

and  $\sum_{t=1}^T F_{\lambda,t}^A(x) \geq \sum_{t=1}^T F_{\lambda,t}^B(x) \quad \forall x \in [0, z^*], T \in \{1, 2\}, \lambda \in \mathcal{L}_S \text{ s. t. } \Gamma(\lambda) \subseteq \Gamma(\lambda_S^+).$

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Framework  
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Results  
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Chronic poverty  
●○○○○

Conclusion

# General framework

## General framework

Main assumption: Intertemporal poverty can be additively decomposed into chronic and transient components, that is:

$$P(\lambda) = C(P(\lambda)) + T(P(\lambda)).$$

where  $C()$  is the chronic component and  $T()$  the transient component.

## Different views

- ▶ the “spell” approach: poor are either chronic poor or transient poor.
  - ▶ the “components” approach: poor can cumulate chronic and transient forms of poverty

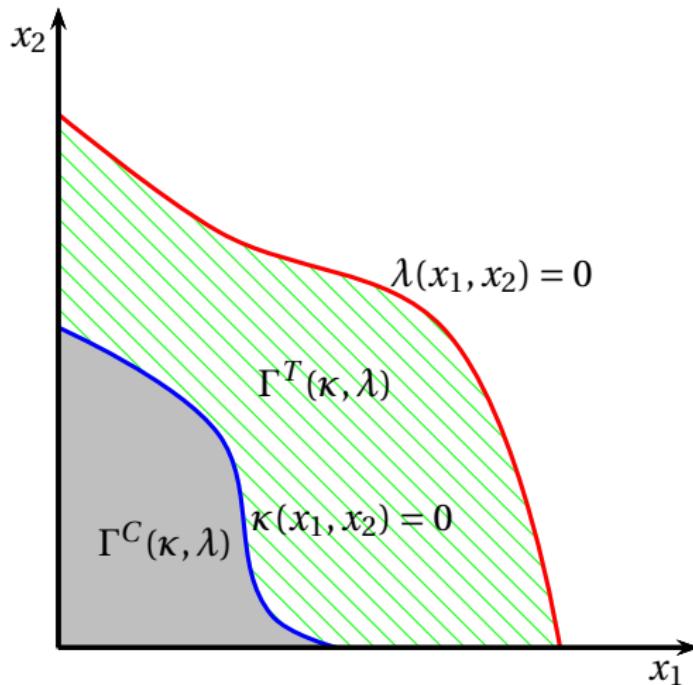
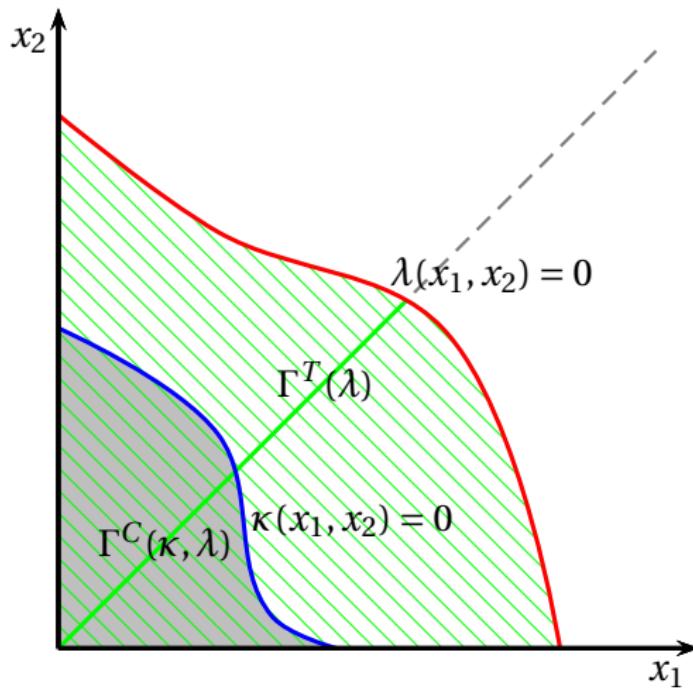


Figure 4: Chronic and transient poverty with the “spell” approach.



**Figure 5:** Chronic and transient poverty with the “component” approach.

## Dominance checks

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In both cases, the chronic poverty domain is the bottom part of the poverty domain.

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⇒ Assuming that  $C(P(\lambda)) \in \Pi(\lambda^+)$ , Theorems 1 to 6 can be used to obtain robust orderings.

Introduction  
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Chronic poverty  
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Conclusion

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- ▶ Implementation.

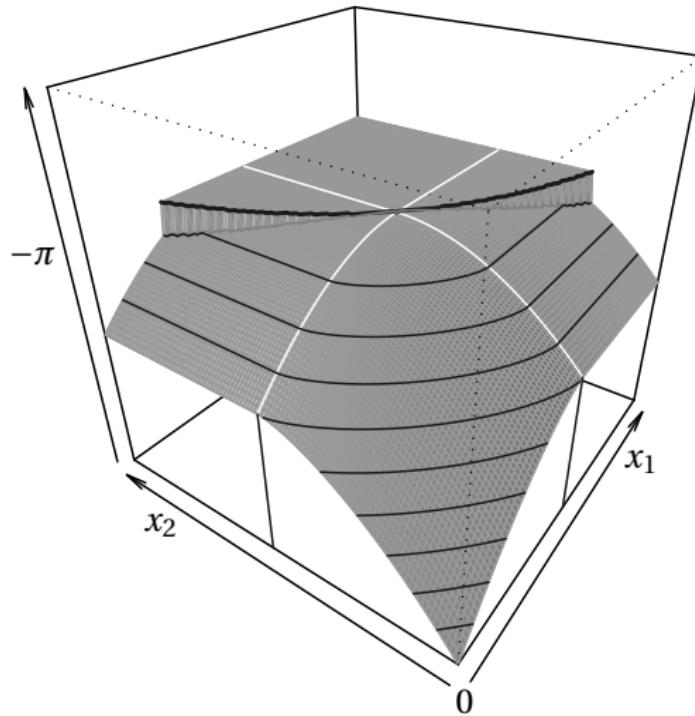
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## To do:

- ▶ Reference to profile  $(z^*; z^*)$  should be generalized to any point on the poverty frontier,
  - ▶ Asymmetry with asymmetric poverty frontiers.
  - ▶ Higher-order of dominance (Pigou-Dalton transfers, transfer-sensitivity...)
  - ▶ Implementation.
  - ▶ Discontinuities within the poverty domain

# The end

**Thank you for your attention.**



Note: the opposite of the individual poverty index is depicted on the figure.

**Figure 6:** A bidimensional poverty index with restricted continuity. [back](#)