Labor market effects of demographic shifts and labor mobility in OECD countries^{*}

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Abstract

The labor force of each industrial country is being shaped by three forces: aging, migration and education. Drawing on a new database for the OECD countries and a standard analytical framework, this paper focuses on the relative and aggregate effects of these three forces on wages across different skill and age groups. The results are unexpected. The change in the age and educational structure of the labor force emerges as the dominant influence on wage changes. The impact is uniform and egalitarian: in virtually all countries, the changes in the age and skill structure favor the unskilled and hurt the skilled across age groups. Immigration plays a relatively minor role, except in a handful of open countries, like Australia and Canada, and merely accentuates the wage-equalizing impact of aging. Emigration is the only inegalitarian influence, especially in Ireland and a few Eastern European countries which have seen significant outflows of labor to European Union countries.

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1 Introduction

The labor force of each industrial country is being shaped by three forces: aging, migration and education. Aging is a powerful, gradual process that evokes concern but little policy action. Immigration is blamed for the predicament of the unskilled and is provoking a political backlash. Education is viewed as a universal panacea to which access is unequally distributed.

Yet one question has not been addressed: what is the relative and combined effect of these three forces on wages across different skill and age groups? This is an especially pertinent question as increased wage inequality across skill and age groups dominate the political and academic debate. This paper addresses the question using a new database for the OECD countries which contains detailed information on the age, education and the place of birth of individuals across two decades. The impact on wages is derived using a simple model of the economy which allows for imperfect substitutability between the skilled and the unskilled, old and young, and natives and foreigners. The change in the age and skill structure of labor forces emerges as the dominant force influencing relative wage changes, with immigration playing a smaller role in most countries.

Aging and international migration are arguably two of the most salient demographic phenomena of the new millennium. According to the National Institute on Aging, individuals over the age of 84 are the fastest growing fraction of the population in many countries, especially among the OECD countries. Data from the 2010 rounds of national censuses show that the 65+ age group represent more than 17 per cent of the native working age population in two-thirds of OECD countries, compared to 11 per cent from the 2000 rounds. In countries such as Japan, Germany, Greece and Italy, people above the age of 65 account for over a quarter of the working age population. Even in lower income OECD members, such as Mexico, Chile, Turkey and Poland, the corresponding shares tend to be around 10 per cent or higher. Migration, especially of the young working age population from lower to higher income countries, is another force shaping labor markets in both sending and receiving countries. Measured as a share of the world population, the relative share of migrants grew by close to 14 per cent between 2000 and 2013 - from 2.9 per cent to 3.3 per cent of the global population (United Nations, 2013). Migrants now come from an increasing number of countries, but go to fewer destinations, predominantly high-income OECD countries (Czaika and de Haas, 2014). The proportion of older individuals (i.e. those over the age of 45) amongst the working age population is lower among immigrants than natives in 23 out of the 33 OECD destination countries. The difference is over 19 percentage points in 10 of these countries. Similar patterns emerge when we compare the age distribution of the native population in these countries to that of their emigrant populations, indicating the young are also moving between high-income countries.

The observed shifts in the age structure of populations have typically provoked concerns about economic growth and fiscal policy sustainability (see Leers et al, 2003; Blake and Mayhew, 2006; Bloom et al, 2010; Lee, 2014; and Sheiner, 2014). Some studies have looked at the relationship between age and productivity at the micro level, while others have gone further to ask how this compares with the relationship between age and wages (see Mahlberg et al. (2013)). The implications of an aging workforce and international mobility of labor for the wages of natives at different points of the age and skill distribution have been explored separately (see discussion below) but not together.

A new database of the OECD countries helps us to measure how the education and age structure of the native population has changed between the years 2000 and 2010. The key stylized facts are depicted in Figure 1 where each quadrant shows the change between 2000 and 2010 in each pair of labor groups as labelled in the axis. The first observation is that the number of the high skilled increased relative to the low-skilled among both the old (quadrant II) and young people (quadrant IV) in virtually all countries. The growth of the high-skilled, however, has generally been faster among the old than among the young (quadrant I), mainly due to the retirement of the older cohorts who were significantly less educated. Among the low-skilled, there is a shaper contrast: the number of the old lowskilled has increased and the number of the young low-skilled has declined (quadrant III). This relatively rapid educational upgrading at different rates across age groups has important implications for wage distributions.

[FIGURE 1 HERE]

Another component of the labor force with rapid and varying growth rates are the migrants. The stock of immigrants has increased across all countries in all categories but with generally faster growths among the skilled (Figure 1). The stock of emigrants shows a similar pattern of increases, but the magnitudes are much smaller. The database also helps us to identify how the age and skill attributes of immigrants and emigrants have changed relative to those of the natives in different countries (Figure 2). While there is significant heterogeneity across countries, we summarize here only the main features: the share of the old is generally lower among immigrants (Panel B) and in many cases also among emigrants (Panel D) compared to natives. Furthermore, most points in both Panels B and D are below the 45-degree line, indicating that the age difference has grown for both groups between 2000 and 2010. The patterns with regard to education are less clear. Emigrants tended to be more skilled relative to the natives (Panel C), especially among those from certain Eastern European countries like the Czech Republic, Hungary and Poland, but the gap is lower in 2010 than 2000 as most countries are below the 45-degree line. In panel A, most observations are located around 1, implying that the immigrants tend to have a skill composition broadly similar to that of the natives. In several important destination countries, such as Australia, Canada, New Zealand and United Kingdom, immigrants are more skilled. Finally, the relative proportion of the skilled among immigrants has declined in more countries than those in which it has increased or stayed the same.

[FIGURE 2 HERE]

This paper derives the impact of the changes in the age, education and migration composition of the labor force of the OECD countries using a simple model of the economy that allows for imperfect substitutability in production between the skilled and the unskilled, old and young, and natives and foreigners. If workers of different ages were perfectly substitutable, aging within a given group of workforce would have no implications for wages. However, available empirical evidence suggests that workers of different ages are imperfect substitutes (Card and Lemieux, 2001). Empirical evidence, mainly from the USA and the UK, also suggests that immigrants and natives are imperfect substitutes across age-education cells (see Ottaviano and Peri (2012) for the USA and Manacorda et al (2012) for the UK). Therefore, the effects of immigrants but their respective elasticities of substitution with natives.

We use a CES production function with multiple nests and consider an economy which produces a homogenous good using eight types of labor: (1) a given worker is either highskilled or low-skilled; (2) within each of these two categories, a worker is either old or young; and (3) finally, within each of the resulting four categories, a work is either nativeborn or foreign-born. The nested CES production function with a homogenous good has generally been the standard in the literature; it is widely used in labor economics to examine substitutability between workers of different ages and to gauge the implication of immigration for the labor market outcomes of natives. For example, Card and Lemieux (2001) use a similar framework to demonstrate that the decline in the relative supply of the college educated explains the widening gap in the wages of college and non-college educated workers over the last three decades of the 20th century in Canada, the UK, and the USA. Other examples include Borjas (2003), Card (2009), Manacorda et al (2012) and Ottaviano and Peri (2012) who examine the effect of immigration on the wages of natives.

Our main finding is that the change in the age and skill structure of the work force is the dominant influence on wage changes across the world. The impact is surprisingly uniform and egalitarian: in virtually all countries, the changes in the age and skill structure actually favors the unskilled and hurts the skilled across all age groups. However, the relative impact of aging differs across countries depending on the magnitude of migration. The OECD countries can be divided into three broad groups. In most of Western Europe, Japan and the United States, aging and changes in the stock of skills, account for most of the changes in wages, and migration plays a marginal role. In the Anglo-Saxon countries (Australia, Canada, New Zealand, the United Kingdom) and Switzerland, which have relatively liberal and skill-biased migration policies, immigration accentuates the impact of aging on wages. In countries which have seen significant labor outflows, such as Ireland and certain Eastern European countries that now have access to Western European labor markets (the Czech Republic, Slovakia, Hungary and Poland) the impact of aging is partially offset by emigration.

Our study relates to the literature on demographic changes in the labor force, in which some papers have argued that these changes explain the growth of wage inequalities observed over the last decades. This started with a simple demand and supply perspective, whereby the relative increase (decrease) in the share of a particular group of workers in the labour force decreases (increases) their relative wages (Freemann, 1979; Katz and Murphy, 1992; Murphy and Welch, 1992; Card and Lemieux, 2001). Later work emphasized the role of demand-side factors, in particular Skill-Biased Technological Change (SBTC) which is said to have favoured skilled workers and been the main causes of wage inequalities (Acemoglu, 2002; Autor et al., 2003; 2008). The debate continued with other authors suggesting that changes in institutional settings, instead, such as the decline in the real value of minimum wage and the decline in the bargaining power of trade unions explain most of the observed wage inequalities (Bound and Johnson, 1992; Lee, 1999; Card and DiNardo, 2002; Lemieux, 2008). Lemieux (2008) and Autor et al. (2008) provide thorough reviews of the literature on the topic, although with competing positions. We acknowledge the importance of these other factors, but choose to highlight the striking demographic changes in the workforce due to aging, education and international labor mobility. In other words, we assume a world where technology and institutions are not changing, and the only changes that occur are the demographic changes we specify.

The second strand of the literature relevant to our study pertains to the impact of immigration on the labor market outcomes of natives. Papers examining the impact of immigration on the wages of natives using a framework similar to ours include Ottaviano and Peri (2012) in the context of the USA, and Manacorda et al (2012) and Dustmann et al (2013) in the case of the UK. These papers only focus on immigration, although emigration is particularly high in some OECD countries such as Ireland and Poland. Docquier et al. (2014), the closest study to ours, fill this gap by looking at the joint effect of emigration and immigration on the wages and employment probabilities of high-skilled and low-skilled natives. The paper does not however account for possible heterogeneity of these effects across age groups. This is particularly important because immigrants and emigrants tend to be younger than natives, and there is evidence that the young and old are imperfect substitutes in the production function, as e.g. in Card and Lemieux (2001). Our study differs from Docquier et al (2014), referred to as DOP hereafter, in the following manner: we consider the effects of aging, emigration and immigration jointly and relax the assumption that the young and old are perfect substitutes in production; we allow for the elasticities of substitution between immigrants and our four groups of natives to differ; and we exploit recent data from the 2000 and 2010 censuses which contain detailed information on the age-education-nativity distribution of the workforce across OECD countries. We find that while immigration benefits the low-skilled on average, emigration appears to have negative wage effects on the low-skilled, but the aging effect dominates both of these in magnitude.

The rest of the paper is organised as follows. Section 2 presents the aggregate production and labour supply framework where we derive wages effects of ageing, education and migration shocks. Section 3 describes the construction of our data set and provides simple summary statistics of the labour force and migrant data and their age and educational composition changes. Section 4 presents the discussion of the main parameter values used in the analysis. Section 5 discusses the basic results of the simulated wage effects of aging, educational changes and migration nusing our model and the range of parameters available from the literature. Section 6 concludes the paper.

2 The analytic model

We construct a standard aggregate model of an economy where workers differ by their migration status, educational attainment (i.e. skills) and age group. In terms of migration status, we distinguish the native-born from the foreign-born. In terms of educational attainment, we have the tertiary educated (or high-skilled) and the non-tertiary educated (or low-skilled), following the literature in labor economics (Freeman, 1976; Katz and Murphy, 1992; Card and Lemieux, 2001). Finally, in terms of age groups, we split the labor force between young (25-44 years of age) and old (45-64 years of age) workers.

2.1 Aggregate production function

We consider a CES function comprising three nests and eight types of workers. The model permits us to examine changes in wages and employment levels of the natives arising from aging, immigration and emigration across these eight different subgroups of the labor force. The CES function has been widely used in the literature examining the impact of immigration on the labor market outcomes of natives (see Borjas, 2003; Card, 2009; Ottaviano and Peri, 2012; Manacorda et al. 2012, Docquier et al, 2014). It is also used in the labor economics literature to examine the drivers of wage inequality (Katz and Murphy, 1992, Murphy and Welch, 1992; Card and Lemieux, 2001; Acemoglu, 2002). These studies view labor in efficiency units (i.e. q), which is assumed to be a nested function of different types of workers. Output is produced using this composite labor (q) and physical capital (k).

$$(1) \quad y = A_0 f(k,q)$$

where A is the total factor productivity (TFP) parameter. If we assume capital is internationally mobile and each country is too small to affect the world capital markets, then the price of the capital is equalized across countries and is taken to be fixed. Since the production function has constant returns to scale, we can write the aggregate output as a linear function of the aggregate composite output q:

$$(2) \quad y = A * q$$

where $A = A_0 f[f'^{-1}(R/A_0)]$ and depends on the TFP parameter A_0 as well as the returns to capital R. Following the labor literature, we assume q is a nested CES function of high skilled and low skilled workers:

(3)
$$q = \left[\rho_s q_h^{\frac{\delta_s - 1}{\delta_s}} + (1 - \rho_s) q_l^{\frac{\delta_s - 1}{\delta_s}}\right]^{\frac{\delta_s}{\delta_s - 1}}$$

where q_h and q_l denote high skilled and low skilled labor respectively. ρ_s and $1 - \rho_s$ are the respective relative productivity parameters of these two skill groups. δ_s is the elasticity of substitution between the skill groups. Our framework does not make a distinction between those with a college degree and those with a higher level of qualification, or between those with and without a high-school diplomas. These simplifying assumptions are reasonable in light of the empirical evidence showing a high degree of substitutability between workers with a high-school diploma and those with no diploma, but a very low degree of substitutability between workers with and those without tertiary education (Goldin and Katz, 2008; Card, 2009; and Peri, 2012).

The empirical evidence suggests imperfect degree of substitution between older and younger workers (Card and Lemieux, 2001) within each education group. If younger workers and older workers were perfectly substitutable, the overall measure of aggregate labor in each education group would simply be the sum of these two types of workers. Given imperfect substitution, q_h and q_l are each assumed to be nested CES functions of younger workers and older workers:

(4)
$$q_s = \left[\rho_a q_{so}^{\frac{\delta_a - 1}{\delta_a}} + (1 - \rho_a) q_{sy}^{\frac{\delta_a - 1}{\delta_a}}\right]^{\frac{\delta_a}{\delta_a - 1}} \qquad s \in \{h, l\}$$

where q_{so} is the stock of older workers of type s and q_{sy} is the corresponding stock of younger workers (Note that $s \in \{h, l\}$ is the label for the relevant skill level). ρ_a and $1 - \rho_a$ are the respective relative productivity parameters of the old and the young. δ_a is the elasticity of substitution between workers of different age group. We assume these elasticities are identical within each education group.

The final distinction in the composition of labor is the split of workers into natives and immigrants within each of the the four education-age cells. This is motivated by the literature that explores the degree of substitutability between immigrants and natives in a labor market (Card, 2009; Manacorda et al., 2012; Ottaviano and Peri, 2012). There are numerous reasons for this imperfect substitutability, such as the time it takes for immigrants to become familiar with the host country's customs and acquire human capital and language skills specific to the host labor market (Chiswick, 1978). Additionally, there is nascent empirical evidence showing that immigrants and natives are employed in different occupations/industries (Peri and Sparber, 2009; Patel and Vella, 2013). Therefore, for each group of workers of educationage type sa = (ho, lo, hy, ly), we have:

(5)
$$q_{sa} = \left[\rho_m q_{san}^{\frac{\delta_m - 1}{\delta_m}} + (1 - \rho_m) q_{saf}^{\frac{\delta_m - 1}{\delta_m}}\right]^{\frac{\delta_m}{\delta_m - 1}} \qquad s \in \{h, l\} \qquad and \qquad a \in \{o, y\}$$

where q_{san} represents the stock of natives of type sa and q_{saf} represents the corresponding stock of immigrants. ρ_m and $1 - \rho_m$ are the respective relative productivity parameters of the two groups. δ_m is the elasticity of substitution between immigrants and natives.

Finally, in light of the importance of human capital for TFP and the externalities of schooling (Lucas, 1988 and Docquier et al., 2014), we express TFP level within the labor market of a country as:

$$(6) \quad A = A_0 e^{\lambda f_h}$$

where A_o represents the independent component of human capital externality of TFP, f_h denotes the share of high skilled individuals in the workforce and λ denotes the semi-elasticity of TFP with respect to f_h .

Our nest structure is now be illustrated by Figure 3:

We proceed in a general equilibrium framework to analyze the wages and employments effects of changes in the age, education and migrant composition of the labor markets.¹ The process simply involves equating demand and supply conditions and solving for their equilibrium values.

¹For comparison, see Card and Lemieux (2001), Manacorda et al (2012), Ottaviano and Peri (2012) and Docquier et al (2014).

2.2 Labor demand

The model assumes each country is a single labor market with eight types of workers as presented in Figure 3. Recall that each type of worker is identified by three indices, sam, where $s \in \{h, l\}$ denotes their skill level, $a \in \{o, y\}$ denotes their age group and $m \in \{n, f\}$ denotes their migration status (native or foreign-born). The marginal productivity of each type of native worker, san, can be obtained by substituting (3), (4) and (5) into (2) and then by differentiating the expression with respect to q_{san} . The demand for native workers of type san is given by:

$$\begin{bmatrix} w_{hon} \\ w_{hyn} \\ w_{lyn} \\ w_{lyn} \end{bmatrix} = \begin{bmatrix} A\rho_s\rho_a\rho_m & 0 & 0 & 0 \\ 0 & A\rho_s(1-\rho_a)\rho_m & 0 & 0 \\ 0 & 0 & A(1-\rho_s)\rho_a\rho_m & 0 \\ 0 & 0 & 0 & A(1-\rho_s)(1-\rho_a)\rho_m \end{bmatrix} \begin{bmatrix} \left[\frac{q}{q_h}\right]\frac{1}{\delta_s}\left[\frac{q_h}{q_{ho}}\right]\frac{1}{\delta_a}\left[\frac{q_{ho}}{q_{hon}}\right]\frac{1}{\delta_m} \\ \left[\frac{q}{q_h}\right]\frac{1}{\delta_s}\left[\frac{q_h}{q_{hy}}\right]\frac{1}{\delta_a}\left[\frac{q_{ho}}{q_{hyn}}\right]\frac{1}{\delta_m} \\ \left[\frac{q}{q_l}\right]\frac{1}{\delta_s}\left[\frac{q_l}{q_{lo}}\right]\frac{1}{\delta_a}\left[\frac{q_{ho}}{q_{lon}}\right]\frac{1}{\delta_m} \\ \left[\frac{q}{q_l}\right]\frac{1}{\delta_s}\left[\frac{q_l}{q_{lo}}\right]\frac{1}{\delta_a}\left[\frac{q_{lo}}{q_{lon}}\right]\frac{1}{\delta_m} \\ \left[\frac{q}{q_l}\right]\frac{1}{\delta_s}\left[\frac{q_l}{q_{lo}}\right]\frac{1}{\delta_a}\left[\frac{q_{lo}}{q_{lon}}\right]\frac{1}{\delta_m} \\ \left[\frac{q}{q_l}\right]\frac{1}{\delta_s}\left[\frac{q_l}{q_{ly}}\right]\frac{1}{\delta_a}\left[\frac{q_{lo}}{q_{lon}}\right]\frac{1}{\delta_m} \\ \left[\frac{q}{q_l}\right]\frac{1}{\delta_s}\left[\frac{q_l}{q_{ly}}\right]\frac{1}{\delta_a}\left[\frac{q_{lo}}{q_{lon}}\right]\frac{1}{\delta_m} \\ \left[\frac{q}{q_l}\right]\frac{1}{\delta_s}\left[\frac{q_l}{q_{ly}}\right]\frac{1}{\delta_a}\left[\frac{q_{lo}}{q_{lon}}\right]\frac{1}{\delta_m} \\ \left[\frac{q}{q_l}\right]\frac{1}{\delta_s}\left[\frac{q_l}{q_{ly}}\right]\frac{1}{\delta_a}\left[\frac{q_{lo}}{q_{lon}}\right]\frac{1}{\delta_m} \\ \left[\frac{q}{q_l}\right]\frac{1}{\delta_s}\left[\frac{q_l}{q_{ly}}\right]\frac{1}{\delta_a}\left[\frac{q_{lo}}{q_{lon}}\right]\frac{1}{\delta_m} \\ \left[\frac{q}{q_l}\right]\frac{1}{\delta_s}\left[\frac{q_{lo}}{q_{lon}}\right]\frac{1}{\delta_m} \\ \left[\frac{q}{q_l}\right]\frac{1}{\delta_s}\left[\frac{q}{q_{lon}}\right]\frac{1}{\delta_m} \\ \left[\frac{q}{q_l}\right]\frac{1}{\delta_s}\left[\frac{q}{q_{lon}}\right]\frac{1}{\delta_m} \\ \left[\frac{q}{q_l}\right]\frac{1}{\delta_s}\left[\frac{q}{q_{lon}}\right]\frac{1}{\delta_m} \\ \left[\frac{q}{q_l}\right]\frac{1}{\delta_m} \\ \left[\frac{$$

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The next step is to obtain the (percentage) change in the marginal productivity of a native worker which comes from (percentage) changes in the stock of immigrants of a given type (i.e. \hat{q}_{saf}), natives of given type (i.e. \hat{q}_{san}) or both. We define these percentage changes as $\hat{x} = \Delta x/x$. We proceed by taking the total differential of each element on the left hand side of equation (7) with respect to variations (Δ) in the employment stock of each worker of type, sam. This leads to the following expression:

$$\frac{\partial ln(w_{san})}{\partial w_{san}} \Delta w_{san} = \frac{1}{\delta_s} \left[\frac{\partial q}{\partial q_{hof}} \frac{\Delta q_{hof}}{q} + \frac{\partial q}{\partial q_{hyf}} \frac{\Delta q_{hyf}}{q} + \frac{\partial q}{\partial q_{lof}} \frac{\Delta q_{lof}}{q} + \frac{\partial q}{\partial q_{lyf}} \frac{\Delta q_{lyf}}{q} \right] \\ + \frac{1}{\delta_s} \left[\frac{\partial q}{\partial q_{hon}} \frac{\Delta q_{hon}}{q} + \frac{\partial q}{\partial q_{hyn}} \frac{\Delta q_{hyn}}{q} + \frac{\partial q}{\partial q_{lon}} \frac{\Delta q_{lon}}{q} + \frac{\partial q}{\partial q_{lyn}} \frac{\Delta q_{lyn}}{q} \right] \\ (8) + \left(\frac{1}{\delta_a} - \frac{1}{\delta_s} \right) \left[\frac{\partial q_s}{\partial q_{sof}} \frac{\Delta q_{sof}}{q_s} + \frac{\partial q_s}{\partial q_{syf}} \frac{\Delta q_{syf}}{q_s} \right] + \left(\frac{1}{\delta_a} - \frac{1}{\delta_s} \right) \left[\frac{\partial q_s}{\partial q_{san}} \frac{\Delta q_{son}}{q_s} \\ + \frac{\partial q_s}{\partial q_{syn}} \frac{\Delta q_{syn}}{q_s} \right] + \left(\frac{1}{\delta_m} - \frac{1}{\delta_a} \right) \left[\frac{\partial q_{sa}}{\partial q_{saf}} \frac{\Delta q_{saf}}{q_{sa}} \right] + \left(\frac{1}{\delta_m} - \frac{1}{\delta_a} \right) \left[\frac{\partial q_{sa}}{\partial q_{san}} \frac{\Delta q_{san}}{q_{sa}} \right] - \frac{1}{\delta_m} \frac{\Delta q_{san}}{q_{san}} + \lambda \Delta f_h$$

where $s \in \{h, l\}$ and $a \in \{o, y\}$

In equilibrium, each type of worker is paid his marginal productivity. Since labor is the only factor of production, the share of the total wage bill, θ_k for all workers of type k can be expressed as $\theta_k = \frac{w_k q_k}{y}$ where w_k is their wage level and q_k is their quantity. Using this information, we derive the following expression (see appendix for the derivation):

(9)

$$\hat{w}_{san} = \begin{bmatrix} \hat{q}_{san} & \hat{q}_{sa'n} & \hat{q}_{s'an} & \hat{q}_{s'a'n} \end{bmatrix} \begin{bmatrix} \frac{\theta_{san}}{\delta_s} + \left(\frac{1}{\delta_a} + \frac{1}{\delta_s}\right)\frac{\theta_{san}}{\theta_s} + \left(\frac{1}{\delta_m} + \frac{1}{\delta_a}\right)\frac{\theta_{san}}{\theta_{sa}} - \frac{1}{\delta_m} \\ \begin{bmatrix} \frac{1}{\delta_s} + \left(\frac{1}{\delta_a} - \frac{1}{\delta_s}\right)\frac{1}{\theta_s}\end{bmatrix}\theta_{sa'n} \\ & \frac{\theta_{s'an}}{\delta_s} \\ & \frac{\theta_{s'a'n}}{\delta_s} \end{bmatrix} +$$

$$\begin{bmatrix} \hat{q}_{saf} & \hat{q}_{sa'f} & \hat{q}_{s'af} & \hat{q}_{s'af} \end{bmatrix} \begin{bmatrix} \frac{\theta_{saf}}{\delta_s} + (\frac{1}{\delta_a} + \frac{1}{\delta_s})\frac{\theta_{saf}}{\theta_s} + (\frac{1}{\delta_a} + \frac{1}{\delta_a})\frac{\theta_{saf}}{\theta_{sa}} \\ [\frac{1}{\delta_s} + (\frac{1}{\delta_a} - \frac{1}{\delta_s})\frac{1}{\theta_s}]\theta_{sa'f} \\ \frac{\theta_{s'af}}{\delta_s} \\ \frac{\theta_{s'a'f}}{\delta_s} \end{bmatrix} + \begin{bmatrix} \lambda \Delta f_h \\ \lambda \Delta f_h \\ \lambda \Delta f_h \\ \lambda \Delta f_h \end{bmatrix}$$

s'=l when s=h and vice-versa; and a'=y when a=o and vice-versa.

In equation (9), \hat{q}_{saf} is the change in the employment stock of foreign-born individuals and \hat{q}_{san} is the corresponding change among natives resulting from emigration, changes in education levels or the changes in the age structure of natives, or both. We also see in equation (9) that the wages of all given types of workers can change due to changes in the stock of any type of labor that constitutes the composite labor force.

For the moment let's focus on the vector that multiplies $[\hat{q}_{san} \ \hat{q}_{s'an} \ \hat{q}_{s'an} \ \hat{q}_{s'a'n}]$ in equation (9). For each type of native worker, the first expression of the matrix captures the effect that results from changes in the stock of its own-type of labor. The second expression captures the effect resulting from changes in the quantity of labor of different age group in the same skill group. The last two expressions capture the effects arising from changes in the stock of labor of different skill groups. Now, when we look at the vector that multiplies $[\hat{q}_{saf} \ \hat{q}_{s'af} \ \hat{q}_{s'af} \ \hat{q}_{s'a'f}]$, we can interpret its elements in the same manner. The only difference is that the changes captured are those due to immigration. Lastly, the term $\lambda \Delta f_h$ captures the economy wide productivity spillovers generated by high skilled labor, and this affects wages across all labor groups.

2.3 Labor supply

Labor supply of natives is determined through the work-leisure trade off in the model. Each native worker i allocates his (or her) unit of time between l_i units of work and $1 - l_i$ units of leisure. This allocation maximizes an instant utility function, which depends positively on consumption, c_i , and negatively on the amount of labor supplied, l_i :

(10) $U_i = \rho_c c_i^{\delta} - \rho_l l_i^{\eta}$

The parameters ρ_c , ρ_l , δ and η are assumed to be identical across all types of workers for simplicity. Furthermore, we assume that individuals consume all their labor income, such that $c_i = l_i w_i$.² Substituting this constraint into equation (10) and maximizing with respect to l_i leads to:

(11) $l_i = w_i^{\gamma} \phi$

where $\phi = \left[\frac{\theta_c \delta}{\theta \eta}\right]^{\frac{1}{\eta-\delta}}$ and $\gamma = \frac{\delta}{\eta-\delta}$ are both constants. We should note that γ captures the elasticity of labor supply, which is assumed to be positive. Aggregate labor supply of native workers of type *san* is obtained by multiplying (11) by Q_{san} :

(12)
$$q_{san} = l_{san}Q_{san} = \phi w_{san}^{\gamma}Q_{san}$$

In other words, Q_{san} is the total population of type san and q_{san} is its share in employment. Finally,³

(13)
$$\hat{w}_{san} = \frac{1}{\gamma} [\hat{q}_{san} - \hat{Q}_{san}]$$

We make the simplifying assumption that all working-age immigrants supply a constant amount of labor (DOP, 2014). This assumption implies that γ , the elasticity of labor supply, is zero for immigrants. A given percentage change in the immigrant population therefore leads to the same percentage change in the labor force. Denoting this constant proportion of immigrant labor supplied as τ , total supply among immigrants of type saf is given by:

 $^{^{2}}$ Allowing consumption to be a constant share of labor income does not alter the implications

³One could also allow γ to vary across worker types. In this case we have $q_{san} = \phi Q_{san} w_{san}^{\gamma_{san}}$

(14)
$$q_{saf} = \tau Q_{saf}$$

2.4 Equilibrium effects of demographic shifts and labor mobility

There are three main sources of changes in the labor force: (1) changes in the number of immigrants of each age-skill type as they enter/exit the country; (2) changes in the number of natives of each age-skill type due to ageing; and (3) changes in the number of natives of each age-skill type due to emigration. Ageing is due to some of the natives moving from one age group to another age group and emigration results from natives leaving/returning to the country. In equilibrium, the wages and employment levels of native workers of each type adjust as they respond to these changes. Equating the supply equation (13) to the demand equation (9) means we have the following equality for each native worker of type san:

(15)

$$\begin{bmatrix} \hat{q}_{san} & \hat{q}_{sa'n} & \hat{q}_{s'an} & \hat{q}_{s'a'n} \end{bmatrix} \begin{bmatrix} \frac{1}{\gamma} - \frac{\theta_{san}}{\delta_s} - (\frac{1}{\delta_a} - \frac{1}{\delta_s})\frac{\theta_{san}}{\theta_s} - (\frac{1}{\delta_m} + \frac{1}{\delta_a})\frac{\theta_{san}}{\theta_{sa}} + \frac{1}{\delta_m} \\ & -[\frac{1}{\delta_s} + (\frac{1}{\delta_a} - \frac{1}{\delta_s})\frac{1}{\theta_s}]\theta_{sa'n} \\ & & -\frac{\theta_{s'on}}{\delta_s} \\ & & -\frac{\theta_{s'yn}}{\delta_s} \end{bmatrix} = \hat{M}_{san}^f + \frac{1}{\gamma}\hat{Q}_{san}$$

where

(16)
$$\hat{M}_{san}^{f} = \frac{1}{\delta_{s}} [\theta_{hof} \hat{Q}_{hof} + \theta_{hyf} \hat{Q}_{hyf} + \theta_{lof} \hat{Q}_{lof} + \theta_{lyf} \hat{Q}_{lyf}] + (\frac{1}{\delta_{a}} - \frac{1}{\delta_{s}}) [\frac{\theta_{sof}}{\theta_{s}} \hat{Q}_{hof} + \frac{\theta_{syf}}{\theta_{s}} \hat{Q}_{hyf}] + (\frac{1}{\delta_{m}} - \frac{1}{\delta_{a}}) [\frac{\theta_{saf}}{\theta_{sa}} \hat{Q}_{haf}] + \lambda \Delta f_{hf}$$

 \hat{M}_{san}^{f} is the change in marginal productivity of native workers of type san resulting from

changes in the stock of immigrants.

Next, we define the following expressions:

$$(17) \quad \alpha_{san} = \frac{1}{\gamma} - \frac{\theta_{san}}{\delta_s} - (\frac{1}{\delta_a} - \frac{1}{\delta_s})\frac{\theta_{san}}{\theta_s} - (\frac{1}{\delta_m} - \frac{1}{\delta_a})\frac{\theta_{san}}{\theta_{sa}} + \frac{1}{\delta_m}$$

(18)
$$\pi_{sa'n} = -\left[\frac{1}{\delta_s} + \left(\frac{1}{\delta_a} - \frac{1}{\delta_s}\right)\frac{1}{\theta_s}\right]\theta_{sa'n}, \ \beta_{s'an} = -\frac{\theta_{s'an}}{\delta_s}, \ \beta_{s'a'n} = -\frac{\theta_{s'a'n}}{\delta_s}$$

and

(19)
$$\Theta_{san} = \hat{M}_{san}^f + \frac{1}{\gamma}\hat{Q}_{san}$$

Then each line in equation (15) can be simply expressed as:

(20)
$$\alpha_{san}\hat{q}_{san} + \pi_{sa'n}\hat{q}_{sa'n} + \beta_{s'an}\hat{q}_{s'an} + \beta_{s'a'n}\hat{q}_{s'a'n} = \Theta_{san}$$

where s'=l when s=h and vice-versa; and a'=y when a=o and vice-versa.

The equilibrium values are obtained by simultaneously solving equation (20) for all four types of native workers: hon, hyn, lon and lyn. The general form of the equilibrium outcomes can be expressed as the following expression. The detailed solutions of which are outlined in the appendix.

(21)

$$\hat{q}_{san}^{*} = \begin{bmatrix} \beta_{s'a'n}\beta_{sa'n}(\pi_{s'an} - \alpha_{s'an}) + \beta_{s'an}\beta_{sa'n}(\pi_{s'a'n} - \alpha_{s'a'n}) + \alpha_{sa'n}(\alpha_{s'an}\alpha_{s'a'n} - \pi_{s'a'n}\pi_{s'an}) \\ \beta_{s'a'n}\beta_{sa'n}(\alpha_{s'an} - \pi_{s'an}) + \beta_{s'an}\beta_{sa'n}(\alpha_{s'a'n} - \pi_{s'a'n}) + \pi_{sa'n}(\pi_{s'a'n}\pi_{s'an} - \alpha_{s'an}\alpha_{s'a'n}) \\ (\alpha_{sa'n} - \pi_{sa'n})(\beta_{s'a'n}\pi_{s'an} - \beta_{s'an}\alpha_{s'a'n}) \\ (\pi_{sa'n} - \alpha_{sa'n})(\beta_{s'a'n}\alpha_{s'an} - \beta_{s'an}\pi_{s'a'n}) \end{bmatrix} \begin{pmatrix} \Theta_{san} \\ \Theta_{sa'n} \\ \Theta_{s'a'n} \\ \Theta_$$

where

$$\Omega = \beta_{lyn} [\pi_{hy1n} \beta_{hon} - \alpha_{hyn} \beta_{hon} + (-\alpha_{hon} + \pi_{hon}) \beta_{hyn}] (\alpha_{lon} - \pi_{lon}) -$$

$$(22) \quad \beta_{lon} [\pi_{hyn} \beta_{hon} - \alpha_{hyn} \beta_{hon} + (-\alpha_{hon} + \pi_{hon}) \beta_{hyn}] (\pi_{lyn} - \alpha_{lyn}) +$$

$$\pi_{hyn} \pi_{hon} (\pi_{lyn} \pi_{lon} - \alpha_{lon} \alpha_{lyn}) + \alpha_{hon} \alpha_{hyn} (-\pi_{lyn} \pi_{lo1n} + \alpha_{lon} \alpha_{lyn})$$

Again, we have s'=l when s=h and vice-versa; and a'=y when a=o and vice-versa.

Even though these expressions seem rather complicated, they have intuitive explanations. The equilibrium value of \hat{q}_{san} is simply a weighted linear sum of 1) changes in the stock of aworker's own skill and age group; 2) changes in the stock of the same skill but different age group; and 3) changes in the stock of workers of different skill groups. The weights are functions of the elasticity of labor supply, the elasticities of substitutions and the shares of the total wage bill going to different types of workers. The response of \hat{q}_{san}^* to each of these changes is captured by the terms in the first matrix on the right hand side of equation (21). These can also be viewed as slopes, which show that greater weight is given to changes in the stock of labor of own skill and age group. Changes in the stock of labor of different skill group are given the lowest weights. A closer inspection of the equilibrium equation suggests higher degree of substitution between age groups within a native-education cell dampens the impact of changes in age-education stocks over time. With immigrants, it is the opposite: higher

degree of substitution between native and immigrants within a given age-education cell leads to greater effects.

The equilibrium wage effects can simply be expressed as:

(23)
$$\hat{w}_{san}^* = \frac{1}{\gamma} [\hat{q}_{san}^* - \hat{Q}_{san}]$$

2.5 Disentangling the equilibrium employment and wage effects due to aging, immigration and emigration

We start by observing that $\Theta_{san} = \hat{M}_{san}^f + \frac{1}{\gamma}\hat{Q}_{san}$. Note \hat{M}_{san}^f is defined as the change in marginal productivity of a native worker of type *san* that arises solely from changes in the stock of immigrants, as defined in equation (16). \hat{Q}_{san} , the percentage change in the population of type *san* can result from either aging, emigration or both. As noted earlier, aging implies individuals moving from one age group to another, while emigration entails particular group of workers moving out of the country. Thus we write:

(24)
$$\frac{1}{\gamma}\hat{Q}_{san} = \frac{1}{\gamma}\hat{Q}_{san}^{n,a} + \frac{1}{\gamma}\hat{Q}_{san}^{n,e}$$

where $\hat{Q}_{san}^{n,a}$ and $\hat{Q}_{san}^{n,e}$ are changes in the stock of native population that arise from aging and emigration, respectively. This implies:

(25)
$$\Theta_{san} = \hat{M}_{san}^{f} + \frac{1}{\gamma}\hat{Q}_{san} = \hat{M}_{san}^{f} + \frac{1}{\gamma}\hat{Q}_{san}^{n,a} + \frac{1}{\gamma}\hat{Q}_{san}^{n,e}$$

Finally, \hat{q}_{san}^* (i.e. the percentage change in the stock of type *san* workers at equilibrium) can be expressed as a linear sum of changes arising from aging, immigration and emigration as suggested by equation (21):

(26)
$$\hat{q}_{san}^* = \hat{q}_{san}^{*f} + \hat{q}_{san}^{*n,a} + \hat{q}_{san}^{*n,e}$$

3 Data

3.1 Labor Force and Migration

The first data source we use is the Database on Immigrants in OECD countries (DIOC), compiled jointly by the OECD and the World Bank. The database is the output of a project that started in 2000, with the purpose of compiling highly standardized population census and register data on immigrants and natives in OECD countries, based on the 2000/01 census rounds. It was then subsequently extended to include a large number of non-OECD countries (the extension is referred to as DIOC-E). The 2010 dataset draws primary from national censuses, supplemented with population registers and surveys where necessary. Significant efforts were made to standardize these data since the original data sources contain significant variation due to national data collection and dissemination policies (Arslan et al, 2016). Censuses survey the entire population or a representative but larger sub-sample, such as a micro-census of Germany, at a single point in time.⁴ For each OECD country, DIOC provides aggregate stocks by destination, origin, migration status, gender, educational attainment and various age groups. Using this information, we are able to distinguish the native-born from the foreign-born, the tertiary educated from the non-tertiary educated and older from

⁴They often focus on the resident population, both regular and irregular.

younger workers in each OECD country. Since the dataset is on a bilateral format, we can also construct the emigrant labor stock of the OECD countries. This is a very unique and valuable feature of DIOC, since we could not obtain emigration numbers from datasets based on censuses of individual countries.

Table 1 presents age distribution by migration status across OECD countries and the age distribution of emigrants from these countries for the year 2010. The first set of columns refer to native-born individuals, followed by a set of columns pertaining to immigrants. Two key observations emerge from Table 1: 1) immigrants and emigrants are generally much younger than natives; and 2) the age distribution of immigrants and natives differ across all countries. We see that in countries such as Japan, Italy, Germany and Greece, the 65+ old comprise a quarter or more of natives of working age. In 28 out of 33 countries, they account for no less than 15 percent. In comparison, migrants are younger in most of these countries. A close inspection of Table 1 reveals that the shares of individuals between the ages of 45 and 64 exceed the corresponding shares among immigrants in 20 out of the 33 countries.

In order to empirically examine how demographic changes affect the relative wages of natives of working age across different age-education cells, we look at individuals between the ages of 25 and 64. The lower bound of 25 is chosen to reduce the incidence of having foreign-born individuals who migrated for the purpose of education in the data. We recap the definitions and groupings used in our analysis below:

Young: individual between the age of 25 and 44

Old: individual between the age of 45 and 64

High-Skilled: individual who completed tertiary education

Low-Skilled: individual with a qualification below tertiary education level

Both the young and the old can be either high skilled or low skilled. The relative changes in the stock of labor force of a given type i between 2000 and 2010 is computed as:

$$(27) \quad \hat{Q}_i = \frac{\Delta Q_i}{Q_{i_{2000}}}$$

More precisely, ΔQ_i is the difference between the total number of individuals of type *i* joining the labor force and the total number of individuals of type *i* leaving the labor force. For both immigrants and emigrants we define these as $\Delta Q_i = Q_{i_{2010}} - Q_{i_{2010}}$. For natives, however, a large proportion of 15 to 24 year olds in 2000 who later become 25 to 34 in 2010 would not yet have finished their education in 2000. In fact this stock is quasi-low-skilled, mainly because the teenagers in this age group and those in their earlier 20s are almost all low-skilled, based on our categorization. Therefore, we are likely to over-count (under-count) changes in the share of younger low (high) skilled workers. Since the 15 to 24 year olds in 2000 are the 25 to 34 year olds in 2010, we are able to address this issue. In a similar way, those between the age of 45 and 54 in 2010 would have been the 35 and 44 year olds in 2000. Although there should be little change in educational distribution within this group, we use 2010 data for consistency. When examining the implications of ageing for younger workers we have:

(28)
$$\hat{Q}_{syn} = \frac{\Delta Q_{syn}}{Q_{syn_{2000}}} = \frac{\sum_{i=25}^{34} Q_{i_{2010}} - \sum_{i=45}^{54} Q_{i_{2010}}}{Q_{syn_{2000}}}$$

For older workers,

(29)
$$\hat{Q}_{son} = \frac{\Delta Q_{son}}{Q_{son_{2000}}} = \frac{\sum_{i=35}^{44} Q_{i_{2000}} - \sum_{i=55}^{64} Q_{i_{2000}}}{Q_{son_{2000}}}$$

The relative changes in the stocks of interest are given in Table 2. Columns 1 to 4 show relative changes resulting from changes in the age-education structure of the native labor force, columns 5 to 8 show relative changes in the stock of immigrants in the country and columns 9 to 12 show relative changes resulting from natives emigrating or returning to their home country. The negative sign on values for emigration imply that there was a reduction in the stock of the native workforce due to increased rate of emigration between 2000 and 2010.

Among natives, the stocks of old high-skilled labor force exhibit the highest rates of relative increase. This is the case in almost all countries in our data, in particular, in European countries where the rate of increase is often about 40 percent or higher. For example, the increase in the size of this group was 95 percent in Spain, 65 percent in Ireland, 57 percent in France and 41 percent in the UK. The exceptions are Denmark, Sweden and Germany where we see an increase of 24 percent or less, arguably due to the fact that the stock of old high skilled labor force might already have been large in 2000. Other non-European OECD countries also show similarly large changes; the USA, Canada, Australia and New Zealand all have a relative increase of over 40 percent. The stocks of older low skilled labor force also exhibit noticeable rates of relative increase between 2000 and 2010, although the magnitude of these are generally much lower in comparison to the older high skilled groups. Japan is the only exception with a relative rate of decline of 17 percent. In short, natives in all countries saw an increase in the old age group in their labor force, with a relatively larger increase among the higher-skilled older people. In line with the discussion in the introduction there is also an increase in the stock of young high skilled workers in the labor force in most countries between 2000 and 2010. The stocks of younger low-skilled workers, in contrast, show declines, except in the Czech Republic, Mexico, Slovakia and Turkey. This, in combination with the fact that the stocks of old high skilled in the labor force significantly increased in all countries, suggest an overall shift toward a more educated and older labor force relative to 2000. This is the main defining feature of the OECD labor markets that will dominate our results.

Columns 5 to 8 of Table 2 point to a general increase in the stock of immigrants, both in the overall population and across age-education groups (with the exception of Poland, Czech Republic, Slovakia and Mexico). However, the relative changes in stocks are not uniform across age-education groups. They mostly appear to be much higher among the high-skilled. Furthermore, it is unclear whether the increase among the older high-skilled dominates the increase among the younger high-skilled groups.

Emigration patterns show similar characteristics as those observed for immigration. The relative changes among the high-skilled dominate and patterns of return migration are visible among the low-skilled. New Zealand, Ireland, Poland and Slovakia particularly stand out as emigration countries. While the first two countries are well known as migrant-sending countries, for the last two countries this is related to their joining of the EU in 2004 and their citizens obtaining freedom to work and live in Ireland, Sweden and the UK from then onward. In comparison to changes arising from ageing, the relative changes in the age-education structure of the native labor force due to emigration, although generally in the opposite direction, are quite small.

[TABLES 1 and 2 ARE HERE]

3.2 Key Parameters

The simulation of the equilibrium wage effects of aging, educational changes and migration require the following key parameters as discussed in the previous section on the analytical model.

1. Elasticity of substitution between high skilled and low skilled workers, δ_s

Most studies estimating the elasticity of substitution between high skilled and low skilled workers find a value no greater than 2. While Katz and Murphy (1992), Murphy et al. (1998) and Caselli and Coleman (2006) estimate δ_s to be between 1.3 and 1.4, Fallon and Layard (1975), Angrist (1995) and Ciccone and Peri (2005) find values between 1.5 and 1.75, and Ottaviano and Peri (2012) find a value of about 2. An exception is Fitzenberger et al. (2006), who find values between 4.9 and 6.9, although they acknowledge this range is much higher than that found in prior studies. We set δ_s to 1.75 in our main scenario, the value DOP use in their intermediate scenario. We then examine how our simulated effects change when we allow δ_s to take the value of 5, close to the lower bound value in Fitzenberger et al. (2006). Even though this value is far greater than the one commonly found in the literature, our aim is to gauge the effect of a higher degree of substitutability between skill groups on the labor market outcomes of interest.

2. Elasticity of substitution between the old and the young workers, δ_a

Katz and Murphy (1992), one of the first studies to estimate this parameter, find a value of about 3. Card and Lemieux (2001), arguably one of the most influential papers on the extent of substitutability between young and old workers, estimate δ_a across three countries. Using data on the US, Canada and the UK, they find values between 4 and 6. In more recent studies, Manacorda et al. (2012) estimate a value of around 5 using UK data, Ottaviano and Peri (2012) find a similar value using the US data and, Glitz and Wissmann (2016) estimate values greater than 7 using the German data. We

set $\delta_a = 5$ in our main scenario, which is in the mid-range of these available estimates. Compared with our central estimate of δ_s , this assumption implies that workers in different age groups within the same skill group are more subtitutable than workers in different skill groups, although still imperfectly so. We later allow for different values of δ_a in order to explore the sensitivity of our simulated labor market effects to the degree of substitutability between age groups.

3. Elasticity of substitution between natives and migrants, δ_m

The elasticity of substitution between migrants and natives, δ_m , is a crucial parameter in determining the effect of immigration on the labor market outcomes of natives. As such, this parameter has received a fair share of attention in the literature on the labor market effects of immigration in main destination countries. On one hand, Borjas (2003) suggest perfect sustutability between immigrants and natives, implying a value of δ_m equal to infinity. Card (2009), Ottaviano and Peri (2012) and Manacorda et al. (2012), on the other hand, find that immigrants and natives are not perfect substitutes and find values of δ_m between 6 and 20. In our empirical analysis, we set δ_m to 20, the value DOP use in their intermediate scenario. We again examine the implications of perfect substitutability on labor market outcomes in the extensions section.

4. Externality of high skilled labor, λ

The parameter λ measures the extent of productivity spillovers from having a higher share of highly skilled workers in the economy, as defined in equation (6). Accemoglu and Angrist (2001) and Moretti (2004) put λ at 0 and 0.75, respectively. We assume an intermediate value of 0.45 in the main analysis.

5. Elasticity of labour supply, γ

 γ captures how labor supply responds to changes in wages. In their synthesis of the literature, Evers et al. (2008) summarize estimates of γ from various studies. Their meta-analysis suggests that 0.1 is a reasonable measure of this parameter among men

and 0.5 among females. When simulating our labor market effects of interest we use a conservative value of 0.1 in our main scenario.

Table 3 below summarizes values of the parameters we use in the main simulation.

[Table 3 HERE]

A key varaible required in the analysis of the equilibrium outcomes is the share of total labor wage bill going to each type of worker in the labor force. These wage bill shares are generally computed by (1) using individual wages and total labor stocks from detailed labor force surveys; (2) using individual hours worked and total labor stocks from detailed labor force surveys; or (3) using total labor stocks and normalized wages, with one group of workers as the base group. Card and Lemieux (2001) and Manacorda et al (2012) present an application of these different approaches. The desired labor force surveys are publicly available in such detail only in Canada, the USA and the UK. The available surveys for EU countries provide wage brackets, not the exact wages of individuals. We proceed with using method (3) in most countries. We set the wage of a low skilled young native worker (i.e. w_lyn) to 1 and use the following to calculate the average wages of workers in other groups:

$$w_{hon} = (\text{age premium})^*(\text{skill premium})^* w_{lyn}$$

$$w_{hyn} = (\text{skill premium})^* w_{lyn}$$

 $w_{lon} = (age premium)^* w_{lyn}$

 $w_{hof} = (\text{immigrant premium})^*(\text{age premium})^*(\text{skill premium})^*w_{lyn}$

 $w_{hyf} = (\text{immigrant premium})^*(\text{skill premium})^*w_{lyn}$

 $w_{lof} = (\text{immigrant premium})^*(\text{age premium})^* w_{lyn}$

 $w_{lyf} = (\text{immigrant premium})^* w_{lyn}$

The share of total wage bill going to a particular age group is the product of the size of that group and the normalized wage going to a given individual in that group, divided by the total wage bill of the entire economy. We use skill premium data from Hendricks (2004) for each country. Average immigrant wage premium for each country is taken from DOP, where they use figures from a number of papers published between 1993 and 2005. We compute age premium ourselves from the American Community Survey data and use this for all countries.

4 Equilibrium wage effects

4.1 Aging and Education Composition

The wage effects due to changes in the age-education structure on non-migrant natives are reported in Table 4 and the first set of graphs in Figure 4. The ordering of the countries start with Anglo-Saxon countries, followed by continental European countries, Scandinavian countries, Eastern and Central European countries and ends with Turkey. Table 4 also reports average wage effects across skill groups and across age groups in the last four columns. In Figure 4, the wage effects on older high skilled workers are illustrated by the blue line, the effects on older low skilled are illustrated by the gray line, those on younger high skilled workers are illustrated with the orange line and those on their low skilled counterparts are illustrated by the yellow line.

Several clear patterns emerge from both Table 4 and Figure 4. First, older high skilled workers are the group most hurt by changes in the age-education structure of the labor force, followed by younger high skilled workers. Second, younger low skilled workers are the biggest beneficiaries of changes in the age-education structure of labor force in all countries. This is not particularly surprising in light of the patterns observed in Table 2, which depicted an increase in the relative stocks of older high skilled workers and a decline in the relative stocks of younger low skilled workers between 2000 and 2010. Older high skilled workers face increased competition due aging and educational upgrading while younger low skilled workers have the exact opposite experience. The negative wage effects are more pronounced in continental European countries, with the magnitude of the effect being as high as 22 percent in some countries (e.g. Spain). In a traditional old-age country such as Japan, the magnitude of the effect of increased aging is 15 percent, arguably because the base stock of older people was already large in 2000. In Anglo-Saxon countries, the effects are generally less than 10 percent, with Ireland being the exception. In line with the changes in the ageeducation structures presented in Table 2, the magnitude of the effects in countries such as Mexico, Poland and Turkey, i.e. those going through a demographic transition, are about 20 percent.

The wage effects resulting from changes in the age-education structure of the labor force on younger high skilled workers are almost negative throughout all countries, with the exception of the USA (+1.5 percent), Canada (+0.3 percent), Germany (+1.9 percent) and Finland (+0.6 percent). These effects are, however, much less pronounced in comparison to the effects on older high skilled workers. In fact the median effect on the high skilled is -4 percent for the wages of the young as opposed to -9 percent for the old. In contrast, the median wage effect is +2.45 for older and +6.81 for the younger low skilled workers. The clear wage gains for younger low skilled workers are visible from the yellow line which is consistently above the x-axis in Figure 4. Although there have been increases in the relative stocks of older low skilled workers between 2000 and 2010 in most countries, the magnitudes of these changes are much lower in comparison to the changes in the number of the older highly skilled. Additionally, this former group benefits from the productivity spillovers generated from having more high skilled workers in the workforce. We also see from the equilibrium solution in equation (21) that there is a positive relationship between the wages of the low skilled and changes in the quantity of the high-skilled workers, even in the absence of a productivity spillover. These factors counteract the negative wage effects that would have resulted solely from increases in their (own-type) relative stocks. Therefore, similar to their younger counterparts, older low skilled workers experience wage gains, though in a few countries and in lower magnitudes.

Looking at wage effects across skill groups and across age groups, it is evident from the last four columns of Table 4 that the low skilled are gainers, and so are younger workers. The high-skilled are clear losers and the average effects vary from one country to another for older workers. The latter is driven by the fact that older low skilled workers account for a larger proportion of the labor force in the base year and so do their wage bills. Therefore, a small positive gain for them may outweigh large losses for the high-skilled when computing average wage effects across age groups.

[TABLE 4 HERE]

[FIGURE 4 HERE]

4.2 Immigration

The effects on the wages of natives due to changes in the stock of immigrants are reported in Table 5 and in Figure 5. The ordering of the countries remains the same as in Table 4. We also present average wage effects across skill groups and age groups in the columns on the right panel of Table 5. The graphs illustrating the wage effects in Figure 6 are of the same colors as those depicting the wage effectsaging. In all countries, with the exception of Belgium and Finland, the increase in the stock of immigrants (as observed in Table 2) has negative wage effects for the highly skilled regardless whether they are old and young. Similarly, immigration has positive effects on the low skilled workers regardless of their age group. In countries such as Canada, Australia New Zealand and the UK with historically liberal immigration policies that attracting high skilled migrants, the wage gains for the low skilled natives tend to be higher. The wage gain, for example, is 8 percent in Australia. This findings are in line with the results found in DOP for the immigration flows of the previous decade. And they are in relatively stark contrast to the popular perception that immigration has negative effects on the wages of low skilled native workers. On the contrary, these groups of low-skilled workers are the main beneficiaries of relatively high skilled immigration in term of their wages.

Our model allows for differential wage effects of immigration on native workers across different age groups which is different than the set-up in DOP where there are no age groups. Nevertheless, the results show that wage gains and losses are relatively symmetric across age groups within the same skill group in the majority of the OECd destination countries. There are some noticeable differences in Canada, the UK, Ireland, Spain and Portugal, suggesting that the (implicit) assumption of symmetric effects in DOP may not necessarily hold. This becomes even more apparent when one allows the elasticities of substitution between immigrants and natives to vary across age groups, as we perform as an extension.

[TABLE 5 HERE]

[FIGURE 5 HERE]

4.3 Emigration

The wage effects due to emigration show different patterns to those that result from changes in the stocks of immigrants. The low skilled non-migrant natives now suffer wage losses and the high-skilled are beneficiaries emigration. This is illustrated in columns 5 and 6 of Table 6 and Figure 6. The heterogeneity in wage effects from emigration is also apparent, with countries which experienced low levels of relative changes in the age-education structure of natives due to emigration having almost no wage effects. All of these results are due to the high skill intensity of emigration flows. The positive relationship between the wages of the low skilled and changes in quantities of high skilled labor means wage losses resulting from emigration among the low-skilled are more pronounced in migrant sending countries like Ireland and Poland. In the former case, older low skilled workers suffer a wage loss of 7 percent and their younger counterparts suffer a wage loss of 5 percent. For Poland, the corresponding losses are 3 percent and 2 percent, respectively for the old and the young. Because the low-skilled account for a larger share of the total labor wage bill and the wage losses they experience dominate the wage gains to the high-skilled, emigration ends up having negative average wage effects on the labor force.

[TABLE 6 HERE]

[FIGURE 6 HERE]

4.4 Overall Effect

The aggregate impact on wages of each of the effects discussed above, changes in the ageeducation composition, immigration and migration, is depicted in Figure 7. The ordering of the countries is as before. For each country we see the impact in sequence on the old high skilled, old low skilled, young high-skilled and young low-skilled. The impact of changes in the age-education composition are shown in blue, of emigration in orange and immigration in grey. The most striking feature is the general dominance of blue, reflecting the finding that for most countries changes in the age-education structure are the dominant influence on wages. Moreover, in virtually all countries, these changes lead to higher wages for the unskilled and lower wages for the skilled across all age groups. Only in the United States and Germany do we see the young high skilled benefitting marginally from the changes.

The relative impact of aging-education differs across countries depending on the magnitude of migration. In most of Western Europe, Japan and the United States, aging and changes in the stock of skills, account for most of the changes in wages, and migration plays a marginal role - note the relative insignificance of the grey and orange bars. In the Anglo-Saxon countries (Australia, Canada, New Zealand, the United Kingdom) and Switzerland, which have relatively liberal migration policies, immigration - the grey bar - reinforces the impact of aging on wages. In countries that have seen significant labor outflows, such as Ireland and certain Eastern European countries (the Czech Republic, Slovakia, Hungary and Poland) the impact of aging is partially offset by emigration, the orange bar. However, the impact of emigration never completely offsets the combined effect of the changing age-skill composition and immigration. Therefore, the aggregate effect of all these changes is egalitarian: for both young and old, the wages of the unskilled are higher and wages of the skilled are lower than they would have been if these changes had not happened.

[FIGURE 7 HERE]

5 Extensions

We examine various scenarios in this section by allowing values of the key parameters of the model to change from their values in the main scenario. We start by changing the value of one parameter at a time. The median wage effects resulting from these changes are presented in Tables 7 and 8, with the minimum and maximum values in brackets. The first scenario allows for a much higher degree of substitutability between skill groups in the production function where we set $\sigma_s = 5$. With the new elasticity level, the wage effects due to ageing, immigration and emigration change. In comparison to the main scenario, the resulting wage losses effects from ageing are relatively smaller for the high skilled workers. To put it differently, as they become better substitutes for low-skilled workers, the higher quantity of the high-skill labor now has a reduced effects on their relative wages. In the same vein, the low-skilled workers, both young and old, experience smaller wages gains. Similarly, immigration has reduced negative effects on the wages of the high-skilled and the gains to the low skilled decrease significantly. This results are again due to the fact that the relatively higher skilled concentration among the immigrants is now less detrimental to the high skilled natives and less advantageous for their low skilled counterparts. Changes in the wage effects from emigration are governed by similar mechanisms.

In the second scenario, presented in the third panel of Table 7, we increase the degree of

substitutability between age groups. The wage effects resulting from ageing become ambiguous across age groups among the high skilled. Both the young and the old often experience positive changes in their labor stocks, but now having the greatest level of relative change matters less for either group as workers in one group become more substitutable with workers in the other group. In comparison to the main scenario, the young low skilled workers experience a clear decline in their wage gains because the substitutability between them and older low skilled workers increases, which dampens the gains from the large reductions in their stocks. The scenario leads to little change in the wage effects arising from immigration or emigration, in comparison to the main scenario.

The third scenario makes the immigrants almost perfectly substitutable with natives. This magnifies the negative wage effects of immigration on the high-skilled workers, but the wage gains to the low-skilled remain fairly stable. Because the elasticity parameter we have changed has little importance in the slope of the wage responses to changes in the quantity of the native labor force, the resulting wage effects from ageing and emigration are negligible. The fourth exercise consists of removing productivity spillovers from high skilled workers, i.e. $\lambda=0$. This change leads to a constant reduction in wages across all groups of workers, in comparison to the main scenario, since the spillovers were relatively uniformly spread.

[TABLE 7 HERE]

The next set of extensions allows the elasticity of substitution between immigrants and natives to differ across skill groups and age groups. This exercise is of particular interest because selective immigration policies, such as point based systems used in Canada and Australia, tend to give additional preferences to younger individuals or older individuals. In light of the perception that immigration hurts younger natives, we consider a relatively pessimistic scenario where young immigrants and young natives are perfectly substitutable. In contrast, we maintain the degree of imperfect substitutability between older immigrants and older natives at the same level as in our main scenario. Patterns of the resulting wage effects are similar to those we observed earlier when immigrants and natives were perfectly substitutable across all age groups. The major difference here, however, is that the negative wage effects on younger high skilled workers become amplified and those on older high skilled workers are subdued. This is particularly the case in Anglo-Saxon countries, which tend to be major immigrant receiving countries. Low skilled workers, both young and old, remain beneficiaries of wage gains. The effects resulting from ageing and emigration, in comparison to the main scenario, remain mostly unchanged.

In the final exercise, we explore how our simulated wage effects respond to having different degrees of substitutability between age groups across our two skill groups. We proceed by setting $\sigma_a = 3$ among the high-skilled, the lower bound of values described earlier, and $\sigma_a = 7$ among the low skilled, implying a somehow higher degree of substitutability. Consistent with our expectations, making younger and older worker less substitutable relative to the main scenario leads to changes in the relative stock of a given type of workers in the labor force becoming more internalized in the wages of that group of workers. The negative wage effects increase among older high skilled workers but decline among their younger counterparts. The reverse pattern occurs among the low-skilled since older workers and younger workers become more substitutable, in comparison to the main scenario; wage gains decline among younger workers but move to right on the number scale among older workers.

[TABLE 8 HERE]

6 Conclusion

While the effects of aging, education and migration on wages has been studied before, no previous analysis has considered considered all three together. This integrated treatment helps us to assess the relative and aggregate force of these three developments. Our most striking conclusion is that changes in the age and skill structure of the population dwarf the effects of immigration and emigration in most countries. For the most part, these big demographic changes boost the wages of the low skilled, on the one hand by making them relatively scarce and on the other hand by increasing their productivity - because the low skilled can work with relatively more skilled people. Immigration, the villain in much political discourse, turns out to be a relatively feeble and actually positive phenomenon.

Whereas in most of Western Europe, Japan and the United States, aging and changes in the stock of skills, account for most of the changes in wages, migration plays a more significant role in two groups of countries. In the Anglo-Saxon countries (Australia, Canada, New Zealand, the United Kingdom) and Switzerland, which have relatively liberal and skillbiased immigration policies, immigration reinforces the impact of aging on wages - because the immigrants tend to be more skilled on average than natives. In countries that have seen significant labor outflows, such as Ireland and certain Eastern European countries (the Czech Republic, Slovakia, Hungary and Poland) the impact of aging is partially offset by emigration, because emigrants too tend to be more skilled than natives. However, the impact of emigration never completely offsets the combined effect of the changing age-skill composition and immigration. Therefore, the aggregate effect of all these changes is egalitarian: for both young and old, the wages of the unskilled are higher and wages of the skilled are lower than they would have been if these changes had not happened.

We recognize that our findings emerge from a model that abstracts from other important changes, especially in technology. Nevertheless, there is value in isolating and highlighting the relative impact of forces that are shaping the wage-structure in industrial economies, both to inform the political debate and to identify reform priorities. Figures and Tables



Figure 1: Relative change in labor stocks



Figure 2: Labor shares in 2000 and 2010 by nativity

Figure 3: Composition of labour

















Figure 7: Simulated aggregate wage effect

(a)



(b)

		Nat	\mathbf{ives}			Immig	grants		Emigrants				
	15-24	25-44	45-65	65 +	15-24	25-44	45-65	65 +	15-24	25-44	45-65	65 +	
USA	18.8	31.5	32.9	16.8	12.3	43.9	31.1	12.7	17.7	31.9	33.5	17	
CAN	17.9	31.5	35.1	15.5	9.8	34.1	35.8	20.3	9.5	32.9	34.9	22.8	
AUS	19.4	34.6	30.3	15.7	11	34.9	33.6	20.5	12.4	51.6	26.3	9.6	
NZL	18.4	30.3	33.1	18.3	15.4	36.2	31.2	17.3	12.5	43.7	34.3	9.4	
GBR	16.3	30.6	32	21.2	13.9	48.3	25.3	12.4	6.4	26.2	39.8	27.6	
IRL	16	36.4	30.5	17.1	15.5	56.8	22.3	5.4	4.4	26.4	31.2	38	
DEU	13.4	28	32.8	25.8	6.5	40.4	37.6	15.5	11.1	38.4	29.8	20.7	
AUT	16.4	35.1	36.5	12	11.8	46.7	33.2	8.2	4.9	19.8	37.3	38	
CHE	18.1	33.2	36.3	12.4	9.3	47.3	34.9	8.6	11.8	46.2	29.2	12.7	
\mathbf{FRA}	16.3	32.4	30.9	20.4	8.6	33.3	37.7	20.5	9.6	45.6	28.6	16.3	
BEL	15.1	30.3	32.6	21.9	11.9	42.7	31.5	13.9	9.3	35.8	34.9	20	
NLD	15.4	30.7	34.3	19.61	11.2	45.1	34.4	9.4	7	24.7	38.6	29.6	
LUX	18.3	28.8	30.9	21.9	10	46.4	32.8	10.8	17.5	34.2	29.4	18.9	
ITA	11.4	30.1	32.5	25.9	13.4	53	27	6.6	2.7	18.8	39.4	39.2	
ESP	11.5	35.3	31.2	22	15.1	53.3	24.5	7.1	6.2	28.9	34.4	30.4	
\mathbf{PRT}	12.7	31.8	31.7	23.9	13.5	53.1	26.2	7.1	6.1	35.1	43.9	15	
GRC	12.5	32	30.4	25	14.3	51	26.6	8	3.5	17.4	41.7	37.4	
DNK	15.1	29.9	33.27	21.8	16.5	47.8	27.2	8.5	7.4	31.2	31.9	29.6	
SWE	16.7	29.2	30.6	23.4	11.9	41.2	31.5	15.5	14.6	44.9	26.4	14.1	
NOR	16.3	31.2	32.27	20.4	14.7	53.4	25.6	6.3	13.4	27.6	28.8	30.2	
CZE	14.3	35.1	32.1	18.3	10.7	40.6	28.8	19.9	6.1	33.1	30.8	30	
FIN	14.6	28.8	34.8	21.8	16	54.5	23.6	5.9	4	18.6	44.2	33.2	
HUN	13.3	34.6	32.4	19.7	9.6	37.5	26	27	5.5	32.9	25.1	36.6	
POL	15.8	35	34	15.2	4.8	8.7	11.4	75	8	47.6	30.5	13.9	
SVK	16.6	37.3	31.6	14.5	5.8	23.8	42.3	28.2	7.8	39.8	27.8	24.5	
JPN	11.2	30.7	31.3	26.8	19.7	50.7	22.1	7.6	11.2	44	32.5	12.2	
MEX	26.4	40.9	23.4	9.2	32.4	36.4	20.8	10.3	15.4	53.6	24.8	6.1	
TUR	23.4	45.1	26.7	4.8	6.9	39.8	41.5	11.8	6.3	49.7	33.6	10.4	

Table 1: Age distribution of populations in 2010 (15+ only), in %

Emigrant populations refer to those at OECD destinations only

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tion	lucated	Young	-0.2	1.0	0.2	0.6	2.0	4.9	-0.1	-0.5	3.2	0.8	0.7	-0.7	13.7	0.3	1.1	5.3	0.7	0.5	-0.2	0.5	2.9	-1.5	-1.9	-7.9	-4.8	0.0	-5.4	-4.7
emigra	Less ec	Old	-0.5	-1.1	-1.2	-11.0	-0.6	13.6	0.2	-0.6	-6.3	-0.6	-1.5	-0.3	8.9	0.3	0.1	-6.6	2.2	0.4	-0.1	0.4	1.5	-1.1	0.6	-5.3	1.3	-0.2	-10.9	-7.7
ives, from	y educated	Young	-0.4	-1.1	-3.8	-10.5	-2.4	-11.5	-2.9	-6.9	-4.7	-3.6	-2.8	-2.2	29.7	-3.1	-0.7	-8.8	-1.7	-2.5	-3.6	-2.6	-0.7	-10.0	-13.5	-25.0	-26.3	-0.3	-4.9	-9.5
Nati	Tertiar	Old	-0.8	-5.2	-3.8	-17.7	-12.3	-19.8	-4.0	-7.2	-9.2	-4.9	-5.9	-5.0	27.9	-5.3	-2.2	-21.0	-5.6	-3.2	-2.5	-3.6	-2.3	-6.6	-1.7	-18.9	-8.0	-0.9	-12.2	-12.8
	lucated	Young	13.4	-20.3	0.9	27.0	39.1	102.0	2.0	20.6	-9.9	4.9	67.6	-13.6	24.0	102.9	145.5	21.9	22.2	29.4	14.4	52.3	101.8	46.3	46.7	-86.5	-84.5	19.1	-13.4	56.6
rants	Less ed	Old	47.6	-0.6	11.9	43.9	15.1	129.4	9.4	21.9	0.3	16.7	44.4	10.1	55.7	214.8	238.0	117.0	86.5	45.8	12.5	62.5	150.5	-9.9	17.3	-87.6	-67.7	16.8	-17.0	132.5
Immig	r educated	Young	33.0	56.0	100.3	87.7	149.4	94.8	48.1	100.8	106.7	39.2	2.6	31.7	71.1	117.2	168.0	46.8	11.5	31.5	51.2	143.2	110.6	67.4	107.2	48.5	-57.7	2.2	24.5	121.4
	Tertiary	Old	79.4	76.0	83.3	90.8	121.8	126.3	72.3	84.1	84.6	70.1	46.0	58.8	110.2	179.7	285.2	118.2	98.2	55.9	46.2	91.8	146.3	34.8	31.7	-57.9	-39.7	65.7	45.0	129.6
ng	lucated	Young	-9.4	-17.8	-3.5	-15.6	-13.1	-8.1	-13.9	-13.4	-15.1	-16.6	-17.6	-16.8	-17.3	-18.3	-15.2	-12.6	-8.6	-15.2	-13.1	-17.1	-7.4	1.2	-2.8	-10.2	2.1	-8.1	20.6	15.8
im agei	Less ec	Old	27.9	25.7	27.9	24.4	11.8	11.0	0.0	16.9	14.4	17.3	11.4	18.4	17.3	7.8	11.3	9.6	1.2	3.2	1.3	5.3	3.3	8.5	5.1	22.7	24.3	-17.2	48.6	49.1
atives, fro	educated	Young	-2.5	1.5	13.9	-1.3	7.1	36.5	-20.6	-7.3	-4.6	34.0	8.9	2.4	33.8	24.7	25.9	59.2	26.0	-0.6	13.4	10.8	-7.0	45.5	41.9	85.6	59.9	8.7	28.1	78.3
Z	Tertiary	Old	41.5	54.5	47.0	51.5	41.0	65.0	23.7	53.3	42.9	57.4	53.6	39.6	45.9	42.4	94.6	43.8	60.9	24.4	9.2	30.1	40.1	24.0	20.5	33.9	40.6	55.9	117.0	90.7
			USA	CAN	AUS	NZL	GBR	IRL	DEU	AUT	CHE	FRA	BEL	NLD	LUX	ITA	ESP	PRT	GRC	DNK	SWE	NOR	FIN	CZE	HUN	POL	SVK	JPN	MEX	TUR

Table 3:	Parameter	values	used	in	simu	lation

δ_s : Elasticity of substitution between the high and low skilled	1.75
δ_a : Elasticity of substitution between the old and young	5
δ_m : Elasticity of substitution between immigrants and natives	20
λ : Intensity of tertiary education externality	0.45
γ : Elasticity of labor supply (can vary across groups if we want)	0.10

Parameters used in the intermediary scenario in DOP; δ_a comes from Card and Lemieux (2001)

Table 4:	Baseline	$\operatorname{scenario}$	wage	effects of	of ageing	(in	$\%$), $\delta_s =$	1.75,	$\delta_a = 5,$	$\delta_m =$	$20, \lambda$	=.45
$\& \ \gamma = 0.$	1											

		Eff	Average wage effect						
	Old, HS	Young, HS	Old, LS	Young, LS	HS	LS	Old	Young	
USA	-6.5	1.5	-1.1	5.5	-2.0	2.5	-3.4	3.7	
CAN	-7.6	0.3	1.1	8.1	-2.7	4.7	-2.3	4.3	
AUS	-6.4	-1.9	-0.1	4.3	-3.6	2.4	-1.7	2.4	
NZL	-8.5	-0.3	0.0	6.2	-3.5	3.4	-2.6	3.8	
GBR	-9.1	-3.0	2.3	6.9	-5.4	4.7	-0.8	3.4	
IRL	-12.3	-7.1	8.5	12.0	-8.6	10.2	3.2	3.0	
DEU	-6.5	1.9	0.2	2.7	-2.0	1.5	-1.6	2.5	
AUT	-11.6	-1.1	-0.8	4.5	-5.1	2.0	-2.6	3.3	
CHE	-8.0	-0.7	0.3	4.7	-3.6	2.6	-2.0	2.9	
FRA	-14.7	-10.8	5.2	11.3	-12.0	8.3	1.1	3.7	
BEL	-11.9	-3.9	4.1	9.2	-6.7	6.6	-0.3	3.7	
NLD	-9.4	-2.2	0.5	7.2	-5.2	4.0	-2.1	4.1	
LUX	-9.0	-7.2	1.0	5.8	-7.9	3.5	-0.5	2.9	
ITA	-16.8	-13.4	1.8	6.7	-14.7	4.3	-0.4	3.4	
ESP	-21.9	-9.1	8.0	13.0	-12.3	10.6	2.1	3.7	
PRT	-17.3	-19.5	4.8	8.9	-18.8	6.9	1.8	2.8	
GRC	-16.7	-10.4	6.4	8.1	-12.5	7.3	2.3	2.3	
DNK	-7.3	-2.6	2.1	5.5	-4.7	3.8	-0.5	3.0	
SWE	-4.0	-4.7	2.6	5.1	-4.3	3.8	0.7	1.9	
NOR	-7.5	-4.0	4.6	8.7	-5.4	6.6	0.6	3.1	
FIN	-8.5	0.6	3.5	5.5	-3.3	4.4	-0.7	3.2	
CZE	-9.7	-13.7	3.1	4.5	-11.9	3.8	1.2	1.3	
HUN	-8.8	-12.9	4.1	5.6	-11.0	4.8	1.6	1.6	
POL	-16.5	-26.8	5.8	12.1	-22.8	9.0	2.1	3.2	
SVK	-12.6	-16.3	3.1	7.0	-14.8	5.2	0.3	2.4	
JPN	-14.7	-5.5	12.8	11.0	-8.7	12.1	5.1	2.3	
MEX	-20.2	-3.1	-0.6	4.9	-7.2	3.0	-3.9	2.9	
TUR	-24.0	-21.9	0.8	7.2	-22.4	5.0	-1.7	3.0	

 δ_s is the elasticity of substitution between high-skilled and low-skilled workers, δ_a is the elasticity of substitution between the old and the young, δ_m is the elasticity of substitution between natives and migrants, λ captures the extent of high-skill productivity spillover, and γ is the elasticity of labor supply.

		\mathbf{Eff}	Ave	Average wage effect						
	Old, HS	Young, HS	Old, LS	Young, LS	HS	LS	Old	Young		
USA	-0.5	0.0	0.7	1.2	-0.2	1.0	0.2	0.7		
CAN	-4.3	-2.9	6.9	7.5	-3.4	7.2	2.6	2.5		
AUS	-8.3	-8.8	7.2	7.8	-8.6	7.6	3.1	2.8		
NZL	-3.2	-3.0	4.0	4.9	-3.1	4.5	1.7	2.0		
GBR	-3.3	-4.6	5.1	4.8	-4.1	4.9	2.9	1.5		
IRL	-1.3	-1.3	3.9	2.9	-1.3	3.4	2.6	1.0		
DEU	-0.5	-0.5	0.5	0.6	-0.5	0.6	0.3	0.3		
AUT	-3.2	-3.2	1.7	1.7	-3.2	1.7	0.9	0.7		
CHE	-7.2	-8.0	6.6	7.0	-7.7	6.8	2.9	2.0		
FRA	-1.6	-0.8	1.0	1.2	-1.0	1.1	0.5	0.6		
BEL	1.3	2.1	-0.9	-1.6	1.8	-1.3	-0.3	-0.1		
NLD	-0.6	-0.5	0.4	0.6	-0.5	0.5	0.1	0.2		
LUX	-9.8	-7.7	5.0	6.4	-8.5	5.7	2.7	3.2		
ITA	-0.8	-0.7	0.7	0.5	-0.7	0.6	0.6	0.3		
ESP	-1.2	0.2	1.6	0.9	-0.2	1.3	1.1	0.6		
\mathbf{PRT}	-3.0	-2.2	1.1	1.4	-2.5	1.3	0.6	0.6		
GRC	-0.9	0.6	0.2	0.4	0.1	0.3	0.0	0.5		
DNK	0.0	0.1	0.2	0.2	0.0	0.2	0.2	0.1		
SWE	-0.8	-1.1	1.3	1.2	-0.9	1.3	0.7	0.5		
NOR	-0.3	-1.0	2.0	1.5	-0.7	1.7	1.2	0.4		
FIN	0.4	0.3	0.1	-0.3	0.3	-0.1	0.2	0.0		
CZE	-0.7	-1.1	0.7	0.1	-0.9	0.4	0.5	-0.1		
HUN	-0.6	-1.2	0.6	0.5	-0.9	0.5	0.4	0.1		
POL	-0.7	-1.1	0.4	0.1	-0.9	0.2	0.2	-0.2		
SVK	-2.2	-2.1	0.3	0.3	-2.1	0.3	-0.2	-0.1		
JPN	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
MEX	-0.2	-0.1	0.1	0.1	-0.2	0.1	0.0	0.0		
TUR	-1.9	-1.6	0.3	0.8	-1.7	0.6	0.1	0.4		

Table 5: Baseline scenario wage effects of immigration (in %), $\delta_s = 1.75$, $\delta_a = 5$, $\delta_m = 20$, $\lambda = .45$ & $\gamma = 0.1$

 δ_s is the elasticity of substitution between high-skilled and low-skilled workers, δ_a is the elasticity of substitution between the old and the young, δ_m is the elasticity of substitution between natives and migrants, λ captures the extent of high-skill productivity spillover, and γ is the elasticity of labor supply.

Table 6:	Baseline	scenario	wage	effects	of	emigration	(in	%),	$\delta_s =$	1.75,	δ_a :	= 5,	δ_m	= 20,
$\lambda = .45$ &	$\gamma = 0.1$													

		Eff		Ave	erage	wage	effect	
	Old, HS	Young, HS	Old, LS	Young, LS	HS	LS	Old	Young
USA	0.1	0.01	-0.03	-0.1	0.0	-0.1	0.0	0.0
CAN	0.8	0.2	-0.5	-0.9	0.4	-0.7	0.0	-0.4
AUS	0.7	0.7	-0.4	-0.6	0.7	-0.5	-0.1	-0.2
NZL	2.5	1.4	-0.8	-2.6	1.8	-1.8	0.2	-1.1
GBR	2.9	1.1	-1.3	-1.8	1.8	-1.6	-0.2	-0.8
IRL	5.7	4.2	-6.9	-5.2	4.6	-6.0	-3.7	-0.8
DEU	1.2	0.9	-0.8	-0.7	1.0	-0.8	-0.3	-0.2
AUT	2.2	2.2	-0.9	-0.9	2.2	-0.9	-0.4	-0.2
CHE	1.6	0.9	-0.1	-1.6	1.2	-0.9	0.4	-0.7
\mathbf{FRA}	1.3	1.1	-0.7	-1.0	1.2	-0.8	-0.3	-0.3
BEL	1.2	0.7	-0.7	-1.1	0.9	-0.9	-0.2	-0.3
NLD	1.2	0.6	-0.7	-0.6	0.8	-0.7	-0.2	-0.2
LUX	-3.5	-3.4	1.4	0.9	-3.4	1.1	0.7	-0.1
ITA	1.9	1.5	-0.5	-0.5	1.7	-0.5	-0.2	-0.2
ESP	0.7	0.4	-0.3	-0.5	0.4	-0.4	-0.1	-0.1
\mathbf{PRT}	5.7	3.5	-0.4	-2.6	4.2	-1.6	0.4	-1.3
GRC	1.8	1.1	-1.0	-0.7	1.3	-0.8	-0.5	-0.1
DNK	1.0	0.9	-0.7	-0.7	0.9	-0.7	-0.3	-0.2
SVK	4.0	7.4	-2.9	-1.8	6.0	-2.3	-1.7	0.0
NOR	0.9	0.7	-1.0	-1.0	0.8	-1.0	-0.3	-0.2
FIN	1.0	0.6	-1.0	-1.2	0.8	-1.1	-0.3	-0.3
CZE	2.4	3.0	-0.9	-0.9	2.7	-0.9	-0.5	-0.2
HUN	1.4	3.7	-1.4	-0.9	2.6	-1.2	-0.9	0.1
POL	5.2	6.5	-2.8	-2.3	6.0	-2.6	-1.5	-0.3
SWE	0.7	0.8	-0.7	-0.6	0.8	-0.7	-0.3	-0.2
JPN	0.2	0.0	-0.1	-0.1	0.1	-0.1	0.0	0.0
MEX	0.8	-0.6	0.8	-0.2	-0.2	0.1	0.8	-0.3
TUR	2.3	1.7	-0.1	-0.7	1.8	-0.5	0.2	-0.3

 δ_s is the elasticity of substitution between high-skilled and low-skilled workers, δ_a is the elasticity of substitution between the old and the young, δ_m is the elasticity of substitution between natives and migrants, λ captures the extent of high-skill productivity spillover, and γ is the elasticity of labor supply.

		Tertiary	educated	Less educated				
		Old	Young	Old	Young			
	Ageing	-9.54	-4.31	2.45	6.81			
		(-24.04 ; -3.96)	(-26.77; 1.89)	(-1.15; 12.78)	(2.75; 12.98)			
base	Immigration	-0.86	-1.09	0.73	0.85			
	0	(-9.83; 1.33)	(-8.81 ; 2.1)	(-0.94; 7.18)	(-1.61; 7.84)			
	Emigration	1 25	0.94	-0 71	-0.86			
	Lingration	(-3.47:5.72)	(-3.43:7.43)	(-6.86:1.4)	(-5.19:0.88)			
	Ageing	-5.19	0.70	0.66	4.77			
	0	(-15.17; -0.31)	(-10.54; 3.51)	(-2.34; 7.45)	(2.13; 8.2)			
_	.							
$\sigma_s=5$	Immigration	-0.04	0.02	0.56	0.57			
		(-2.07; 0.95)	(-0.85; 1.17)	(-0.19; 3.84)	(-0.86; 4.5)			
	Emigration	1.40	0.84	-0.74	-0.84			
	0	(-3.54; 6.65)	(-3.39; 8.34)	(-6.62; 1.34)	(-5.41; 0.93)			
	Ageing	-9.66	-4.61	2.94	6.17			
		(-23.61; -4.06)	(-25.63; 0.81)	(-0.14 ; 12.58)	(2.4; 12.33)			
$\sigma = 7$	Immigration	-0.80	-1.03	0.76	0.88			
$o_a - i$	mingration	$(-9.3 \cdot 1.52)$	$(-8.74 \cdot 2)$	$(-1.06 \div 7.31)$	$(-1.48 \cdot 7.72)$			
		(0.0; 1.02)	(0.11 ; 2)	(1.00,1.01)	(1110,1112)			
	Emigration	1.19	0.99	-0.74	-0.84			
	_	(-3.44; 5.42)	(-3.45; 7.03)	(-6.62; 1.34)	(-5.41; 0.93)			
	Ageing	-9.48	-4.25	2.48	6.76			
		(-23.77;-3.91)	(-26.74; 1.84)	(-0.99; 12.78)	(2.68; 12.92)			
$\sigma_{m} = 1000$	Immigration	-1.38	-1.38	0.72	0.76			
0 11 2000	8-001011	(-12.4 ; 1.06)	(-10.54 ; 2.08)	(-1.24:6.94)	(-2.13; 7.82)			
					())			
	Emigration	1.22	0.91	-0.71	-0.86			
		(-2.83; 5.59)	(-2.67; 7.33)	(-6.82; 1.56)	(-5.16 ; 1.19)			
	Ageing	-10.88	-5.97	1.11	5.20			
		(-25.4; -5.26)	(-29.55; 1.41)	(-1.87 ; 9.07)	(2.26; 9.86)			
$\lambda = 0$	Immigration	-1.09	-1.28	0.59	0.67			
	0	(-11.35; 1.85)	(-11.01; 2.62)	(-0.42; 4.98)	(-1.1; 5.64)			
		× / //	× / /					
	Emigration	1.53	1.21	-0.43	-0.53			
		(-3.92; 7.44)	∮ 0 3.88 ; 8.28)	(-5.15; 0.95)	(-3.47; 0.43)			

Table 7: Extensions, median values (minimum; maximum)

 δ_s is the elasticity of substitution between high groups, δ_a is the elasticity of substitution between age groups, δ_m is the elasticity of substitution between natives and migrants, λ captures the extent of high-skill productivity spillover.

		Tertiary	educated	Less educated				
		Old	Young	Old	Young			
	Ageing	-9.55 (-24.04 ; -3.96)	-4.25 (-26.74; 1.84)	$2.45 \\ (-1.15 ; 12.78)$	6.76 (2.68; 12.92)			
$\sigma_{mo}=20 \& \sigma_{my}=\infty$	Immigration	-0.86 (-9.83 ; 1.33)	-1.38 (-10.54 ; 2.08)	0.73 (-0.94; 7.17)	0.76 (-2.13; 7.83)			
	Emigration	1.25 (-3.47; 5.72)	0.91 (-2.67; 7.33)	-0.71 (-6.86; 1.4)	-0.86 (-5.16; 1.18)			
	Ageing	-11.68 (-28.58 ; -3.73)	-3.23 (-29.39 ; 4.38)	2.94 (-0.14 ; 12.58)	$ \begin{array}{c} 6.17 \\ (2.40; 12.33) \end{array} $			
$\sigma_{ah}=3$ & $\sigma_{al}=7$	Immigration	-1.04 (-11.07; 0.90)	-1.22 (-8.97 ; 2.34)	0.76 (-1.06; 7.31)	$\begin{array}{c} 0.88\\ (-1.48\ ;\ 7.72)\end{array}$			
	Emigration	1.40 (-3.54; 6.65)	0.84 (-3.39; 8.34)	-0.74 (-6.62; 1.34)	-0.84 (-5.41; 0.93)			

Table 8: Allowing for elasticities of substitution to vary, wage effects (in %)

 $\overline{\delta_{mo}}$ is the elasticity of substitution between natives and immigrants among older workers, and δ_{my} is the corresponding elasticity among the young. δ_{ah} is the elasticity of substitution between younger and older high-skilled workers, and δ_{al} is the corresponding elasticity among the low-skilled.

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Appendix

At equilibrium, each type of labor is paid its marginal productivity, and as labor is the only factor of production, the share of total wage bill, θ_k , for any k group of workers can be expressed as: $\theta_k = \frac{w_k q_k}{y}$

such that:

(30)
$$\theta_{saj} = \frac{w_{saj}q_{saj}}{y} = w_{saj}\frac{q_{saj}}{y} = \frac{\partial q}{\partial q_{saj}}\frac{q_{saj}}{q}$$

(31)
$$\theta_{sa} = \frac{w_{sa}q_{sa}}{y} = w_{sa}\frac{q_{sa}}{y} = \frac{\partial q}{\partial q_{sa}}\frac{q_{sa}}{q}$$

and

(32)
$$\theta_s = \frac{w_s q_s}{y} = w_s \frac{q_s}{y} = \frac{\partial q}{\partial q_s} \frac{q_s}{q}$$

(6) implies that

(33)
$$\theta_{saj} \frac{\Delta q_{saj}}{q_{saj}} = \frac{\partial q}{\partial q_{saj}} \frac{\Delta q_{saj}}{q}$$

Equations 6 and 32 imply that:

(34)
$$\frac{\theta_{saj}}{\theta_s} \frac{\Delta q_{saj}}{q_{saj}} = \frac{\partial q_s}{\partial q_{saj}} \frac{\Delta q_{saj}}{q_s}$$

(6) and (31) implies that

(35)
$$\frac{\theta_{saj}}{\theta_{sa}} \frac{\Delta q_{saj}}{q_{saj}} = \frac{\partial q_{sa}}{\partial q_{saj}} \frac{\Delta q_{saj}}{q_{sa}}$$

with $j \in \{n, f\}$.

(33) equates to the first set of terms in (9), (34) equates to the second set of terms and (34) equates to the third set of terms. Substituting these equivalences into (9), we get the equilibrium conditions:

$$\hat{w}_{san} = \frac{1}{\delta_s} [\theta_{hof} \hat{q}_{hof} + \theta_{hyf} \hat{q}_{hyf} + \theta_{lof} \hat{q}_{lof} + \theta_{lyf} \hat{q}_{lyf}] + \frac{1}{\delta_s} [\theta_{hon} \hat{q}_{hon} + \theta_{hyn} \hat{q}_{hyn} + \theta_{lon} \hat{q}_{lon} + \theta_{lyn} \hat{q}_{lyn}] + (\frac{1}{\delta_a} - \frac{1}{\delta_s}) [\frac{\theta_{sof}}{\theta_s} \hat{q}_{sof} + \frac{\theta_{syf}}{\theta_s} \hat{q}_{syf}] + (\frac{1}{\delta_a} - \frac{1}{\delta_s}) [\frac{\theta_{son}}{\theta_s} \hat{q}_{son} + \frac{\theta_{syn}}{\theta_s} \hat{q}_{syn}] + (\frac{1}{\delta_m} - \frac{1}{\delta_a}) [\frac{\theta_{san}}{\theta_{sa}} \hat{q}_{san}] - \frac{1}{\delta_m} \hat{q}_{san} + \lambda \Delta f_h$$

Offsetting the wage and employment effects of aging with immigration; partial equilibrium condition

Ceteris paribus, if the only changes we observe are in the stock of immigrants of type saf and the stock of natives of type san, equations ?? and 13 entail the overall employment and wage effect to natives of type san is zero, in absence externality, iff

$$(37) \quad \frac{1}{\delta_s}\theta_{saf}\hat{Q}_{saf} + (\frac{1}{\delta_a} - \frac{1}{\delta_s})\frac{\theta_{saf}}{\theta_s}\hat{Q}_{saf} + (\frac{1}{\delta_m} - \frac{1}{\delta_a})\frac{\theta_{saf}}{\theta_{sa}}\hat{Q}_{saf} = \\ \frac{1}{\delta_s}\theta_{san}\hat{Q}_{san} + (\frac{1}{\delta_a} - \frac{1}{\delta_s})\frac{\theta_{san}}{\theta_s}\hat{Q}_{san} + (\frac{1}{\delta_m} - \frac{1}{\delta_a})\frac{\theta_{san}}{\theta_{sa}}\hat{Q}_{san} - \frac{1}{\delta_m}\hat{Q}_{san} - \frac{1}{\gamma}\hat{Q}_{san}$$

in other words

$$(38) \quad \hat{Q}_{saf}\left[\frac{1}{\delta_s}\theta_{saf} + \left(\frac{1}{\delta_a} - \frac{1}{\delta_s}\right)\frac{\theta_{saf}}{\theta_s} + \left(\frac{1}{\delta_m} - \frac{1}{\delta_a}\right)\frac{\theta_{saf}}{\theta_{sa}}\right] = \\ \hat{Q}_{san}\left[\frac{1}{\delta_s}\theta_{san} + \left(\frac{1}{\delta_a} - \frac{1}{\delta_s}\right)\frac{\theta_{san}}{\theta_s} + \left(\frac{1}{\delta_m} - \frac{1}{\delta_a}\right)\frac{\theta_{san}}{\theta_{sa}} - \frac{1}{\delta_m} - \frac{1}{\gamma}\right]$$

which implies

$$(39) \quad \hat{Q}_{saf}\left[\frac{1}{\delta_s}\theta_{saf} + \left(\frac{1}{\delta_a} - \frac{1}{\delta_s}\right)\frac{\theta_{saf}}{\theta_s} + \left(\frac{1}{\delta_m} - \frac{1}{\delta_a}\right)\frac{\theta_{saf}}{\theta_{sa}}\right] = \\ -\hat{Q}_{san}\left[\frac{1}{\gamma} + \frac{1}{\delta_m} - \frac{1}{\delta_s}\theta_{san} - \left(\frac{1}{\delta_a} - \frac{1}{\delta_s}\right)\frac{\theta_{san}}{\theta_s} - \left(\frac{1}{\delta_m} - \frac{1}{\delta_a}\right)\frac{\theta_{san}}{\theta_{sa}}\right]$$

that is

$$(40) \quad \hat{Q}_{saf} = -\hat{Q}_{san} \frac{\frac{1}{\gamma} + \frac{1}{\delta_m} - \theta_{san} \left[\frac{1}{\delta_s} + \left(\frac{1}{\delta_a} - \frac{1}{\delta_s}\right) \frac{1}{\theta_s} + \left(\frac{1}{\delta_m} - \frac{1}{\delta_a}\right) \frac{1}{\theta_{sa}}\right]}{\frac{1}{\delta_s} \theta_{saf} + \left(\frac{1}{\delta_a} - \frac{1}{\delta_s}\right) \frac{\theta_{saf}}{\theta_s} + \left(\frac{1}{\delta_m} - \frac{1}{\delta_a}\right) \frac{\theta_{saf}}{\theta_{sa}}}$$

$$(41) \quad \hat{Q}_{saf} = -\hat{Q}_{san} \frac{\frac{1}{\gamma} + \frac{1}{\delta_m} - \theta_{san} [\frac{1}{\delta_s} (1 - \frac{1}{\theta_s}) + \frac{1}{\delta_a} (1 - \frac{1}{\theta_{sa}}) + \frac{1}{\delta_m} \frac{1}{\theta_{sa}}]}{\frac{1}{\delta_s} \theta_{saf} + (\frac{1}{\delta_a} - \frac{1}{\delta_s}) \frac{\theta_{saf}}{\theta_s} + (\frac{1}{\delta_m} - \frac{1}{\delta_a}) \frac{\theta_{saf}}{\theta_{sa}}}$$

The above equation tells us that first the net inflow of migrants need to be in the opposite direction as the change in the size of native workers of a given type as long as the numerator is positive, which is the case unless the elasticities of substitution between skill or age groups are far greater than the elasticity of substitution between natives and immigrants. Second, the magnitude of the inflow of immigrants required depends on the share of total labor wage bill going the type of native workers in question, weighted by the share of total labor wage bill going to migrants workers of the same type. Both quantities are weighted by elasticities of substitutions at the various nests of the production function.

Equilibrium change in quantity of labor demand for each worker type

$$\hat{q}_{hon}^{*} = \begin{bmatrix} \beta_{lyn}\beta_{hyn}(\pi_{lon} - \alpha_{lon}) + \beta_{lon}\beta_{hyn}(\pi_{lyn} - \alpha_{lyn}) + \alpha_{hyn}(\alpha_{lon}\alpha_{lyn} - \pi_{lyn}\pi_{lon}) \\ \beta_{lyn}\beta_{hyn}(\alpha_{lon} - \pi_{lon}) + \beta_{lon}\beta_{hyn}(\alpha_{lyn} - \pi_{lyn}) + \pi_{hyn}(\pi_{lyn}\pi_{lon} - \alpha_{lon}\alpha_{lyn}) \\ (\alpha_{hyn} - \pi_{hyn})(\beta_{lyn}\pi_{lon} - \beta_{lon}\alpha_{lyn}) \\ (\pi_{hyn} - \alpha_{hyn})(\beta_{lyn}\alpha_{lon} - \beta_{lon}\pi_{lyn}) \end{bmatrix} \begin{bmatrix} \Theta_{hon} \\ \Theta_{hyn} \\ \Theta_{lon} \\ \Theta_{lyn} \end{bmatrix} \Omega^{-1}$$

$$\hat{q}_{hyn}^{*} = \begin{bmatrix} \beta_{lon}\beta_{hon}(\alpha_{lyn} - \pi_{lyn}) + \beta_{lyn}\beta_{hon}(\alpha_{lon} - \pi_{lon}) + \pi_{hon}(\pi_{lyn}\pi_{lon} - \alpha_{lon}\alpha_{lyn}) \\ \beta_{lon}\beta_{hon}(\pi_{lyn} - \alpha_{lyn}) + \beta_{lyn}\beta_{hon}(\pi_{lon} - \alpha_{lon}) + \alpha_{hon}(\alpha_{lon}\alpha_{lyn} - \pi_{lyn}\pi_{lon}) \\ (\alpha_{hon} - \pi_{hon})(\beta_{lyn}\pi_{lon} - \beta_{lon}\alpha_{lyn}) \\ (\pi_{hon} - \alpha_{hon})(\beta_{lyn}\alpha_{lon} - \beta_{lon}\pi_{lyn}) \end{bmatrix}' \begin{bmatrix} \Theta_{hon} \\ \Theta_{hyn} \\ \Theta_{lon} \\ \Theta_{lyn} \end{bmatrix} \Omega^{-1}$$

$$\hat{q}_{lon}^{*} = \begin{bmatrix} (\pi_{lyn} - \alpha_{lyn})(\alpha_{hyn}\beta_{hon} - \pi_{hon}\beta_{hyn}) \\ (\pi_{lyn} - \alpha_{lyn})(\alpha_{hon}\beta_{hyn} - \pi_{hyn}\beta_{hon}) \\ \beta_{lyn}\beta_{hyn}(\pi_{hon} - \alpha_{hon}) + \beta_{lyn}\beta_{hon}(\pi_{hyn} - \alpha_{hyn}) + \alpha_{lyn}(\alpha_{hon}\alpha_{hyn} - \pi_{hyn}\pi_{hon}) \\ \beta_{lyn}\beta_{hyn}(\alpha_{hon} - \pi_{hon}) + \beta_{lyn}\beta_{hon}(\alpha_{hyn} - \pi_{hyn}) + \pi_{lyn}(\pi_{hyn}\pi_{hon} - \alpha_{hon}\alpha_{hyn}) \end{bmatrix}' \begin{bmatrix} \Theta_{hon} \\ \Theta_{hyn} \\ \Theta_{lon} \\ \Theta_{lyn} \end{bmatrix} \Omega^{-1}$$

$$\hat{q}_{lyn}^{*} = \begin{bmatrix} (\alpha_{lon} - \pi_{lon})(\pi_{hon}\beta_{hyn} - \alpha_{hyn}\beta_{hon}) \\ (\alpha_{lon} - \pi_{lon})(\pi_{hyn}\beta_{hon} - \alpha_{hon}\beta_{hyn}) \\ \beta_{lon}\beta_{hon}(\alpha_{hyn} - \pi_{hyn}) + \beta_{lon}\beta_{hyn}(\alpha_{hon} - \pi_{hon}) + \pi_{lon}(\pi_{hyn}\pi_{hon} - \alpha_{hon}\alpha_{hyn}) \\ \beta_{lon}\beta_{hon}(\pi_{hyn} - \alpha_{hyn}) + \beta_{lon}\beta_{hyn}(\pi_{hon} - \alpha_{hon}) + \alpha_{lon}(\alpha_{hon}\alpha_{hyn} - \pi_{hyn}\pi_{hon}) \end{bmatrix}' \begin{bmatrix} \Theta_{hon} \\ \Theta_{hyn} \\ \Theta_{lon} \\ \Theta_{lon} \\ \Theta_{lyn} \end{bmatrix} \Omega^{-1}$$

The expression for Ω^{-1} is given in the main text, in equation (22).