Zipf's law, Gibrat's law and international migration *

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This note details recent efforts to appraise the existence of Zipf's Law and Gibrat's Law when applied to international migration. In comparison with previous work, we examine aggregate and high-skilled immigration and emigration in levels and as percentages of the population. We provide evidence that a number of statistical regularities, including Zipf's Law, hold, which although often documented elsewhere in a variety of different contexts, nevertheless provide several statistical curiosities. These are interesting not least because the theoretical models used to explain Zipf's and Gibrat's laws elsewhere are not truly applicable when the unit of observation is a sovereign country, the borders of which are arguably controlled by the Government.

In urban economics, Zipf's Law and Gibrat's Law occupy a special place in explaining the relationships between cities in terms of their relative sizes and growth rates. The role of this note is to establish some stylized facts and preliminary analysis to link international migration to urban economics, especially the two issues of population growth and distribution. More specifically, we draw on two of the most recent advances in the development of bilateral migration data, Özden et al. (2011) and Artuç et al. (2014), to examine the relevance of Gibrat's and Zipf's Laws for immigrant and emigrant numbers and densities in aggregate, as well as examining migration patterns for the high-skilled.

Zipf's law for cities states that the city size distribution within a country can be approximated by a power law distribution. More specifically, if we were to rank the cities in terms of their sizes, the nth largest city is 1/n of the size of the largest city. Another way to look at this rule is by running a regression with the natural log of *city size rank* as the dependent variable and the natural log of city size as the main explanatory variable. A coefficient of 1 provides evidence that Zipf's Law holds. This is typically the case with a high level of precision, especially when larger cities are considered.

Gibrat's law, on the other hand, is about growth rates and was initially noted for French firms (Gibrat, 1931). When applied to cities, it states that growth processes have a common mean and are independent of initial sizes. The seminal paper of Gabaix (1999) establishes that, when cities grow according to Gibrat's law, then, in steady state, their size distribution will follow a Pareto distribution with a power exponent of one. This, as well as numerous results from across the urban economics literature, which also show that Zipf's Law holds, provides some evidence against the applicability of Gibrat's Law, which predicts that the resulting distribution is log-normal. These patterns are observed across the world and are discussed in great detail in many other places (such as Gabaix, 2004).

The natural question then is to ask what mechanisms exist that might link Gibrat's and Zipf's laws. This question is especially important when divergent rates of population growth across locations are the norm. Among the causes that would lead to differing growth rates are climate, natural disasters and resource endowments. There are numerous examples of cities that disappear from history or boom after the discovery of a natural resource. People however, have the ability to move from one location to another, especially when some locations become overcrowded and resources are restrained due to higher natural growth rates. Under these circumstances, the convergence of growth rates via migration would naturally take place within a larger geographic area (such as a country) with no internal barriers to population mobility.

There have been few studies that take the analysis beyond national boundaries since the theoretical models used to explain Zipf's and Gibrat's laws are not truly applicable when the unit of observation is a sovereign country. Governments exercise considerable power over their international (as opposed to internal) borders and can dictate who can enter and, to a certain extent, exit the country. The most prominent study is Rose (2005), who tests these theories using countries as the relevant geographical entities. He concludes that the `hypothesis of no effect of size on growth usually cannot be rejected'. When testing for Zipf's law for countries, he shows that it also strongly holds in the upper tail of the distribution of country sizes, as is the case with city sizes. The only study that links Gibrat's and Zipf's laws to international immigration is the work by Clemente et al. (2011), which uses aggregate immigration numbers and densities and finds, for the biggest 50 countries, that Zipf's Law holds only for immigrant numbers.

We draw upon two recent bilateral migration databases, Özden et al. (2011) and Artuç et al. (2014). These yield two advantages. First, global bilateral migration data allow us to calculate country level emigrant stocks, as opposed

4

to simply immigrant stocks. Secondly, the latter database, which reports bilateral migration stocks by education levels, allows us to further delineate between patterns of migrants with low and high education levels.

Zipf's law implies a concentrated distribution of the total population among a few large cities. We observe similar patterns in immigration patterns but the converse over time in emigration patterns. In 2000, the top ten receiving countries accounted for no less than 57 percent of the world migrant stock, which is approximately equivalent to the total emigrant stocks of the top 25 emigration countries in the same decade. In 1960, the figure for the top ten receiving nations was 54 percent, which is equivalent to less than the total emigrant stock of the top 9 sending countries in 1960. A similar concentration is observed for individual corridors. In 2000, 90 percent of all bilateral corridors (over 40,000 in total) comprised fewer than 50 migrants each. Together, they accounted for only 0.1 percent of total migrant stock. On the other hand, in the same year, just 505 corridors accounted for over 80 percent of the global migrant stock.

Whether the sum of these bilateral trends holds across countries is the subject of our analysis. We delve deeper into the observed patterns of international migration over the period 1960-2000, by examining the existence of Zipf's and Gibrat's Laws, two empirical regularities that are ubiquitous in the urban economics literature. We therefore adopt an alternative perspective in analyzing to what extent the underlying trends in global migration patterns are converging or diverging.

The results are both interesting and encouraging. With regards to Zipf's law, we find that it holds in the upper tail of the distribution with the Pareto coefficient very close to unity for both immigration and emigration for all time periods. The results are less precise for high skilled migration but even so we cannot reject the hypothesis of the coefficient being unity. When the whole sample is included, we find a

log-normal distribution for immigrant numbers, which are very similar to the results found for cities and countries in the literature. With respect to Gibrat's law, our results are somewhat less uniform. We find evidence of convergence with the growth rates being linked negatively to initial levels for both aggregate and high-skilled immigration and emigration numbers and densities. In our non-parametric analysis, we find some evidence in favor of Gibrat's Law holding for immigration stocks, i.e. that the growth in stocks is independent of their initial values and stronger evidence that immigration densities are diverging over time. Conversely, emigrant stocks are converging in the sense that countries with smaller emigrant stocks are growing faster than their larger sovereign counterparts. These are all surprising regularities given that government policies and other physical and cultural barriers impose strong restrictions on international migration patterns.

▶ Zipf's law in migration

We begin our analysis with an examination of the existence of Zipf's Law, the rank-size rule, which describes the (upper tail) of the population distribution of the geographical entity under investigation. Typically the existence of Zipf's Law is analyzed graphically and/or using regression techniques. Both approaches first rank the (population) size variable of interest S, from highest to lowest. Then the natural logarithm of the rank variable is analyzed with respect to the natural logarithm of the (population) size variable.

The left hand two panels in Figure 1 show the graphical scatter plots of the natural logarithms of the rank of *total* immigrant and emigrant levels for the 50 countries in the top tail of the various distributions. Although we only plot these for the year 2000, these graphs are also representative of other years. The data in the right hand two panels refer to high-skilled migrants, defined as having completed at least

one year of tertiary education. Clear linear trends are evident, but the line imposed demonstrates that some deviations from Zipf's Law clearly exist in the data. For a more robust analysis, simple regression analysis is performed the details of which are presented in the background paper of this note. The results from the regression analysis show that the Pareto coefficient is remarkably close to unity for the 50 largest countries in the upper tail of the size distribution, although Zipf's Law is resolutely rejected across the entire distribution since the coefficient is 0.385 for the year 2000. Turning to the high-skill migration numbers, the point estimates on the Zipf coefficient are substantively different from unity but still within the confidence intervals.

When we consider the entire distribution of bilateral migrant stocks, we easily reject Zipf's law. The question is whether the distribution would satisfy log-normality assumptions. This would imply that Gibrat's law held and that an independent growth process might lead to this distribution. Our empirical analysis shows that both immigrant numbers and densities are often log-normally distributed.

► Gibrat's law in migration

As opposed to the static analysis presented when discussing Zipf's Law, an examination of Gibrat's Law with growth rates, requires a dynamic analysis of global migration movements. Should the growth of geographical entities be independent of their size, their growth will subsequently result in a log-normal distribution. Of course, this does not mean that a log-normal distribution implies that Gibrat's Law necessarily holds. Both immigration and emigration are determined by government policies and limited to the extent by which individuals are willing and able to move. Internal mobility, on the other hand, is far less restricted and is typically thought to follow Zipf's and Gibrat's laws. Thus, it is difficult to argue that some underlying law of nature might determine the underlying

growth and distribution processes of populations over national boundaries and across the globe.

In our empirical analysis, we estimate parametric and non-parametric kernel regressions to test for the applicability of Gibrat's Law. These regress the size of migrant populations (or in our case densities) on their growth. Across all decades and across all measures, the parametric regression results are negative and statistically significant or else statistically insignificant. These results are indicative of convergence over time or indeed of Gibrat's Law holding. The major exceptions are those countries in the lower end of the distribution, which tend to grow faster than the other countries. The composition of high skilled immigrant and emigrant levels and densities exhibit signs of convergence, albeit to differing degrees. These patterns are consistent with the onset of the global competition for international talent, an increasing global supply of high-skilled workers across all origins, limited supplies per origin as well as the imposition of selective immigration policies by an increasing number of destination countries.

5

Figure 1. Zipf Plots for Aggregate and High-skilled Immigrants and Emigrants, 2000
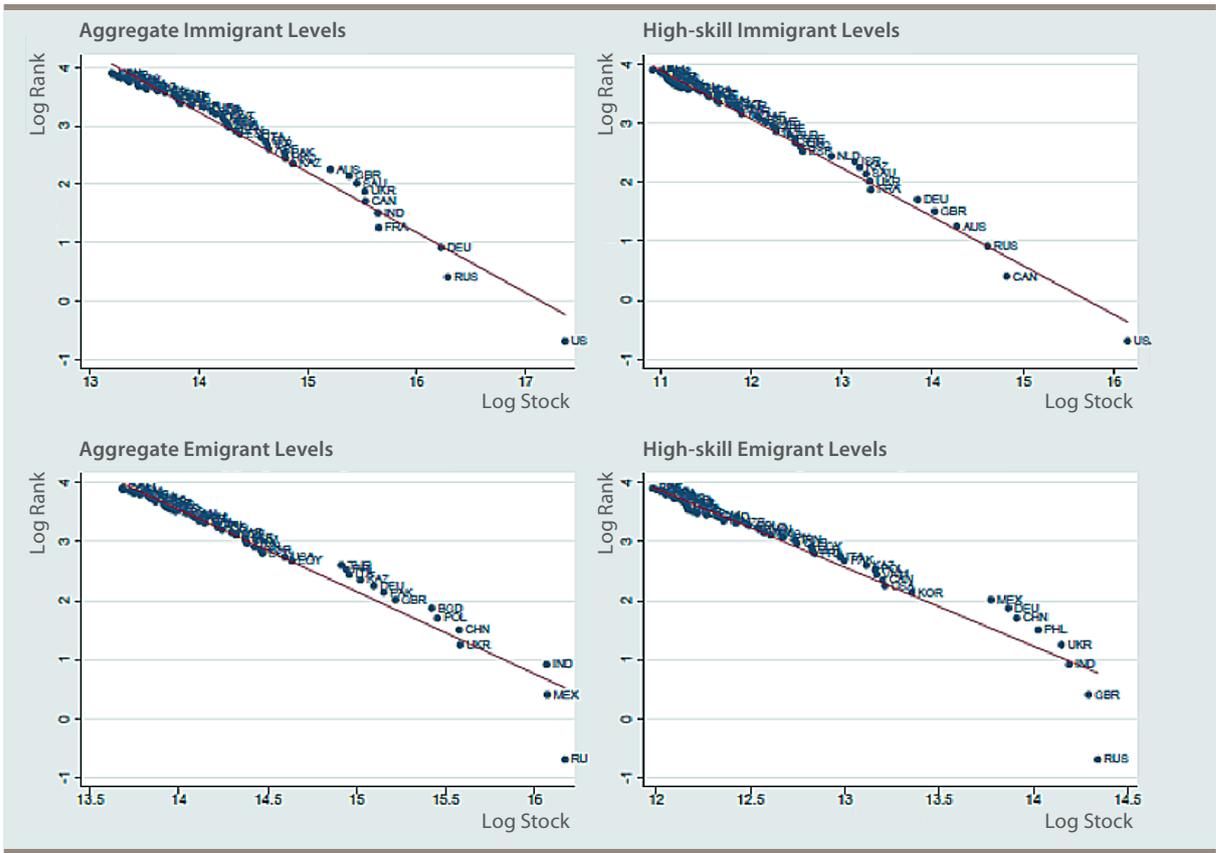
▶ Conclusion

Gibrat's and Zipf's laws are among the most studied and well established phenomenon in various contexts including linguistics, firm sizes and urban agglomerations. The linkages between population growth and the distribution across geographic space are key to all economic analysis since economic activity cannot be analyzed in isolation from location. Thus, it is important to study and identify the underlying processes that determine the growth rates of populations over time and their allocation across various locations. Even though Zipf's and Gibrat's laws have been extensively analyzed in the literature, there are fewer studies on the role of population movements and there are even fewer studies that focus on mobility across national boundaries. It is therefore natural to search for a relationship between such population laws and migration patterns.

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101
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