

Stochastic Dominance and Equity

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Main message

- Second-degree stochastic dominance is associated to measurement of inequality since Atkinson 1970
 - ⇒ He demonstrated the link between Lorenz dominance and SSD
- First degree is not viewed as an instrument to detect inequality
- Whereas First-degree SD is already a powerful tool in equity measurement

What equity is if different from equality?

- Equity is likely to be multidimensional rather than unidimensional
⇒ Is there a trade-off between the different dimensions?
- Equity refers both to what you can do and what you actually do
⇒ Both ex-ante and ex-post dimensions.

Capabilities vs Equality-of-Opportunity

- Capability-set literature refers to this ex-ante perspective
- Equality-of-opportunity moral philosophy also refer to this ex ante perspective
- It also refers to an ex-post perspective when freedom has been exercised

- Suppose that opportunity sets have been equalized.
- The capability approach will say that it is enough
- The EOp will say that it is not enough

- For instance, full equality of outcome is not precluded by capability approach
- Whereas, in general, it is by the EOp approach

Equality of opportunity

- Some inequalities may seem morally or socially acceptable
⇒ need to analyze the process that generates inequality

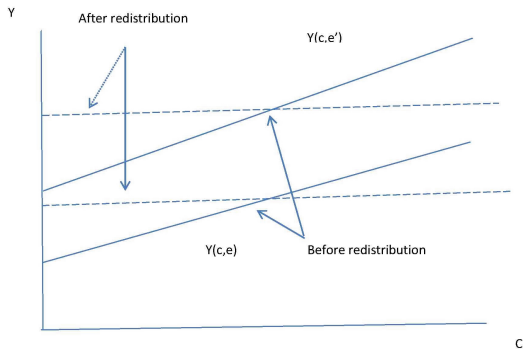
Philosophical background

- Responsibility-sensitive egalitarianism: Dworkin, Arneson, Cohen
- Cohen (1989) : *“eliminate involuntary disadvantage, i.e. disadvantage for which the sufferer cannot be held responsible, since it does not appropriately reflect choices that he has made”*
- Legitimate/illegitimate inequalities: responsibility cut
- Responsibility-sensitive egalitarianism ⇒ Distinction between **effort** and **circumstances**

Two principles

- The *compensation* principle: The effect of circumstances on outcome should be neutralized for a given effort: *ex post perspective*
- The *reward* principle: The effect of effort on outcome should be respected (fully or partially) for a given circumstance: *ex ante perspective*
- EOP is satisfied when the two principles are satisfied (sometimes, only the first principle, Vandenbroucke (2001), Hild and Voorheve (2004))
- A conflict between both principles in full generality (Fleurbaey (1994), Bossert (1995) and Fleurbaey and Peragine (2013))

illustration



Some theoretical undetermination

- How should circumstances and effort be defined ?
- For instance, Roemer: circumstances are defined “*by society*”; effort is the residual determinant
- No self-content theory of reward.

- Fleurbaey (2008) closes up with giving priority to a principle over the other
- The dominated principle should be respected for at least a reference level of effort (for the compensation principle) or a reference level of circumstance (for the natural reward principle)
- Roemer closes up with some inequality aversion among people exerting the same level of effort.
- More on the difference between both views in Roemer and Trannoy (JEL)

Non Observability

- Much more difficult to implement than equality of outcomes
- Private information/Public information: effort is private knowledge
- Difficult to describe all circumstances
- The implementation of EOP is plagued with problems of identification
- Roemer (1993, 1998): a first attempt of a *pragmatic* view of EOP
- Pragmatic: taking account for non-observability of some factors

⇒ **Issue:** *How can we test EOP when some circumstances and effort are not publicly observable?*

Contribution

- Building upon a joint work with *Arnaud Lefranc* and *Nicolas Pistolesi*
- "*Equality of opportunity and Luck: definitions and testable conditions with an application to income in France*" *JPubE* 2009
- Stochastic definition of equality of opportunity, introducing random factors explicitly/ Fleurbaey (2008) Deterministic model
- Stochastic dominance useful for detecting violations of the compensation principle
- Van de Gaer (PhD thesis 1993, the first appeal to SD to define EOP)
⇒ A step further when some factors are unobservable
- Previous contribution: Only the compensation principle
- Extension to the reward principle

Outline

- A Stochastic definition of EOP (JpubE paper + new): an upcoming WP with

Arnaud Lefranc

- Three views about the correlation about effort and circumstances, Barry, Roemer, Swift

(Coming from the joint work with Florence Jusot and Sandy Tubeuf Effort or Circumstances: Does the correlation matters for inequality of opportunity in health? Health Economics 2013)

- Non-observability of effort \Rightarrow Empirical identification of *Compensation* principle with Roemer's view (JPubE paper)
- Non-observability of circumstances \Rightarrow Empirical identification of *Reward* principle with Swift's view (New)

In the background

- Perfect Information: Full observation of the process generating disposable income
- Post-intervention income with the impact of all state interventions - y
- Three boxes of income determinants
- Background describing the origin, circumstances - c
- Action (in a game theoretic framework, all moves of the players) - e supposed to be a scalar
- Random factor = luck (all events that bring gain or loss of income and occurs with some frequency) - \tilde{l} distributed according to some CDF $F_{c,e}(\cdot)$
- The set up: $y = Y(c, e, l)$ for each realization of the random variable
- The mechanisms featured by the function y are quite complex and involve time dimension.

Compensation Principle: Definition

- Ex ante, for any c, e , \tilde{y} is a random variable distributed according to the conditional CdF $F(y|c, e) = F_{c,e}(Y^{-1}(c, e, l))$
- Luck is really what moves individual along $F(y|c, e)$
- Since inequalities related to effort are morally acceptable, the requirement of **equality of opportunity should only apply among individuals with similar effort**
- *“Those who are at the same level of talent and ability, and have the same willingness to use them, should have the same prospects of success regardless of their initial place in the social system”* (Rawls, 1971)

Definition EOP (Compensation)

Compensation Principle is satisfied iff:

$$\forall(c, c') \forall e, \quad F(.|c, e) = F(.|c', e) = H(e)$$

Interpretation: luck is even-handed w.r.t circumstances for a given level of effort

Why is it important to distinguish random from deterministic factors?

- In the deterministic EOp theory, two kind of operations: compensation or laissez-faire
- Random factors: one more operation
- Neutralization of the correlation between \tilde{l} and the distribution of c
- Random factors should be independently distributed = "*even-handed*"

Flexible Principle

- **1-** Suppose that $F(\cdot|c, e)$ degenerates into a mass point which only depends on effort
- Then the compensation principle translates into the requirement that for a given effort, the income must be deterministic; the luck effect is erased
- **2-** Suppose that outcome y is determined by two sets of factors: circumstances c and luck l . No more effort

Definition EOP (Compensation)

Compensation principle is satisfied iff : $\forall(c, c'), F(y|c) = F(y|c') = F(y)$.

Interpretation : luck is even-handed w.r.t circumstances

- **3-** The support of the distribution $H(e)$ is a degree of freedom. The definition is still compatible with quite different conceptions of EOP and it can be as close as we want from equality of outcomes
- **4-** As the set of circumstances becomes larger, the role for residual luck is reducing and the support of $H(e)$ becomes smaller.

Reward Principle: Definition

- First suppose that the compensation principle is not satisfied

Definition EOP (Reward without compensation)

Assuming that the Compensation Principle does not hold, the principle of natural reward is satisfied iff: $\forall (e, e'), \forall c, e \geq e' \Leftrightarrow F(\cdot|c, e) \succeq_{FSD} \text{ or } \succeq_{SSD} F(\cdot|c, e')$.

- In this version, the two principles are stated independently
- Second suppose that the compensation principle is satisfied

Definition EOP (Reward with compensation)

Assuming that the Compensation Principle holds, the principle of natural reward is satisfied iff: $\forall (e, e'), e \geq e' \Leftrightarrow H(e) \succeq_{FSD} \text{ or } \succeq_{SSD} H(e')$.

- If this principle holds, then full EOP holds

Flexible principle

- Compatible with reward principles in the literature:
- Natural reward principle: if individuals were fully responsible for their characteristics, no redistribution should take place
- A weakening of natural reward principle: if individuals were fully responsible for their characteristics, full equality is not allowed
- Utilitarian reward principle: redistribution that may take place among the same type is that induced by the maximization of a utilitarian social welfare function
- Utilitarianism is known to correspond to zero aversion to inequality. Inequality shouldn't be erased within a type.

Defining inequality of opportunity

- Violation of one or both principles
- Violation of compensation: There exists at least two circumstances c, c' such that for any level of effort e , $F(.|c, e) \succeq_{FSD} \text{ or } \succeq_{SSD} F(.|c', e)$
- Violation of reward principle: There exists at least two effort levels, e, e' , $e > e'$ such that for at least c , $F(.|c, e) = F(.|c, e')$

- Duality

	<i>Compensation</i>	<i>reward</i>
<i>Equality</i>	$CDF =$	$F(S)SD$
<i>Inequality</i>	$F(S)SD$	$CDF =$

Correlation between effort and circumstances

- These two determinants cannot be assumed to be independent
- **1/** What kind of effort?: Debate between Roemer and Barry about the case of an Asian student
- **2/** What kind of circumstances? Intergenerational inconsistency of EOP

Effort: Roemer against Brian Barry

- *“Asian children generally work hard in school and thereby do well because parents press them to do so. The familial pressure is clearly an aspect of their environment outside their control”*
- Roemer said that we should respect the individual effort *“if we could somehow disembody individuals from their circumstances”*
- Effort should be purged of any contamination coming from circumstances
- Barry argues that *“the fact that their generally high levels of effort were due to familial pressure does not make their having expended high levels of effort less admirable and less deserving of reward than it would have been absent such pressure”*
- True effort should be respected (effort in the incentives literature)
- Do we hold sons of smokers less responsible to smoke than sons of non-smokers?

Four channels of transmission

- The second issue concerns the impossibility to respect the principles of compensation and natural reward for all generations
- Roemer (2004) considered that parents affect the opportunities of their children through four channels:
 - Provision of social connections and tangible resources,
 - Formation of beliefs and skills in children through family culture and investment
 - Genetic transmission of ability
 - Formation of preferences and aspirations in children = *parental effort*
- The first three should be deemed circumstances. The status of the fourth category is more debatable. It is both an effort for the parents and a circumstance for the children

Parental effort: EOP against of with the family?

- If we give priority to the young generation in the application of EOp, we should consider that the whole initial background represents circumstances
- If we give priority to the past generation in the application of EOp, we should consider that parental effort must be respected whatever its consequences to the next generation
- For Adam Swift (2002), the principle of natural reward for the past generation is viewed as more important than the principle of compensation for the young generation
- “To the extent that the reproduction of inequality across generations occurs through the transmission of cultural traits, it does so substantially (though not exclusively) through intimate familial interactions that we have reason to value and protect. Preventing those interactions would violate the autonomy of the family in a way that stopping parents doing spending their money on, or bequeathing money to their kids would not”

Three views

- "Barry's view: Circumstances are past variables, efforts are current variables which somehow depend on the free will of individuals
- Roemer's view: Circumstances are past variables, effort must be cleaned from any contamination coming from circumstances
- Swift's view (An extremist Swift's view!): Circumstances are past variables which must be cleaned from any correlation with effort of children, the part of circumstances that is correlated with children represents parental effort to educate children.

The identification problem

- Assessing whether EOP-Compensation or EOP-reward are satisfied requires, in general, that both circumstances and effort be observable
- This may not be the case in most data sets. Effort may not be observable. Circumstances may only be partially observable
- We don't know the process of generating incomes
- **Case 1:** Ex post, we only have data on income and circumstances not on effort or luck
- **Case 2:** Ex post, we only have data on income and effort not on circumstances or luck (no records)

The identification problem (case 1)

- We know circumstances but not effort
- If ex post we have enough data and if we are in a stationary state, thanks to a strong law of large numbers, the ex post conditional distribution of income corresponds to the ex ante conditional distribution:

$$F_{ep}(y|c) = F(y|c) = \int_e F(y|c, e) dG(e|c)$$

- We can only analyze $F(y|c)$

Definition

The "type approach" (terminology Vito Peragine) means looking at the conditional distribution of outcome and requiring full equality of these distributions: $\forall(c, c'), F(|c) = F(|c')$.

Roemerian effort

- Can EOP (compensation) be assessed in this case?
 - It depends on the property of the conditional distribution of effort wrt circumstances $G(e|c)$
 - in general: **No**
 - if e is distributed independently of c or if we retain the definition of relative effort of Roemer: **Yes**

Definition

Relative effort is defined by $e_r = G(e | c)$

- The distribution of *relative* effort is the same across types
- Roemer: *"If we could somehow disembodify individuals from their circumstances, then the distribution of the propensity to exert effort would be the same in every type"*

Implementation of compensation: a necessary condition

Theorem

If either $\forall c, G(e|c) = G(e)$ or if we substitute e_r for e in the definition of compensation, then compensation \implies Type approach

- Why?
- EOP-Compensation requires that outcome prospects, given effort, are similar for all types
- If this is true and if effort is independent of type, by aggregation over effort, the distribution of outcome should be the same for all types that is the type approach

Test of inequality of opportunity

- By the same token, we obtain also a necessary condition for the existence of IO_p

Theorem

If either $\forall c, G(e|c) = G(e)$ or if we substitute e_r for e in the definition of the violation of the compensation, then if there exists at least two circumstances c, c' such that for any level of effort e , $F(.|c, e) \succeq_{FSD}$ (or \succeq_{SSD}) $F(.|c', e)$ implies that for those c, c' , $F(.|c) \succeq_{FSD}$ (or \succeq_{SSD}) $F(.|c')$

Partial observability of circumstances

- $c = \{c_1, c_2\}$ and we can only observe c_1 . c_2 the invisible circumstance
- Can we assess compensation?
 - Under the independence of effort or Roemerian view, a necessary condition is the type approach on observable circumstances
 - This does not require that c_1 and c_2 be independently distributed

The identification problem (case 2)

- We know effort but not circumstances (no record case)
- If ex post we have enough data and if we are in a stationary state, thanks to a strong law of large numbers, the ex post conditional distribution of income corresponds to the ex ante conditional distribution:

$$F_{ep}(y|e) = F(y|e) = \int_c F(y|c, e) dG(c|e)$$

- We can only analyze : $F(y|e)$

Definition

The "tranche approach" (terminology Vito Peragine) means looking at the conditional distribution of outcome wrt to effort and checking that $\forall (e, e')$, $e \geq e' \Leftrightarrow F(\cdot|e) \succeq_{FSD} F(\cdot|e')$

Swiftian circumstances

- Can EOP be assessed in this case?
 - It depends on the property of the conditional distribution of circumstances wrt effort $G(c|e)$
 - in general: **No**
 - if c is distributed independently of e or if we retain the definition of relative circumstances of "Swift": **Yes**

Definition

Relative circumstance is defined by $c_r = G(c | e)$

- The distribution of *relative* circumstance is the same across tranches

Implementation of reward: a necessary condition

Theorem

If either $\forall e, G(c|e) = G(c)$ or if we substitute c_r for c in the definition of reward then reward \implies Tranche approach

Proof.

For a given y : $F(y|e) = \int_{c_r} F(y|c_r, e) dG(c_r|e) = \int_{c_r} F(y|c_r, e) dJ(c_r)$. Similarly $F(y|e') = \int_{c_r} F(y|c_r, e') dJ(c_r)$.

$F(y|e) - F(y|e') = \int_{c_r} (F(y|c_r, e) - F(y|c_r, e')) dJ(c_r)$.

By assumption $F(y|c_r, e) - F(y|c_r, e') < 0$ for all c_r .

We deduce $F(y|e) - F(y|e') < 0$. Q.E.D □

- The analogue for a violation of the reward principle.

Conclusion

- Switch from a deterministic theory of EOP to a stochastic theory of EOP
- Provide testable conditions to assess EOP, without observing individual effort or the set of circumstances
- We can test (necessary condition) for the compensation principle with the type approach but Roemerian effort is needed
- We can test (necessary condition) for the reward principle with the tranche approach but Swiftian circumstance is needed
- A common feature (more than a wild guess) is that most empirical results on EOP are obtained using FSD (a meta analysis to be done)
- What a social advantage means if it does not translate into a gain in terms of FSD?
- If there is a crossing, redefine the types