

# Quality, Quantity and Duration of Lives

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## Abstract

The evaluation of development processes and of public policies often involves comparisons of social states that differ in income distributions, population sizes and life longevity. This may require social evaluation principles to be sensitive to the quality, the quantity and the duration of lives. This paper 1) reviews some of the normative issues at stake, 2) proposes and discusses some specific methods to address them in a generalized utilitarian framework, and 3) briefly illustrates the application of some of these methods to the global distribution of incomes, population sizes and longevity over the last century. Depending on the approach taken, it is found *inter alia* that global social welfare in 2010 can be deemed to be between 1.8 and 407 times that of 1910, the role given to the quantity of lives being particularly important in that assessment.

**Keywords:** Global welfare; Critical-level utilitarianism; Social evaluation; Longevity; Life expectancy; Population size.

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# 1 Introduction

Much of economic analysis and social evaluation involves an often implicit trade-off between population sizes and “representative” (or *per capita*) welfare. A common example is whether we should give precedence to total GDP or to *per capita* GDP when providing measures of economic growth and when making announcements of recessions/recoveries.<sup>1</sup> More generally, when comparing society’s welfare across time, it is possible to observe both a deterioration in average welfare and an increase in total welfare (through an increase in the number of individuals). The same is true for comparisons of social “illfare” (such as poverty): it is possible to witness simultaneously a fall in *per capita* illfare (or indices of representative poverty) as well as an increase in total illfare (through an increase in the number of individuals). Quantifying the social impact of shocks and policies can also depend on the importance given to population size. Some shocks/policies can for instance increase average welfare but at the cost of a smaller population. This is true, for example, of poverty-correlated mortality rates (by which the rich are more likely to survive some shocks, leading to lower *per capita* poverty but possibly also to lower total welfare) and of policies that lower population size but that may increase *per capita* income (such as policies on contraception, abortion, euthanasia, or immigration/emigration).

The question of which weight should be given to population size in assessing social welfare and in trading off quality and quantity is at the core of the “optimal population problem”.<sup>2</sup> The study of this quality/quantity problem has a long

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<sup>1</sup>The direction of the change in total/*per capita* GDP was opposite in 24 quarters during the last fifty years, based on quarterly *per capita* and total GDP obtained from OECD national accounts data and UN population data.

<sup>2</sup>Much has also been said on what is called the “Principle of population”, a term drawn from the title of Malthus’ famous book (Malthus 1798). Malthus’ basic argument — which was predated by 18th-century economists such as Mirabeau (1756) and Quesnay (1778) — is that a geometric progression of the population cannot be sustained in a world in which resources grow linearly, and will therefore be stopped by the “iron law” that draws wages to subsistence levels. The more modern literature has nevertheless suggested that the quantity and the quality of lives may be complementary, and not necessarily substitutes, at least in some contexts; see for instance Boserup (1965), Kremer (1993), Alesina and Spolaore (1997) and Acemoglu, Johnson,

history; the title of one of de Sismondi (1819)'s chapters (Book 7, Chapter 4) is "What population increase is desirable for a nation?", and, half a century before Malthus' "Principle of population", Cantillon (1959 (1755)) asks (without answering the question) "whether it is better to have a great multitude of inhabitants, poor and badly provided, than a smaller number, much more at their ease: a million who consume the produce of 6 acres per head or 4 millions who live on the product of an acre and a half" (Book 1, chapter 15).

Another dimension of individual and social welfare that has gained particular prominence over the more recent decades is that of life longevity. Incorporating longevity into evaluations of social welfare is indeed in line with recent academic and social advances in measuring development; the objectives of development policy have shifted somewhat from the traditional objective of income and economic growth towards broader human development goals. Health and longevity have been salient elements of this shift, as exemplified in UNDP's first Human Development Report:

"The objective of development is to create an enabling environment for people to enjoy *long*, healthy and creative lives." (Mahbub ul Haq, instigator of UNDP's Human Development Reports, UNDP 1990; our emphasis)

The traditional optimal population problem trades off the number of individuals living at any particular time with the representative welfare of those individuals; an "optimal longevity problem" would analogously be concerned both with the number of years lived by individuals and with the periodic welfare of those years. As in the above quality/quantity trade-off, we might want to consider sums or averages. For instance, is development increasing average welfare over a lifetime? Or is it only increasing total welfare over the lifetime? Qualitatively, what should we say when periodic and total measures evolve in opposite directions, as when lifetimes become longer but with a greater number of years in poor health

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and Robinson (2005).

— or with lower levels of living standards? Quantitatively, even when the two average and total measures move in the same direction, do they change equally rapidly?

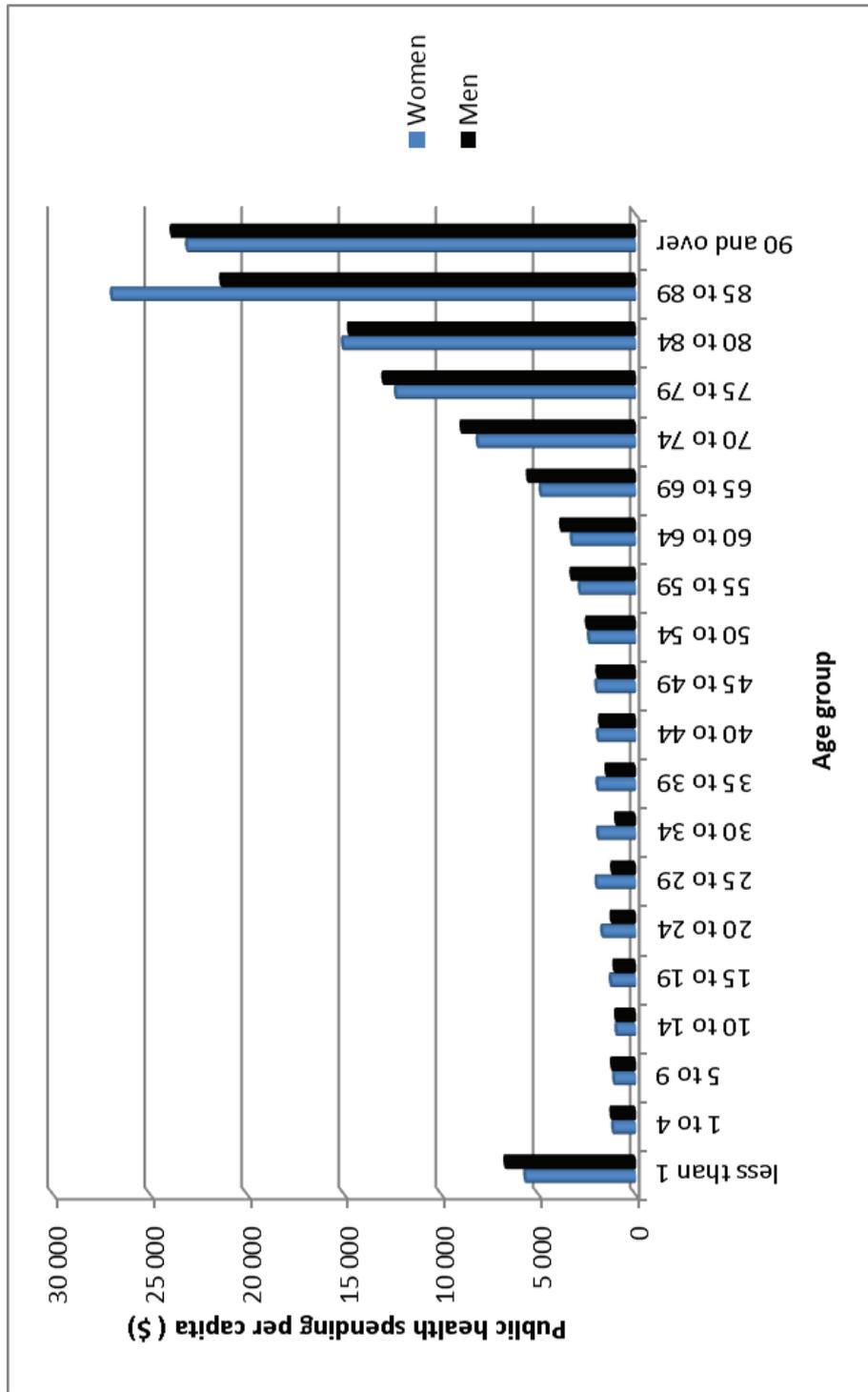
It therefore seems reasonable to admit that there can be social and individual trade-offs in attempting to increase both the duration of lives and the quality of the years actually lived. Figure 1 displays the distribution of public healthcare expenditures by age in Canada (using Canadian Institute for Health Information 2012 data). The Figure shows the marked increase in health-care spending as individuals age beyond 60 years old. Significant resources are also spent on healthcare just before the end of life: “We end up spending about a third of our overall health care resources in the last year of life” (Harding 2010). Some of these resources could presumably be spent for other socially valuable purposes, such as improving the living standards of younger lives:

“More than 10 million children under age 5 still die each year — that’s almost 30,000 a day — almost all in developing countries. Most of these children die from diarrhea, pneumonia, malaria and measles, all of which can be prevented or treated. (...) The tools that can save these lives are not expensive. For example, antibiotics to treat pneumonia can cost as little as 15 cents. A child can be immunized against six major childhood diseases for as little as \$15 and a one-year dose of vitamin A capsules costs just a few cents.” (Save the Children 2005, pp. 1-2)

These sorts of trade-offs are particularly important in the context of the aging of the population and in that of the public finance pressures felt in most of Canada’s provinces. For instance, under reasonable projection scenarios, Quebec’s public health care spending as a percentage of total provincial revenue (with current fiscal parameters) is set to approach 70% (from a current level of 43%) in 2030, *viz*, in less than 15 years — see Clavet, Duclos, Fortin, Marchand, and Michaud (2013). The emergence of such pressures would appear to make it even

more important to set the allocation of resources in an explicit trade-off between life quality and life duration.

Figure 1: *Per capita* public health expenditures, by age group (2010)



Source: Canadian Institute for Health Information (2012)

## Objectives and overview

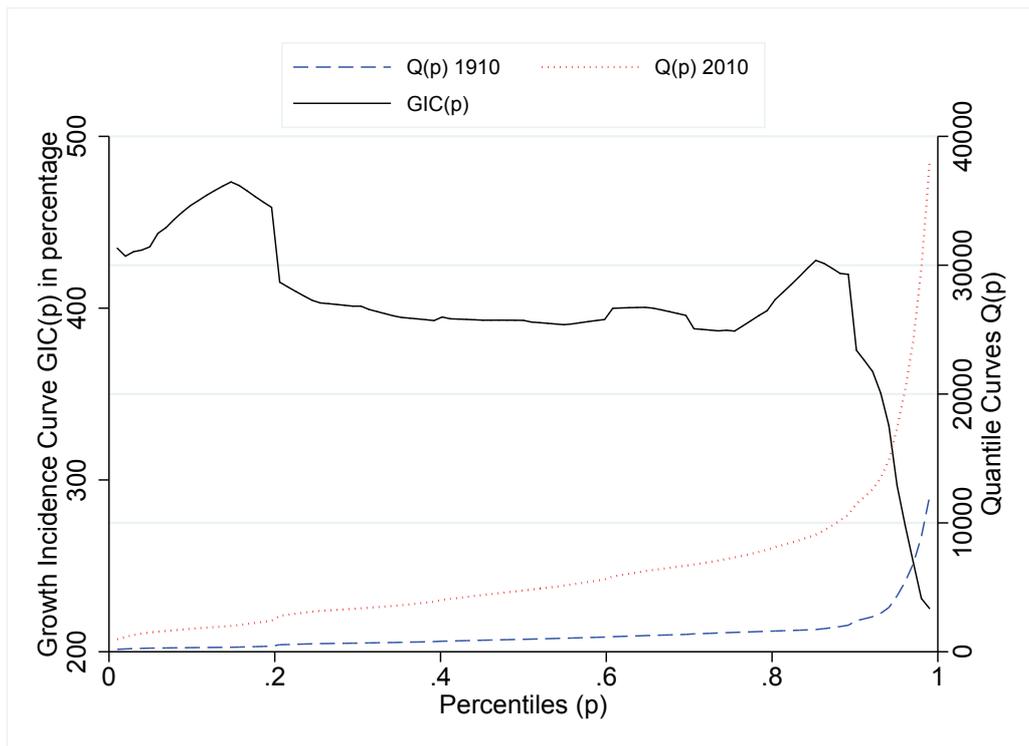
The paper has three main objectives. First, it reviews briefly some of the normative issues at stake in making social trade-offs between the quantity, the duration and the (usual) quality of life dimensions. Second, it proposes and discusses relatively simple methods to incorporate quantity and duration of lives into a social evaluation function. Third, it assesses the empirical importance of those three dimensions in the evolution of global social welfare in the last century.

Before turning to the analytical core of the paper, however, it is useful briefly to take stock of some of the considerable changes that humanity has seen over the last century in each of the dimensions of life quality, life quantity and life duration. (The data sources and data procedures are explained in Section 4.) All three dimensions — the quality, the quantity and the duration — of lives have changed considerably. The right vertical axis of Figure 2 shows the global income quantiles (denoted as  $Q(p)$ ) for 1910 and 2010 at different percentiles of the global population (denoted as  $p$ ). Incomes are in 1990 purchasing-power-adjusted USD, as is the case of all income statistics quoted in this paper (except for the statistics shown in Table 1). The left vertical axis shows the percentage increase (given by the so-called growth incidence curve,  $GIC(p)$ ) observed at each percentile over the past century. Hence, global median income ( $Q(0.5)$ ) was about \$970 in 1910 and \$4700 in 2010. Incomes have clearly undergone a considerable increase at all percentiles over the last century. The percentage increases in quantiles range from about 300% to 500% according to percentiles and are largest for the lowest percentiles.

The change in the quantity of lives is no less striking. The human population is much larger in 2010 (6.9 billion) than it was in 1910 (1.8 billion). Figure 3 shows the global age pyramids for 1910 and 2010. Not only has global population size changed, but the shape of the global pyramid has also evolved significantly over time, with a significantly lower global proportion of younger individuals and with evident population aging.

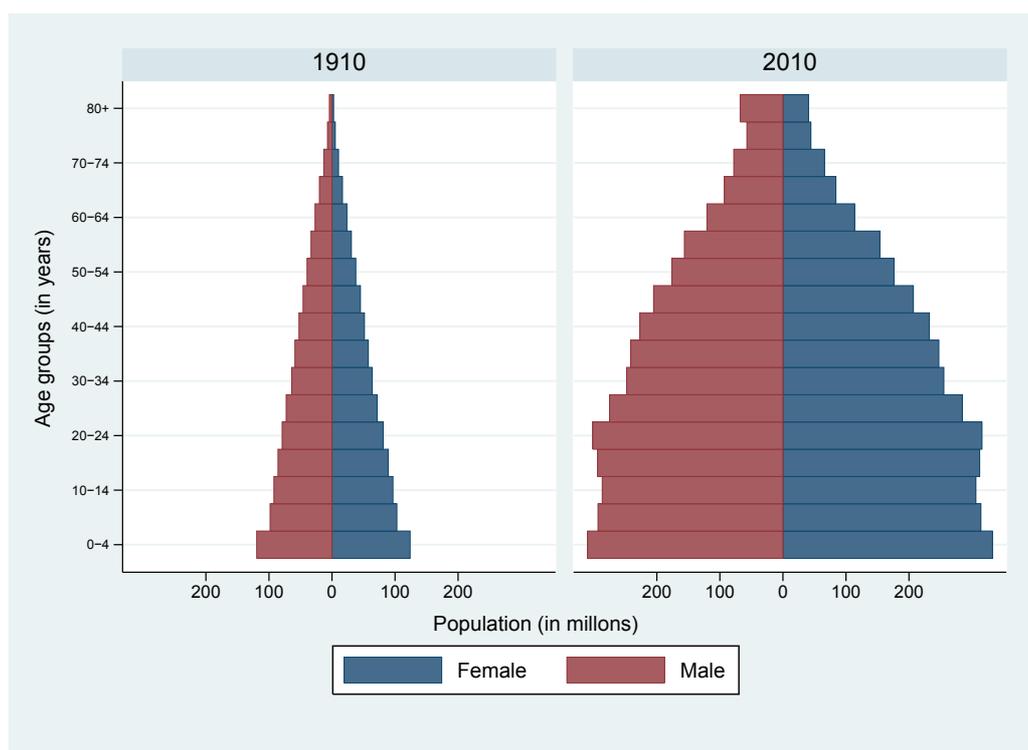
Figure 4 shows the heterogeneity of regional population pyramids between

Figure 2: 1910-2010 income quantiles (1990 USD PPP): levels ( $Q(p)$ ) and total percentage changes ( $GIC(p)$ ) between 1910 and 2010 at different percentiles  $p$



1950 and 2010. Both the less developed and the more developed regions<sup>3</sup> have aged over the last 60 years, with more developed regions aging more rapidly. A clear exception to this aging phenomenon is Sub-Saharan Africa, whose total population size has expanded fivefold over the 1950-2010 period (at an average annual growth rate of 2.6%) and where signs of population aging have not yet become evident.<sup>4</sup>

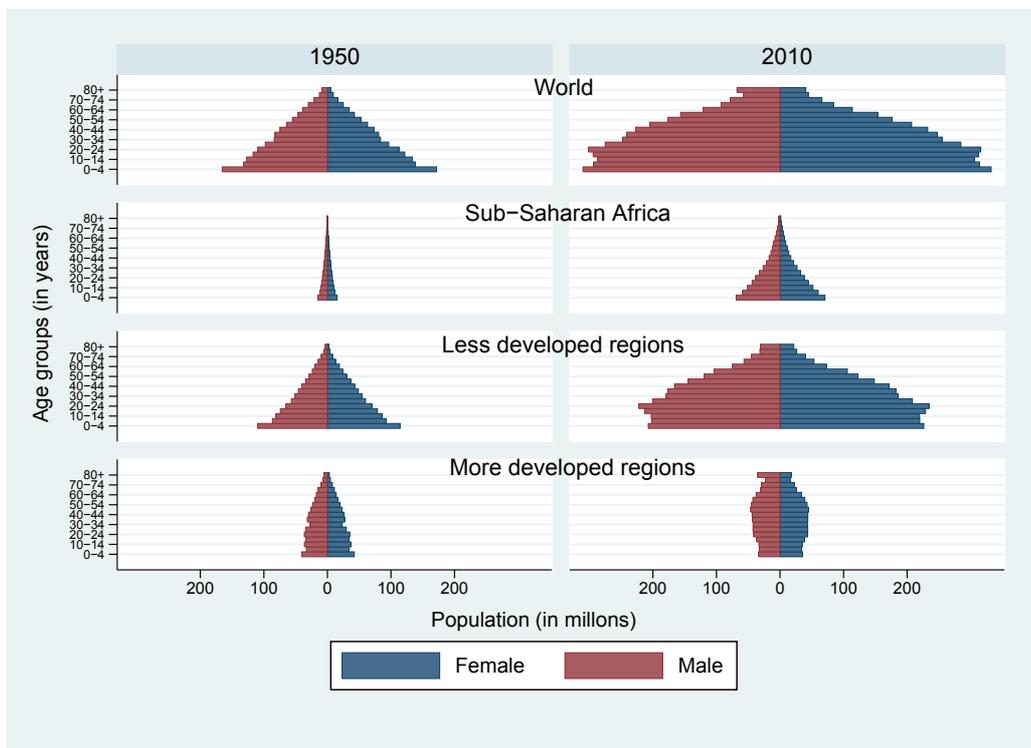
Figure 3: A major global demographic transition



<sup>3</sup>The more developed regions include all countries of Europe plus Northern America, Australia/New Zealand and Japan. The less developed regions include all regions of Africa, Asia (excluding Japan), Latin America and the Caribbean, plus Melanesia, Micronesia and Polynesia (see the UN definition of regions: <http://esa.un.org/wpp/excel-Data/country-Classification.pdf>).

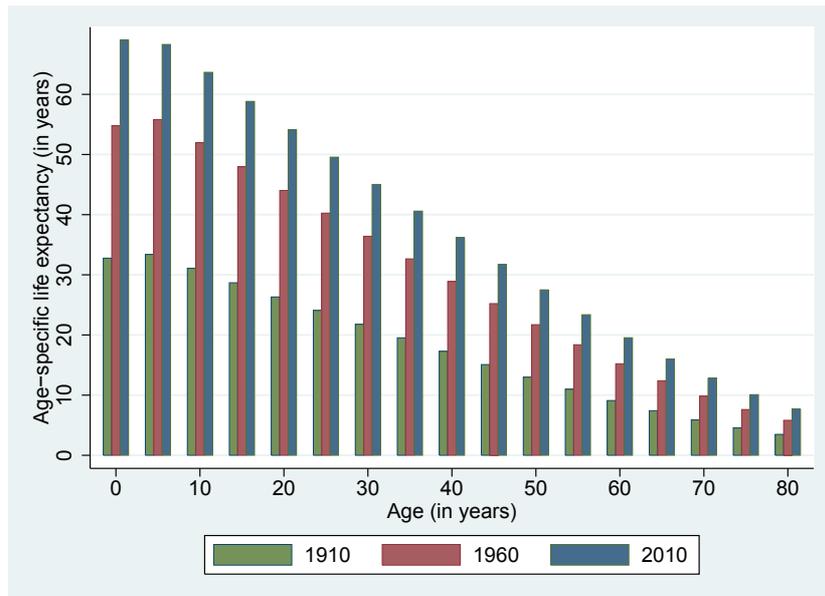
<sup>4</sup>The narrowing of the bottom of the pyramids is an outcome of changes in replication rates in the latter half of the 20th century (see for instance references in Gomez and Foot 2003) and is largely a reflection of individual choice regarding fertility; it could also be interpreted as societal choice regarding quantity of life.

Figure 4: Regional demographic transitions have differed



There have also been large changes in life duration over the last century. Figure 5 displays the evolution of global age-specific life expectancy in 1910, 1960 and 2010. Much of the increase of life expectancy has taken place between 1910 and 1960. The effect of the fall in child mortality has been particularly important, as can be seen by the relatively greater increase in life expectancy at younger ages.

Figure 5: Global age-specific life expectancy



Figures 6 and Figure 7 show the trends and the annual growth rates by decade over the last century in the quality, quantity and duration of lives. Population size and income *per capita* display similarly rapid evolutions, with an average yearly increase of the order of 1.4%. Income growth rates are the highest post-second-World-War; population size growth rates are also the largest in the latter half of the 20th century. Life expectancy shows slower growth of the order of 0.7% per year and increases more rapidly at the beginning of the 20th century.

Figure 6: *Per capita* GDP, population size and life expectancy, 1910-2010

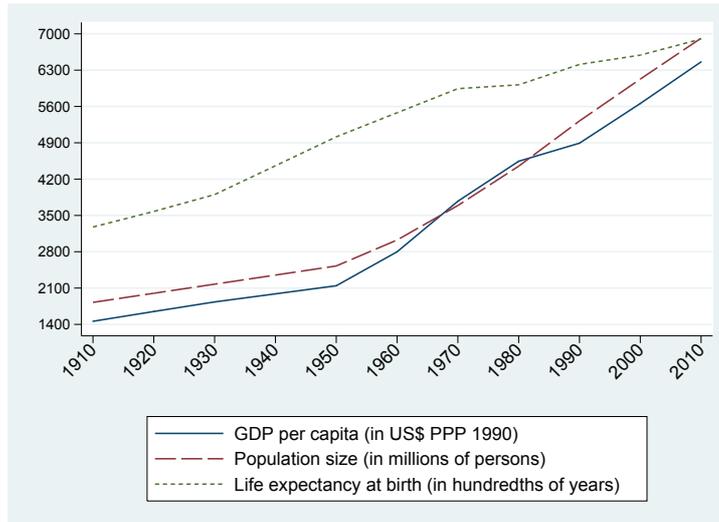
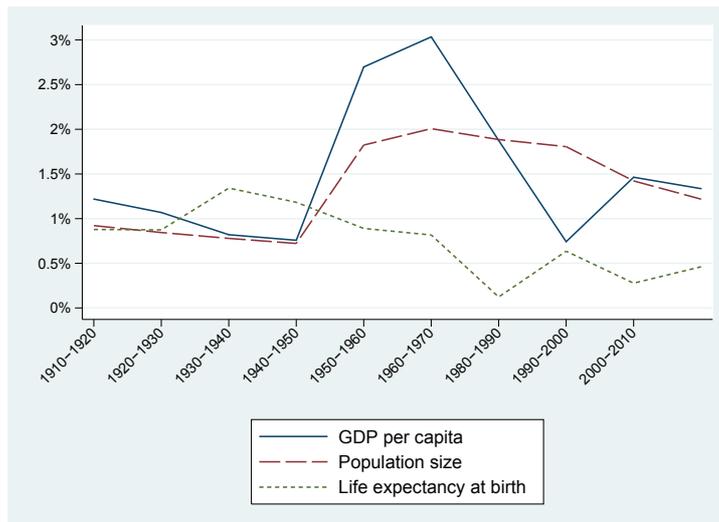


Figure 7: Percentage changes in global *per capita* income, population size and life expectancy, 1910-2010



## 2 “Sizes” in welfare economics

Overall, therefore, the world has changed significantly over the last century in the traditional welfare dimension of the quality of lives (as measured by income)

as well as in two size dimensions, the quantity and the duration of lives. As mentioned above, the consideration of sizes in economics has been historically approached both from a positive and a normative perspective. It is in a normative context that this paper is set; although positive inputs — in the form of causal and empirical relationships between the dimensions of quality, quantity and duration of lives — are necessary for policy guidance, an explicit normative framework is also essential in order to be able to solve the ‘optimal population problem’.

## 2.1 Average and total utilitarianism

Historically, there have been two opposite views on the “ideal” population size. The first view, associated most often to Malthus, has insisted that the optimal population size is small; the State should limit population growth in order to sustain an adequate *representative* level of living standards for humanity, given the earth’s limited resources. Maximizing that representative level of living standards, most often captured by *average* income, is the objective function of most social evaluation optimization exercises in the welfare economics literature (see Say 1840 for an influential supporter), and is in particular the rationale for the use of average utilitarianism as the objective function of the State.

To see this more clearly, let  $N$  individuals form a population with a distribution of individual welfare denoted as  $\mathbf{y} := (y_1, \dots, y_N)$ , where  $y_i$  is individual  $i$ ’s measure of welfare (or utility, which could be income in a simple case). Let  $g(y_i)$  be a transformation of  $y_i$ , most often referred to as  $i$ ’s contribution to social utility in the welfare economics literature. Average (generalized) utilitarianism is then given as

$$W^A = N^{-1} \sum_{i=1}^N g(y_i). \quad (1)$$

(1) boils down to average utility when  $g(y) \equiv y$ .

An “average” formulation of the type seen in (1) is the foundation of most of welfare economics. Welfare economics indeed almost always (and typically im-

explicitly) postulates Dalton’s population principle,<sup>5</sup> which says that an income distribution  $y$  and its  $r$ -times replication (for an arbitrary integer  $r$ ) must yield identical levels of social welfare (as well as inequality and poverty): population sizes do not matter *per se* in traditional social evaluation exercises. Classic foundational examples of this include Kolm (1969), Atkinson (1970), Shorrocks (1983) and Kakwani (1984) for (atemporal) inequality and welfare dominance, explicitly assuming identical population sizes or implicitly relying on Dalton’s population principle to normalize population sizes to a common value.

The second ‘ideal population size’ view has argued that the ideal size is probably rather large. An argument in favor of this view is Bentham’s and Sidgwick’s famous support of total (or classical) utilitarianism (see also de Sismondi 1819 and Godwin 1820), where maximization of the “greatest possible happiness for the greatest number” is proposed as the State’s objective function.<sup>6</sup> According to that second view,

“the point up to which (...) population ought to be encouraged to increase, is not that at which average happiness is the greatest possible (...) but that at which the *product* formed by multiplying the number of persons living into the amount of average happiness reaches its maximum”. (Sidgwick 1966, pp. 415-416, our emphasis)

Total (general) utilitarianism’s objective function is therefore given by

$$W^T = \sum_{i=1}^N g(y_i), \quad (2)$$

whose special case of  $g(y) \equiv y$  is simply the population’s total welfare.

It is well known that both of these views generate social evaluation difficulties. Evaluations using average utilitarianism are subject to “eugenism”: the death of

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<sup>5</sup>See Dalton (1920), p. 357: “inequality is unaffected if proportionate additions are made to the number of persons receiving incomes of any given amount”.

<sup>6</sup>An even stronger argument, based entirely on the quantity of lives, can be found in Bodin (1576 (1955)): “But one should never be afraid of having too many subjects or too many citizens, for the strength of the commonwealth consists in men.” (Book V, chapter II)

anyone with a level of social utility below the mean level will lead to an increase in social welfare. Similarly, the birth of anyone with a level of social utility below the average will reduce social welfare. At the limit, if that situation were feasible, an optimal society would be made only of those persons with the greatest utility, such as Carlos Slim or Bill Gates. The fact that such a society would contain few individuals does not in itself matter for average utilitarianism; it is representative welfare that matters, not total welfare.

Social evaluations based on total utilitarianism exhibit the opposite difficulty of (arguably) giving too much weight to population size and not enough to the quality of lives. The most famous formulation of this difficulty has been in the form of Parfit’s “repugnant conclusion”, which occurs when:

“For any possible population of at least ten billion people, all with a very high quality of life, there must be some much larger imaginable population whose existence, if other things are equal, would be better, even though its members have lives that are barely worth living.”  
(Parfit 1984, p.388).

The evident concern here is that total utilitarianism could too easily dismiss Malthus’ preoccupation and lead to societies with “a great multitude of inhabitants, poor and badly provided”, to quote Cantillon (1959 (1755)) again.

## 2.2 Critical-level utilitarianism

One procedure to address and potentially to avoid both of these difficulties is through a reformulation of average and total utilitarianism called “critical-level generalized utilitarianism” (denoted as CLGU, Blackorby and Donaldson 1984).<sup>7</sup> The CLGU social evaluation function can be defined as

$$W(\alpha) = \sum_{i=1}^N [g(y_i) - g(\alpha)] \quad (3)$$

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<sup>7</sup>Alternative reformulations are also possible, such as Ng (1986)’s “maximization of number-dampened total utility”, but these are often less transparent than critical-level utilitarianism.

where  $\alpha$  is called the *critical level*. A population is socially preferred to another if its  $W(\alpha)$  function is larger.  $\alpha$  is described by Broome (2007) as

“A particular value for what I call the *neutral level for existence*. This neutral level is defined as the level of well-being such that adding to the population a person who has that level of well-being is equally as good as not adding her”. (p. 115)

$\alpha$  is the level of welfare above which human life is *worth living* — from a social welfare perspective, *not* an individual one.<sup>8</sup> As for average and total utilitarianism, CLGU simplifies to critical-level utilitarianism when  $g$  is the identity function. Whenever  $g(\alpha) = 0$ , CLGU is equivalent to total utilitarianism. Positive values of  $g(\alpha)$  in (3) nevertheless avoid the repugnant conclusion, so long as  $\alpha$  is set to a “sufficiently large” value.

Drawing on Atkinson (2014), the CLGU transformation in (3) makes it possible to model a simple tradeoff of quantity/quality of lives. Let total income be fixed and be given by  $Y = \sum_{i=1}^N y_i$ . Assuming that income is equally distributed, each individual gets a value  $Y/N$  and we have

$$W(\alpha) = Nu \left( \frac{Y}{N} \right), \quad (4)$$

where  $u(Y/N) = g(Y/N) - g(\alpha)$ . In such a simple world, it is socially useful to increase population size even with constant total income whenever  $\partial W / \partial N > 0$ , that is, whenever

$$\frac{u(Y/N)}{Y/N} > u'(Y/N). \quad (5)$$

This occurs when the ratio of average utility to average income ( $\frac{u(Y/N)}{Y/N}$ ) exceeds the marginal utility of *per capita* income, ( $u'(Y/N)$ ), namely, when the social

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<sup>8</sup>When  $g(\alpha)$  exceeds  $g(y_i)$ , individual  $i$  reduces social welfare, but it does not mean that  $i$  would prefer not to live.

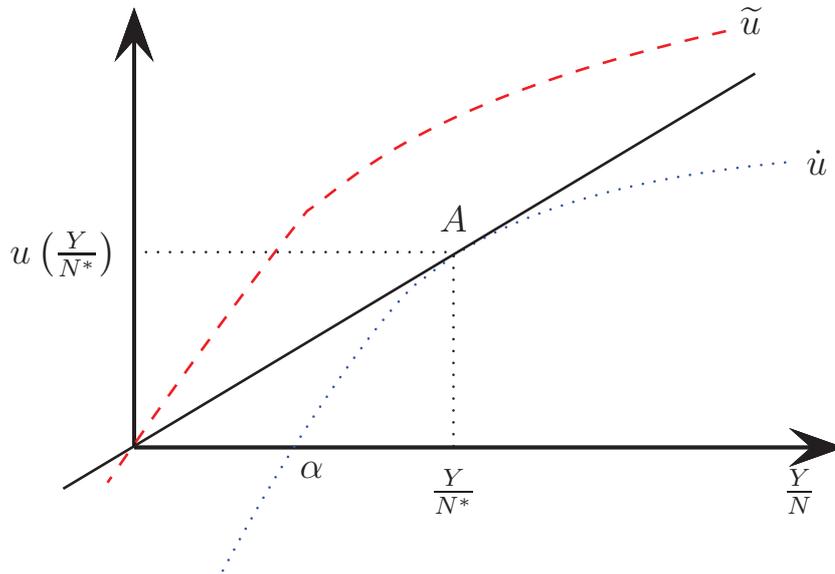
utility of adding one more person is higher than the social utility of preserving the level of welfare of the existing individuals.

Figure 8 illustrates this problem graphically. Drawing on the usual  $g$  concavity assumption, total CLGU utilitarianism with  $g(\alpha) = 0$  is given by dashed line  $\tilde{u}$  (with the normalization  $g(0) = 0$ ). The optimal population size would be infinite in such a setting since the tangent to  $\tilde{u}$  of a line drawn from the origin would be arbitrarily close to  $Y/N = 0$ . If, however, we set instead  $g(0) < 0$  and thus have  $u(\alpha) = 0$  only when  $\alpha > 0$ , then we obtain the dotted line  $\dot{u}$  in Figure 8, resulting in a finite optimal population size given by  $N^*$  below point  $A$ . Intuitively, the greater the value of  $\alpha$ , the larger the optimal value of  $Y/N$  and the lower the optimal population size, since a higher critical level penalizes population size. Conversely, the greater the curvature of  $u$  — or the faster the decrease in marginal utility — and the larger the total amount of resources  $Y$ , the higher the optimal population size. In fact, in this simple model,  $N^*$  is directly proportional to  $Y$ , such that the optimal level of *per capita* income is the same for all values of total resources available.

The use of CLGU as a generalization and as an alternative to average and total utilitarianism is certainly attractive. It does pose, however, important implementation difficulties, the greatest of which is probably the difficulty of assigning a consensual value to the critical level  $\alpha$ . An important additional difficulty includes agreeing on a precise form for  $g$ . The level of  $\alpha$  needs to be high enough to avoid the repugnant conclusion and low enough not to lead to excesses of eugenism.

Fortunately, we can make progress by considering different  $\alpha$  values and different shapes for  $g$  in order to test the robustness of comparisons and conclusions. One way around the difficulty of valuing  $\alpha$  is, for instance, to make social welfare assessments over intervals of critical levels — see for instance Trannoy and Weymark (2009) for such a suggestion. Robustness tests can also be made on the basis of (partial) social orderings over entire classes of CLGU functions. Cockburn, Duclos, and Zabsonré (2014) demonstrate how an extension of well-known population-size-invariant poverty and social welfare dominance procedures makes

Figure 8: Optimum population size and critical level utilitarianism



it possible to test robustness over such classes of functions that may differ from each other by functional form and by value of the critical level.

Such procedures can also lead to the estimation of “robust” lower and upper bounds for allowable intervals of  $\alpha$ . Consider two populations, a larger one  $y$  and a smaller one  $z$ . Figure 9 illustrates the sort of intervals for  $\alpha$  over which we could conceivably rank robustly  $y$  and  $z$ . A smaller value of  $\alpha$  will tend to make  $y$  preferable since it imposes a lower penalty on population size. This is why we might find that, over a lower interval  $[0, \alpha_+]$  of  $\alpha$ , the larger population  $y$  will dominate the smaller one  $z$ , as is shown in Figure 9. The converse is also possible; as illustrated in Figure 9, there may exist an interval  $[\alpha^-, \infty[$  of larger  $\alpha$  (penalizing population size) over which the smaller population  $z$  can be deemed preferable to the larger population  $y$ .

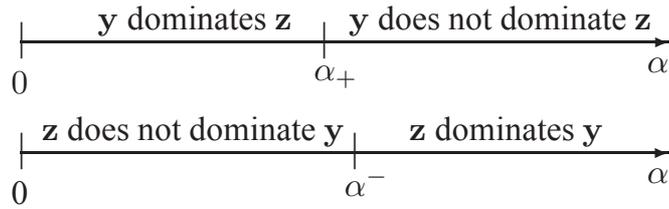
Table 1 (drawn from Cockburn, Duclos, and Zabsonré 2014) shows an example of the application of such a methodology. Although the world and three of its larger regions can be shown to exhibit greater CLGU social welfare in 2005 than

in 1990, this is true only if the values of the critical levels are chosen to be lower than some bounds  $\alpha_+$ , these bounds being considerably smaller in Sub-Saharan Africa (\$230) than for the entire world (\$1,288) and for the Eastern Asia and Pacific region (\$2,242).

Table 1: The (larger) global population in 2005 exhibits greater CLGU social welfare than in 1990 for all critical levels lower than  $\alpha_+$  (in USD PPP 2005)

Regions	Eastern Asia + Pacific	Latin America + Carribean	Sub-Saharan Africa	World
$\alpha_+$	2,242	827	230	1,288

Figure 9: Dominance and non-dominance of a larger  $y$  population over a smaller  $z$  population according to different ranges of critical levels  $\alpha$



### 3 Live or let live?

Performing intertemporal social comparisons in a welfarist framework amounts to ranking a two-dimensional matrix (individuals and time periods) of individual welfare defined across different social states. The social evaluation principles embedded in such a framework enable making trade-offs between quality, quantity and duration of lives. As mentioned in the introduction, an important size problem analogous to the one involved in comparing populations with different sizes also arises when it comes to comparing lives of different lengths. As with the traditional optimal population problem that trades off quality and quantity, the choice

of average *versus* total intertemporal welfare as a metric of individual welfare will bear importantly on the outcome of the social comparisons and on the evaluation of the impact of shocks and policies. The use of average intertemporal utility as a social welfare metric may promote the existence of lives that are too short; the use of total intertemporal welfare might promote the existence of lives that are too long.

It is, for instance, possible to think of an “intertemporally repugnant conclusion” analogous to Parfit’s repugnant conclusion:

“A social ordering leads to a ‘repugnant temporal conclusion’ if it can always judge any population of richer individuals to be intertemporally less desirable than a population of individuals with barely worth living lives so long as these individuals live sufficiently long lives.”

As in (4), to avoid this repugnant conclusion, we can model the choice of quality *versus* quantity using CLGU, this time through interpreting quantity as the number of years lived. For simplicity, let us focus on a single individual and let the intertemporal objective function of that individual be defined as in (4), with  $Y$  the total resources available over a lifetime (given, for instance, by lifetime earnings) and  $Y/N$  the average consumption over that same lifetime. With this framework, the optimal number of years lived is given by (5), *viz.*, at the value of  $N^*$  at which the ratio of average utility to average consumption ( $\frac{u(Y/N^*)}{Y/N^*}$ ) equals the marginal utility of average consumption. This optimal number of years is attained when the utility of adding one more year ( $u(Y/N^*)$ ) is just equal to the fall ( $Y u'(Y/N^*)/N^*$ ) in the utility of the years already lived. The greater the value of  $\alpha$ , the lower the optimal number of years; analogously, the greater the concavity of  $u$  and the higher the total consumption available, the larger the optimal duration of lives.

Figure 10 illustrates graphically the trade-offs involved in the two quantity/duration number problems. The first problem requires deciding whether to assign a value to life quantity *per se*. This can be pondered by comparing populations  $B$  and  $C$

in Figure 10. All individuals in Figure 10 are assumed to enjoy the same level of periodic life quality. Population  $B$  has only one individual living during each of the two periods; population  $C$  has two individuals in each period, each of these two individuals living only one period.

The question is then whether a social welfare analyst should consider  $B$  and  $C$  to be equivalent or not. Traditional social welfare analysis (as would average utilitarianism) would deem the two populations to be socially equivalent, by Dalton's principle of population. Total utilitarianism would clearly favor  $C$ . The CLGU ranking would depend on the value of the critical level; if the individual's welfare were to lie below  $\alpha$ ,  $B$  would be preferred to  $C$ , and *vice versa*.

The second problem requires deciding whether to assign a value to life longevity *per se* (or, alternatively, whether to assign a cost to *life fragmentation*). This can be seen by comparing populations  $A$  and  $B$  in Figure 10. Population  $A$  has only one individual, living two periods; population  $B$  has two individuals, each of them identical, except that individual 1 lives in period 1 only and individual 2 lives in period 2 only.

The question is then whether a social welfare analyst should consider  $A$  and  $B$  to be equivalent or not. One interpretation of Gandhi's famous command, "*Live simply so that others may simply live*", is that limits to longevity should be envisaged in order for others to enjoy a life too — namely, that longer lives should give room to more numerous shorter lives. Such an interpretation stands, however, in opposition to the principle of *favoring unfragmented lives* that characterizes the intertemporal CLGU function proposed in Blackorby, Bossert, and Donaldson (2005), where it is also stated that:

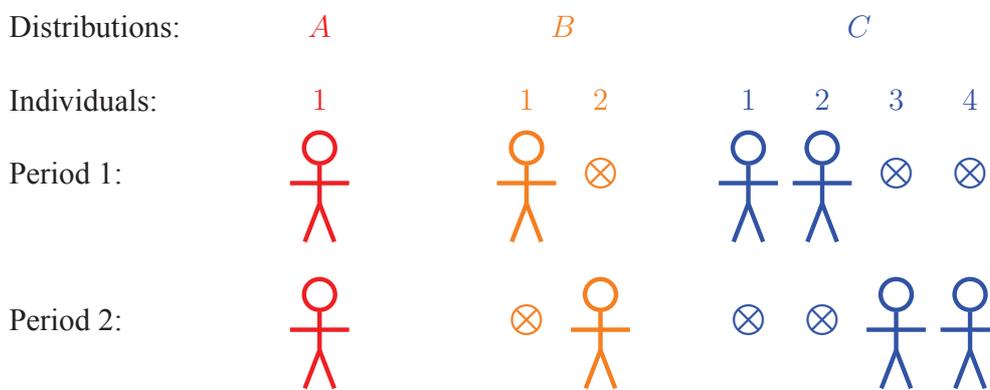
“Preventing someone's death is more important than bringing about new lives when the consequences for total utility are the same” (Blackorby, Bossert, and Donaldson 2005, p.153).

Blackorby, Bossert, and Donaldson (2005)'s formulation of intertemporal CLGU uses a fixed *lifetime* critical level that favors longer lives over combinations of shorter lives, thus also favoring population  $A$  over population  $B$ . Given this strict

preference for life unfragmentation, it could also be that  $A$  would be preferred to  $C$ : 2 periods of lives would then be considered preferable to 4 periods of lives of the same quality. Instead, using a *periodic* critical level would lead to social indifference between population  $A$  and population  $B$ .<sup>9</sup> Whether  $A$  and  $B$  are preferred to  $C$  would then depend on the value of that critical level.

Section 5 proposes a number of alternative methodologies to qualify and quantify these important trade-offs. Before turning to this, however, Section 4 describes the data procedures that are used to illustrate those methodologies.

Figure 10: Optimum population size and critical level utilitarianism



## 4 Data and empirical methodology

The empirical illustration (the term “illustration” is important here, given the obvious caveats on the quality of the data that we use, as will soon become evident) uses three types of data: *i*) annual distributions of income among world citizens from 1820 to 2010; *ii*) demographic and health data, such as age struc-

<sup>9</sup>Giving a value to the unfragmentation of lives (and thus preferring longevity over size) is also inconsistent with aversion to inequality over the total lifetime welfare of existing and potential lives, though not over the average lifetime welfare of those lives — see again Blackorby, Bossert, and Donaldson (2005).

tures of the global population and distributions of life expectancies by age in each period; and *iii*) transition matrices mapping incomes from one period to another.

Data on income distributions come from Bourguignon and Morrisson (2002), who provide historical data for the different regions of the world for the 1820-1992 period in the form of “grouped” income distributions by deciles.<sup>10</sup> We estimate the global income distributions by aggregating regional income distributions using regional population sizes. We extend the dataset to 2010 by using the annual growth rates of *per capita* income published by the World Bank (2013) and by assuming, for simplicity, that inequality levels have remained unchanged between 1992 and 2010. For the purpose of intertemporal analysis, we also make projections of income distributions from 2010 to 2080, assuming that inequality levels will remain unchanged and that the *per capita* income will increase annually at a rate of 1.85% (the annualized growth rate of global *per capita* GDP observed over the period 1950-2010). We generate samples of individual-level microdata from the decile-grouped income distributions by means of Shorrocks and Wan (2009)’s algorithm.<sup>11</sup> This leads to a vector of 1,000 individual observations of income for every year between 1820 and 2080.

Demographic data are drawn from the Population Division of the UN Department of Economic and Social Affairs (2013), which provides the age structures of the world population for different regions and countries between 1950 and 2010 and from Chamie (2001), who provides an age structure for 1900. We then assume a linear population growth for the different age groups to estimate age structures between 1910 and 1950. Life expectancies by age are estimated by combining the historical data on life expectancy at birth provided by Bourguignon and Morrisson (2002) and the World Life Tables obtained from World Health Organization (2012). Income transition matrices (the choice of which has little quantitative impact in our analysis) are set to Britain’s 1991/1992 decile transition matrix provided in Jarvis and Jenkins (1997).

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<sup>10</sup>More details on how these data are constructed and on the different countries and groups that are included can be found in Bourguignon and Morrisson (2002).

<sup>11</sup>This was performed using the Stata DASP package; see Araar and Duclos (2007) for details.

Thus, estimates of the global income/size/longevity distributions for 1910, 1960 and 2010 are constructed using the following procedures:

1. For each year between 1820 and 2080, we apply disaggregating procedures to the annual income data and population size data drawn from Bourguignon and Morrisson 2002 combined with annual growth rates of *per capita* income published by the World Bank (2013) for more recent periods;
2. Using global age structures and life expectancies at various ages, we assign an age and a date of death to each world citizen living in 1910, 1960 and 2010;
3. For each such individual, we generate prospective and retrospective income deciles using current income and decile transition matrices to move backward and forward in time;
4. Given an individual's decile for a given year, a yearly income for the corresponding year is drawn using the income data collected in step 1.

## **5 Quality, quantity and duration: by how much has global welfare changed in the last century?**

### **5.1 Combining quality, quantity and duration: an airport clue?**

Those of us who have traveled through airports in the last few years have often noticed the following HSBC ad (frequently found on those covered walkways that are used to embark on or disembark from airplanes):

“Two-thirds of the people who have ever reached 65 are alive today”.

Many of us travelers have probably also asked ourselves: is this statement saying something good or bad about humanity's global welfare? In suggesting that humanity's welfare has improved, the HSBC statement clearly inputs both life

quantity and life duration aspects. This of course makes sense only if social evaluation should give importance to both of these aspects of human lives, which is not usually the case in traditional welfare economics.

To understand better the potential social welfare effect of those aspects of human lives, we can compute an estimate of their respective importance over the last century. Assume first that total individual intertemporal welfare should enter into global social welfare, but not total population size. In such a circumstance, with no allowance for the quality of lives (to which we turn below), a natural social welfare measure would be the *proportion* of individuals that have reached 65 at some given time. This would say that an individual contributes to social welfare only if he has reached 65. According to this criterion, global social welfare would be deemed to have increased over the last century since the proportion of the population aged over 65 years has risen from 4.3% in 1910 to 7.7% in 2010. This would say that welfare in 2010 is  $7.7/4.3=1.8$  times that of 1910 — which corresponds to an annualized growth rate of 0.59% per year over the century.

Let us now suppose that both total individual intertemporal welfare and total population size should affect social welfare. In such a case, again with no allowance for the quality of lives, HSBC's measure would be appropriate so long as the contribution of an individual to social welfare equals one when the individual's longevity surpasses 65 years and zero otherwise. In our last-century empirical context, this is equivalent to moving from *proportions* of people having reached the age of 65 years towards the *absolute numbers* of such individuals. Doing the computations, we can rewrite HSBC's quote as follows:

“87% of the people who were 65 or older either in 1910 or in 2010 were alive in 2010 (as opposed to 13% in 1910).”<sup>12</sup>

According to this, therefore, welfare in 2010 would be 6.7 times (87/13) that of 1910. Therefore, incorporating population size into HSBC-style social evaluation

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<sup>12</sup>To be clear, these numbers differ from the HSBC's estimates given that we only consider those who have reached 65 either in 1910 or in 2010, while HSBC's statement refers to all those who have *ever* reached 65, no matter when they lived.

increases the value of 2010 relative to 1910 from 1.8 to 6.7.

It is important to note that the traditional quality input into welfare measurement is absent from the HSBC quote: should we not also be concerned about the welfare of individuals, not only about their number and longevity? In keeping with the spirit of that quote, we can still condition the contribution of an individual to social welfare on that individual having lived at least until 65. Four possible measures that incorporate quality then suggest themselves:

- H1:** First, not adjusting for duration and quantity, we can compare the total periodic incomes of those aged 65+ in 1910 and 2010 divided by the total population size in each of those years.
- H2:** Second, adjusting only for duration and not for quantity, we can compare, across 1910 and 2010, the total *lifetime* incomes of those aged 65+ divided by the total population size in each of those years.
- H3:** Third, adjusting only for quantity and not for duration, we can compare, across 1910 and 2010, the sum of the total periodic incomes of those aged 65+ in each of those given years.
- H4:** Fourth, adjusting both for duration and for quantity, we can compare, across 1910 and 2010, the total *lifetime* incomes of those 65 and above in 1910 and 2010.

Table 2 displays the levels of those four measures in 1910 and 2010 as well as their total and annualized percentage changes between those two years. Incorporating quality (the traditional income welfare input) naturally increases significantly the value of global social welfare for 2010 relative to 1910, compared to the non-income HSBC-style assessments presented previously. According to the **H1** measure, which does not adjust for life duration and life quantity, global welfare in 2010 would be 7.8 times that of 1910. This is considerably larger than the 400% increase in the global population's average income between those two years, as observed in Figure 2. Hence, focusing only on the quality of lives of

the elderly (as does the HSBC quote) raises significantly one's valuation of the 1910-2010 change in global social welfare.

The **H2** line adds duration into account. That now indicates that social welfare in 2010 is 6.1 times that in 1910. Since this is smaller than the above ratio of 7.8, life duration has had a proportionately smaller effect on social welfare than life quality, at least with an HSBC-style function. Intuitively, the increase in life duration for those aged 65+ over the last century has been proportionately smaller than their increase in annual income.

The evaluation of the change in global social welfare improves remarkably when life quantity, as measured by population size, is also taken into account. The **H3** line's incorporation of life quantity has the major effect of making 2010's social welfare 29.5 times that of 1910, compared to less than 10 above. The **H4** line's additional incorporation of life duration yields a similar result (23.1), though somewhat smaller — again because life duration has increased proportionately less than life quantity for the 65+. The annualized rates of change in global social welfare are above 3% in both cases. With an HSBC-style social evaluation function, incorporating quantity thus makes a substantial difference to the assessment of the evolution of global welfare over the last century.

Overall, therefore, HSBC-style social welfare — which considers only the contribution to social welfare of those with longer lives — is naturally sensitive to the evolution of life duration. If no account is taken of life quality, duration and quality, HSBC social welfare evaluation says that 2010 is 1.8 times better than 1910; when life quality is taken into account, that ratio rises to 7.8; when life duration is further incorporated, the ratio falls to 6.1; when all three aspects of life quality, duration and quantity are entered into an HSBC-style social welfare function, 2010 is deemed 23.1 times as good as 1910.

Table 2: HSBC-style global social welfare estimates from 1910 to 2010

	Levels		Changes	
	1910	2010	Ratio	Annualized % change
<b>H1</b>	93.96	730.29	7.8	2.07%
<b>H2</b>	5748.42	35006.54	6.1	1.82%
<b>H3</b>	1.71e+11	5.05e+12	29.5	3.44%
<b>H4</b>	1.05e+13	2.42e+14	23.1	3.19%

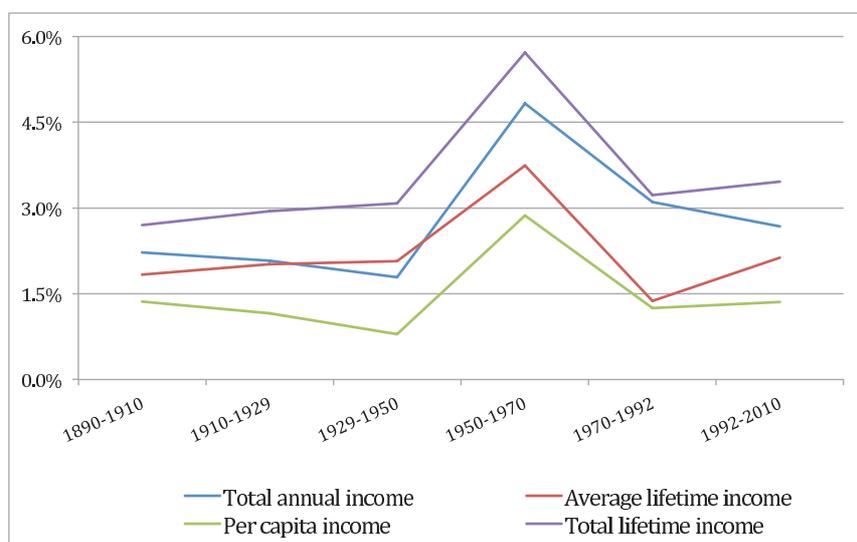
## 5.2 Measures of total income

In line with HSBC's airport quote, the above estimates measure global welfare by discretely separating those who are 65 or above and the others, and by assigning no social welfare value to those aged less than 65. Should we not, however, be concerned about the social welfare contribution of all individuals, not only about that of those individuals that have reached 65? One simple way to do this is to compute measures of average and total income for the *entire* population. We can add up incomes either across individuals or across time, and we can also add up incomes simultaneously both across individuals and across time. This leads to four alternative simple measures of average and total income:

1. A first measure is the usual *per capita* income measure;
2. A second measure is total annual income, given by *per capita* income times population size;
3. A third measure is the average lifetime income of those individuals living in some particular year;
4. A fourth measure is the total lifetime income of all those individuals living in some particular year; this is given by the average lifetime income of those living in that year times the population size of that year times the average life expectancy of those individuals living in that year.

The estimates of the annualized growth rates of those four income measures over six different periods between 1890 and 2010 are shown in Figure 11. Overall, global *per capita* income has increased over the last century at an average annualized growth rate of 1.5%. The highest growth (at an annualized rate of 2.9%) is observed during the 1950-1970 period, which corresponds roughly to the post World-War-II period of *the glorious thirty* (1945-1975). The growth rates of total annual income, average lifetime income and total lifetime income are naturally higher than that of *per capita* income given that they incorporate the effect of increases in population size and life duration. Again, the effect of the increase in life quantity dominates almost everywhere throughout the century that of the increase in life duration. The combined effects of life quality, quantity and duration lead to an annualized growth rate of total lifetime income of around 4% throughout the century — almost three times that of the growth in *per capita* income and surpassing the 3.19% growth of the HSBC-style social welfare function of Section 5.1.

Figure 11: Annualized growth rates of global income between 1890 and 2010



### 5.3 Intertemporal utilitarianism and intertemporal measures of representative income

We now wish to add two final inputs into the above social welfare measurement procedures. First, there is the potential role of a critical level in assigning a cost to life quantity and life duration; without this potential cost, social evaluation can generate the two repugnant (quantity and temporal/duration) conclusions discussed above. Second, should we not also assign a cost to inequality across human beings, as does most of the welfare economics literature with constant population sizes?

To do and to see this, let a set  $\mathcal{S}_t$  contain a number  $N_t$  of individuals living at time (year)  $t$ , and let  $T_i$  and  $b_i$  be respectively the life duration and the time of birth of an individual  $i$ . Also let  $y_{t,i}$  be the income of individual  $i$  at time  $t$ . Assuming no discounting and a critical level  $\alpha$  that is independent of time, the CLGU social evaluation of the population of individuals living at  $t$  can be expressed as

$$W_t(\alpha) = \sum_{i \in \mathcal{S}_t} \frac{\sum_{s=b_i}^{b_i+T_i-1} g(y_{s,i}) - T_i^\lambda g(\alpha)}{T_i^\tau N_t^\eta}, \quad (6)$$

where  $\eta$ ,  $\tau$  and  $\lambda$  are parameters whose values can help distinguish between various variants of utilitarianism:

- $\eta = 0, 1$  respectively differentiates between population-total utilitarianism and population-average utilitarianism;
- $\tau = 0, 1$  respectively distinguishes between duration-total utilitarianism and duration-average utilitarianism;
- $\lambda = 0, 1$  respectively differentiates between “lifetime” critical level (where the cost of a lifetime is given by  $g(\alpha)$ ) and “periodic” critical level (where each period of life has a cost  $g(\alpha)$ ).

With total utilitarianism (either  $\eta = 0$  or  $\tau = 0$ ), setting  $g(\alpha) = 0$  would lead either to a quantity or to a temporal repugnant conclusion. The combination of

$\tau = 0$  and  $\lambda = 0$  would also lead to a temporal repugnant conclusion; this is because (6) would then penalize life fragmentation since the critical level cost  $g(\alpha)$  would apply only to the number of lives and not to the number of periods lived. Finally, it would seem inconsistent to have both  $\eta = 1$  and  $\tau = 0$ ; this combination of parameter values would indeed value the duration but not the quantity of lives. Hence, a good set of parameter values would arguably be given by  $\eta = 0$ ,  $\tau = 0$ ,  $\lambda = 1$ , and  $g(\alpha) > 0$ ; that would value consistently both duration and quantity and would avoid both the quantity and the temporal repugnant conclusions.

The formulation in (6) assumes no discounting; time preference may, however, be of normative interest since it relates to the existence or otherwise of a future, to life expectancy uncertainty and to the valuation of anticipated wellbeing in the future, both at the individual and societal level (see for instance Anderson 2005 for a recent normative treatment and application.) Setting  $\eta = 0$ ,  $\tau = 0$  and  $\lambda = 1$ , a formulation of (6) with discounting is given by:

$$W_t(\alpha) = \sum_{i \in \mathcal{S}_t} \sum_{s=b_i}^{b_i+T_i-1} (1+r)^{(t-s)} (g(y_{s,i}) - g(\alpha)), \quad (7)$$

where  $r$  is the discount rate. The larger the value of  $r$ , the greater the relative weight put on earlier years of life of individual welfare. With infinite  $r$ , social welfare evaluation would depend exclusively on the first-year welfare of those having been born first. The illustration below assumes for simplicity that  $r = 0$ , in which case (7) equals (6) when  $\eta = 0$ ,  $\tau = 0$  and  $\lambda = 1$ .

From (6), we can also proceed towards computing an equally distributed equivalent (*EDE*, or representative) income (see Atkinson 1970) for populations with *variable* population sizes and durations. This is usefully done by setting a reference value for the number of lives and periods lived — something that is not needed in a context of constant quantity and duration. To see this, and for expositional simplicity, let us fix the reference number of lives to the number of lives lived in 1910 and suppose for now that duration is constant across populations. We may then ask what level of 1910's *EDE* would yield the same social welfare

as that observed in 2010; denote this as  $EDE_{2010|1910}$ .

Using a constant relative inequality aversion formulation for  $g$  and setting  $g(\alpha) = 0$ , we then have by definition (assuming  $\epsilon \neq 0$ ):

$$(1 - \epsilon)^{-1} \sum_{i \in \mathcal{S}_{1910}} EDE_{2010|1910}^{1-\epsilon} = (1 - \epsilon)^{-1} \sum_{i \in \mathcal{S}_{2010}} y_{2010,i}^{1-\epsilon}, \quad (8)$$

which yields

$$EDE_{2010|1910} = \left( \frac{\sum_i^{N_{2010}} y_{2010,i}^{1-\epsilon}}{N_{1910}} \right)^{1/(1-\epsilon)} \quad (9)$$

$$= \left( \frac{N_{2010}}{N_{1910}} \right)^{1/(1-\epsilon)} EDE_{2010|2010}, \quad (10)$$

and where  $EDE_{2010|2010}$  is the usual  $EDE$  measure for 2010. Variations in population sizes therefore have an impact on social welfare through the term  $\left( \frac{N_{2010}}{N_{1910}} \right)^{1/(1-\epsilon)}$ . For  $0 < \epsilon < 1$  (which is a common range of relative inequality aversion), a larger population size in 2010 raises  $EDE_{2010|1910}$ ; the impact is larger the larger the value of  $\epsilon$ . When  $\epsilon > 1$ , the impact of population size increases is reversed: *ceteris paribus*, a larger population size in 2010 *reduces*  $EDE_{2010|1910}$ , because  $y_{2010,i}^{1-\epsilon}$  is then decreasing with income. This counterintuitive result (which we avoid below by setting  $0 \leq \epsilon < 1$ ) is obtained because (8) is then *negative* and falling with population replications.

Now suppose an inequality-neutral proportional change of  $\gamma - 1$  in 2010's incomes. (10) then becomes:

$$EDE_{2010|1910} = \gamma \left( \frac{N_{2010}}{N_{1910}} \right)^{1/(1-\epsilon)} EDE_{2010|2010}. \quad (11)$$

(11) shows clearly the differential impact of growth in population size and growth in average income. This says that social welfare will rise more rapidly with population size when  $\epsilon$  takes larger values. The impact of growth is, however, independent of  $\epsilon$ . From (11), the elasticity of substitution of growth in average

income with respect to growth in population size is given by  $-(1 - \epsilon)^{-1}$ . Thus, the greater the value of  $\epsilon$ , the more effective (compared to income growth) is population growth at raising social welfare.

We may now generalize (8) to CLGU; this gives:

$$(1 - \epsilon)^{-1} \sum_{i \in \mathcal{S}_{1910}} \left( EDE_{2010|1910}^{1-\epsilon} - \alpha^{1-\epsilon} \right) = (1 - \epsilon)^{-1} \sum_{i \in \mathcal{S}_{2010}} \left( y_{2010,i}^{1-\epsilon} - \alpha^{1-\epsilon} \right), \quad (12)$$

which leads to

$$EDE_{2010|1910} = \quad (13)$$

$$\left( \frac{N_{2010}}{N_{1910}} \right)^{1/(1-\epsilon)} \left( \frac{\sum_{i \in \mathcal{S}_{2010}} y_{i,2010}^{1-\epsilon} - (N_{2010} - N_{1910})\alpha^{1-\epsilon}}{N_{1910}} \right)^{1/(1-\epsilon)}. \quad (14)$$

The term within the first right-hand-side parentheses corresponds to the previous social welfare benefit of a population increase. The  $(N_{2010} - N_{1910})\alpha^{1-\epsilon}$  term within the second set of parentheses in (13) is the cost of an increase in population size that is newly introduced by the critical  $\alpha$ . The larger that  $\alpha$ , the lower is  $EDE_{2010|1910}$ . This makes it transparent that population size increases have both a social welfare cost and a social welfare benefit.

An  $EDE$ 's intertemporal welfare formulation is analogous to (12) and (13), summing over both the number of individuals and over the life duration of each individual, and replacing  $N_{2010}$  by the total number of years lived by all those individuals that were living in 2010 (and analogously for  $N_{1910}$ ).  $EDE_{2010|1910}$  is then implicitly given by

$$\sum_{i \in \mathcal{S}_{1910}} T_i \left( EDE_{2010|1910}^{1-\epsilon} - \alpha^{1-\epsilon} \right) = \sum_{i \in \mathcal{S}_{2010}} \left( \sum_{s=b_i}^{b_i+T_i-1} g(y_{s,i}) - T_i g(\alpha) \right). \quad (15)$$

The statistics in Tables 3 to 6 show the levels (in \$) and the rates of growth of those incomes that the global population of individuals living in 1910 would

have needed to enjoy in order to yield the same level of social welfare as that observed in 1960 and 2010; these are therefore  $EDE_{\cdot|1910}$  incomes. The three ‘atemporal’ columns assign these  $EDE$  incomes to years lived in 1910; the three ‘intertemporal’ columns assign these  $EDE$  incomes to every year of the lives of those living in 1910. The results of Tables 3 and 4 use an utilitarian formulation, namely,  $g(x) = x$  or  $\epsilon = 0$ ; those of Tables 5 and 6 use a generalized utilitarian function with  $g(x) = \sqrt{x}$ , or  $\epsilon = 0.5$ . For the critical level formulations, a periodic critical value  $\alpha = \$365$  (equivalent to using a \$1 a day international poverty line) is set for P-CLGU and a lifetime critical-level is used for L-CLGU, that lifetime critical-level being equal to the periodic critical-level times 69 years (the 2010 value of global life expectancy at birth).

All of the social evaluation functions suggest a continuous improvement of global social welfare over 1910-2010, but at considerably different rates. Let us first consider Tables 3 and 4. Take the usual metric of average utilitarianism for a start; the  $EDE$  incomes increase from \$1918 in 1910 to \$9151 a century later. Taking instead average intertemporal utilitarianism (lifetime income averaged across individuals and across periods),  $EDE$ ’s increase from \$1913 in 1910 to \$10168 in 2010. Table 4 shows that the respective annualized growth rates of these measures over the entire century are thus 1.57% (for *per capita* income) and 1.68% (and slightly larger between 1960 and 2010).

Moving in Tables 3 and 4 to total income across individuals (atemporal CU) leads to an annualized growth rate of 2.94% — a significant rise compared to the earlier 1.57% — which is further augmented to 3.88% if we consider the growth in total income across the lifetime and across individuals (intertemporal CU). Using a critical level for penalizing lives (setting  $\alpha = \$1$  a day) at each period (P-CLGU) or for the lifetime (L-CLGU) decreases only marginally that annualized growth rate to 3.85%.

The generalized utilitarian results of Tables 5 and 6 penalize income inequality. The  $EDE$ ’s of Table 5 are therefore lower than those of Table 3. The growth rates of AGU in Table 6 are sometimes lower or greater than those of AU in Ta-

ble 4, depending on whether inequality has fallen or increased (they are larger if inequality has fallen). The growth rates of the other *EDE*'s are, however, considerably larger because a 1% increase in the number (quantity and duration) of lives is worth a 2% increase in average income with  $\epsilon = 0.5$  (the elasticity of substitution has doubled). This means that *EDE*'s have grown at an annual rate of 6.19% when we consider both life duration and quantity; those living in 1910 would have needed to have a level of income of \$543,879 (as opposed to the actual \$1913, see Table 3) to generate a level of social welfare equal to that of 2010. The ratio of social welfare in 2010 to social welfare in 1910 is then an astonishing  $543,879/1334=407$ . With a critical level of \$1 per day, the *EDE* growth rate is slightly lower at 5.73% and 5.30%, but still far larger than the rates of 2.94% and 3.88% obtained with  $\epsilon = 0$ .

Table 3: Levels of global *EDE*,  $g(x) = x$  and  $\alpha = \$365$

	Atemporal			Intertemporal		
	1910	1960	2010	1910	1960	2010
<b>AU*</b>	1918	3596	9151	1913	4259	10168
<b>CU</b>	1918	5965	34693	1913	13379	86180
<b>P-CLU</b>	1918	5724	33674	1913	12574	83419
<b>L-CLU</b>				1913	12873	84037

Table 4: Annualized changes in global *EDE*,  $g(x) = x$  and  $\alpha = \$365$

	Atemporal			Intertemporal		
	1910- 1960	1960- 2010	1910- 2010	1910- 1960	1960- 2010	1910- 2010
<b>AU</b>	1.27%	1.89%	1.57%	1.61%	1.76%	1.68%
<b>CU</b>	2.30%	3.58%	2.94%	3.97%	3.80%	3.88%
<b>P-CLU</b>	2.21%	3.61%	2.91%	3.84%	3.86%	3.85%
<b>L-CLU</b>				3.89%	3.82%	3.85%

Table 5: Levels of global  $EDE$ ,  $g(x) = \sqrt{x}$  and  $\alpha = \$365$

	Atemporal			Intertemporal		
	1910	1960	2010	1910	1960	2010
<b>AGU*</b>	1330	2833	6832	1334	3198	7494
<b>CGU</b>	1330	7793	98187	1334	32130	543879
<b>P-CLGU</b>	1330	5730	67613	1334	18805	351618
<b>L-CLGU</b>				1334	16436	233363

Table 6: Annualized changes in global  $EDE$ ,  $g(x) = \sqrt{x}$  and  $\alpha = \$365$

	Atemporal			Intertemporal		
	1910- 1960	1960- 2010	1910- 2010	1910- 1960	1960- 2010	1910- 2010
<b>AGU</b>	1.52%	1.78%	1.65%	1.76%	1.72%	1.74%
<b>CGU</b>	3.60%	5.20%	4.40%	6.57%	5.82%	6.19%
<b>P-CLGU</b>	2.96%	5.06%	4.01%	5.43%	6.03%	5.73%
<b>L-CLGU</b>				5.15%	5.45%	5.30%

## 6 Conclusion

Development (sometimes steered by public policies) has led to important demographic and economic changes worldwide, most importantly an increase in population size, a growth in longevity and a general growth in living standards. This has had significant impacts on what we term the “quantity”, the “duration” and the “quality” of lives. It seems reasonable to suppose that all such three dimensions of human lives may be inputs into the evaluation of global social welfare. This paper reviews briefly some of the normative issues at stake in incorporating these three dimensions into social evaluations, it proposes and discusses relatively simple methods to quantify their influence, and it assesses the empirical importance of those three dimensions in the evolution of global social welfare over the last century.

The methods that are proposed and implemented are utilitarian in nature, either in the usual average form or in the total (or classical) formulation. Generalized

utilitarianism is also considered in order to allow for possibly decreasing marginal income utilities. A critical level  $\alpha$  — understood as the level of well-being for which adding a new period of life has no impact on social welfare — is also introduced to avoid some of the difficulties to using average and total utilitarianisms, namely, those introduced by quantity and duration “repugnant conclusions”. The use of average utility as a social welfare metric may indeed promote the existence of lives that are too few or too short; the use of total welfare might support the existence of lives that are too many or too long. A critical-level generalized utilitarian (CLGU) social welfare function is then given by the sum of transformed individual utilities net of the same transformation of  $\alpha$ .

In this context, the paper proposes and applies three alternative/complementary measurement approaches to incorporate both quantity and duration of lives into social evaluations. The first approach generates an “HSBC-style” social welfare function that focuses only on the social welfare contribution of individuals with longer lives (65+). The second approach uses measures of average and total income in a context in which both the quantity and the duration of all lives may matter. (This leads to four simple measures: *per capita* income, total annual income, lifetime income averaged across individuals, and lifetime income summed across individuals.) The third approach constructs measures of equally-distributed-equivalent income that takes into account inequality in individual welfare and the possible influence of critical levels and whose elasticity of substitution between life quality and life quantity will exceed one if marginal income utilities are decreasing.

All of these tools show a continuous improvement in global social welfare over 1910-2010, but at considerably different rates and with different implications. The evaluation of the change in global social welfare is particularly sensitive to the incorporation of life quantity. The effect of life duration is smaller, mainly because life duration has increased proportionately less than life quantity over the last century.

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